

EE 320 Introductory Mathematical Economics

Homework 4

Due 4 April 2013

There are five questions in total, each worth equally.

1. Given the following supply and demand functions:

$$Q_D = 125 - 3P$$

$$Q_S = 80 + 2P$$

- a) Find the equilibrium price and equilibrium quantity in this market.
- b) Suppose that now the government collect a specific tax t baht per unit of this good from the producer. If the government's objective is to maximize the total tax revenue, at what tax rate should the government set? Assume that the total cost function is given by: $TC = 15 + 6Q$.

2. Suppose that there are two firms selling an identical product in the market. The total cost of each firm is exactly the same and is equal to $TC_i = 25Q_i$, $i = 1, 2$. Both firms face the same market demand function

$$P = 56 - 4Q, \quad \text{where } Q = Q_1 + Q_2.$$

- a) Suppose that each firm maximizes its own profit, and take the other's quantity as given. Determine the profit-maximizing quantity for each firm.
- b) Now, suppose the two firms collude in order to maximize their combined profit (as one monopolist would). Determine the profit-maximizing quantities of the two firms.

3. The demand for a monopolist's two products are determined by the equations

$$p_1 = 30 - q_1 \quad \text{and} \quad p_2 = 28 - 2q_2$$

where p_1 and p_2 are prices per unit of the two goods, and q_1 and q_2 are the corresponding quantities. The costs of producing and selling q_1 units of the first good and q_2 units of the second good are

$$C(q_1, q_2) = 2q_1^2 + 4q_1q_2 + q_2^2.$$

- a) Find the monopolist's profit $\pi(q_1, q_2)$ from producing and selling q_1 units of the first good and q_2 units of the second good.
- b) Find the values q_1 and q_2 that maximize $\pi(q_1, q_2)$. Show the sufficient conditions for profit maximization.

4. Suppose that a monopolistic firm sells a single product in three separate markets (say, three different countries), and therefore faces three different demand functions:

$$P_1 = 30 - 1.5Q_1$$

$$P_2 = 72 - 3Q_2$$

$$P_3 = 40 - 2Q_3$$

The total cost function is given by: $TC = 60 + 20Q$, where $Q = Q_1 + Q_2 + Q_3$.

- a) Show the first-order necessary conditions for the firm's profit maximization.
- b) Find the profit-maximizing output levels of Q_1 , Q_2 , and Q_3 , and determine the maximum profit. Verify that the second-order sufficient condition is met.

5. Let the production function of a competitive firm be:

$$Q = 2K^{1/4}L^{1/2}$$

where Q = output, K = capital input, and L = labor input. Suppose that the market price of this output is P per unit, and the input prices per unit for capital and labor are r and w ($r, w > 0$), respectively.

- a) Determine the values of K^* and L^* (as functions of the output and input prices) that maximize the firm's profit.
- b) Find the comparative statics derivatives $\frac{\partial K^*}{\partial r}$ and $\frac{\partial L^*}{\partial r}$, evaluate the signs, and interpret their economic meanings.