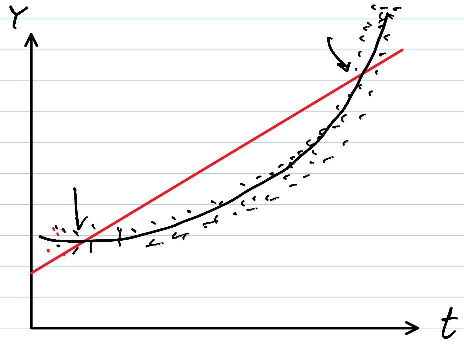
CHOOSING FUNCTIONAL FORMS

- THE WORLD IS NOT FLAT: THINGS WOULD BE RELATIVELY EASY IF WE COULD ALWAYS PRESUME THAT ALL POPULATION RELATIONSHIPS WERE LINEAR.

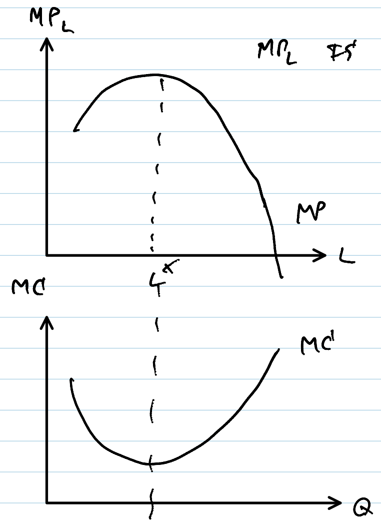
AT THE HEART OF ECONOMICS, LINEARITY IS OFTEN A POOR APPROXIMATION OF THE TRUTH.

EX:

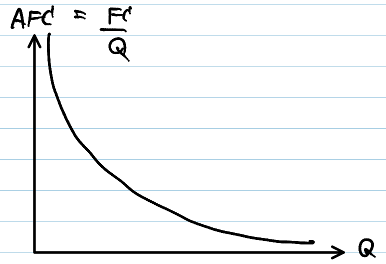


THE ESTIMATED (STRAIGHT LINE) REGRESSION LINE ONLY CORRECTLY REPRESENTS THE POPULATION LINE AT TWO POINTS!

EX



MP_L IS DIMINISHING WHEN $L > L^*$.



FUNCTIONAL FORMS OF REGRESSION MODELS

① THE LOG-LINEAR MODEL OR LOG-LOG MODEL OR DOUBLE-LOG MODEL

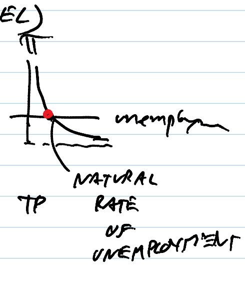
: $$\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i .$$

② SEMILOG MODELS

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i . \quad \text{(LOG-LIN MODEL)}$$

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i . \quad \text{(LIN-LOG MODEL)}$$

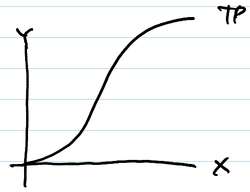
③ RECIPROCAL MODELS → EX: AFC, PHILLIPS CURVE



$$Y_i = \beta_1 + \beta_2 \left[\frac{1}{X_i}\right] + u_i .$$

④ THE LOGARITHMIC RECIPROCAL MODEL

$$\ln Y_i = \beta_1 - \beta_2 \left[\frac{1}{X_i}\right] + u_i$$



EX: SHORT RUN PRODUCTION FUNCTION

I. DOUBLE-LOG MODEL OR LOG-LINEAR MODEL.

CONSIDER $Q = a \cdot K^{\beta_1} L^{\beta_2}$ (COBB-DOUGLAS PRODUCTION FN.)

a : MEASURE THE CURRENT STAGE OF TECHNOLOGY

IF WE TAKE THE LOG OF BOTH SIDES AND ADD AN ERROR TERM, WE ARRIVE AT

$$\ln(Q) = \ln(a) + \beta_1 \ln(K) + \beta_2 \ln(L) + \varepsilon$$

OR

$$\ln(Q) = \beta_0 + \beta_1 \ln(K) + \beta_2 \ln(L) + \varepsilon.$$

WHERE $\beta_0 = \ln(a)$

$\hat{\beta}_k \Rightarrow$ THE ELASTICITY OF Q W.R.T THE EXPLANATORY VARIABLES.

$\beta_1 + \beta_2 = 1 \rightarrow$ CRS

$\hat{\beta}_1 + \hat{\beta}_2 > 1 \rightarrow$ IRS " DOUBLING OF ALL INPUTS GIVES MORE THAN

$\hat{\beta}_1 + \hat{\beta}_2 < 1 \rightarrow$ DRS

EX:

CONSIDER DEMAND FUNCTION

$$Y_i = \beta_1 X_i^{\beta_2} e^{u_i}$$

$\Rightarrow$ EXPONENTIAL REGRESSION MODEL.

BY LINEAR TRANSFORMATION:

$$\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i.$$

NOTE: $\ln$ = natural log (log to the base e, $e = 2.718$)

$$\boxed{\ln Y_i = \alpha + \beta_2 \ln X_i + u_i}$$

$\alpha = \ln \beta_1$

WHERE

$$\boxed{Y_i^* = \alpha + \beta_2 X_i^* + u_i}$$

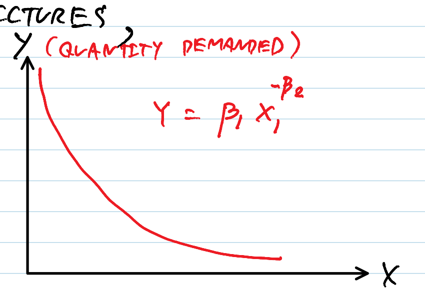
WHERE $Y_i^* = \ln Y_i$, $X_i^* = \ln X_i$

VIA OLS $\hat{\alpha}$, $\hat{\beta}_2$ OBTAINED.

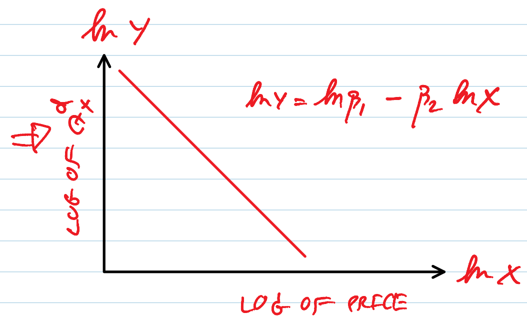
$\hat{\alpha}$: OLS ESTIMATOR OF α

$\hat{\beta}_2$: OLS ESTIMATOR OF β_2

IN PICTURES,



"CONSTANT ELASTICITY" MODEL



$$\ln Y = \beta_1 + \beta_2 \ln X$$

$$\frac{d\ln y}{dx} = \beta_2 \frac{d\ln x}{dx}$$

$$\frac{1}{Y}\frac{dy}{dx} = \beta_2 \cdot \frac{1}{X}$$

$$\beta_2 = \frac{dy}{dx}\cdot\frac{X}{Y} = \frac{\overbrace{\frac{dy}{Y}}^{\% \Delta Y}}{\underbrace{\frac{dx}{X}}_{\% \Delta X}} = \frac{\frac{\Delta Y}{Y}\times 100}{\frac{\Delta X}{X}\times 100}$$

$\frac{Y_2 - Y_1}{Y_1}\times 100$

$\frac{x_2 - x_1}{x_1}\times 100$

THE SLOPE COEFFICIENT β_2 MEASURE THE ELASTICITY OF Y W.R.T X : WHEN X CHANGES BY 1 %, Y CHANGES BY __ %.

$\beta_2 > 1 \Leftrightarrow \% \Delta Y > \% \Delta X$

(DEMAND IS PRICE-ELASTIC)

CAN YOU FIND SLOPE ?

$$\frac{dy}{dx} = ?$$

YOU HAVE $\beta_2 = \frac{dy}{dx}\cdot\frac{X}{Y}$.

THEREFORE, $\frac{dy}{dx} = \beta_2\left(\frac{Y}{X}\right)$

MEASURES MARGINAL EFFECT OF Y W.R.T. X: ONE UNIT CHANGE IN X LEADS TO ___ UNIT CHANGE IN Y.

OR IT IS ALSO CALLED

" MARGINAL EFFECT OF X UPON Y.

II SEMILOG MODELS

- LOG-LIN MODEL
- LIN-LOG MODEL

LOG-LIN MODEL : $\ln Y_i = \beta_1 + \beta_2 X_i + u_i$.

USED FOR ⇒ GROWTH MODEL

HOW? ⇒ CONSIDER $Y_t = Y_0 (1+r)^t$.

TAKE THE LOG OF BOTH SIDES:

$$\ln Y_t = \ln Y_0 + t \ln(1+r).$$

NOTE: $\ln(A\cdot B) = \ln A + \ln B$.

NOW, DENOTE $\beta_1 = \ln Y_0$

$\beta_2 = \ln(1+r)$

SO, $\boxed{\ln Y_t = \beta_1 + \beta_2 \cdot t}$.

MATHEMATICAL MODEL

SO, $\boxed{\ln Y_t = \beta_1 + \beta_2 \cdot t}$.

MATHEMATICAL MODEL

$$\boxed{\ln Y_t = \beta_1 + \beta_2 \cdot t + u_t .}$$

ECONOMETRIC MODEL

$$\frac{d \ln Y_t}{dt} = \beta_2 \cdot \frac{dt}{dt}$$

$$\frac{1}{Y} \frac{dY_t}{dt} = \beta_2$$

SO $\beta_2 = \dfrac{\frac{dy}{Y}}{dt}$ [LIKE $\dfrac{\frac{dy}{Y}}{dx}$.]

$$\boxed{\beta_2 = \frac{\text{RELATIVE CHANGE } Y}{\text{ABSOLUTE CHANGE IN } t}}$$

IN GENERAL: YOU HAVE $\ln Y = \beta_1 + \beta_2 X$

$$\frac{d \ln Y}{dX} = \beta_2 \cdot \cancel{\frac{dX}{dX}}^{\,1}$$

$$\frac{1}{Y}\frac{dY}{dX} = \beta_2$$

$$\beta_2 = \frac{\frac{dy}{Y}}{dX} \quad \text{OR} \quad \frac{\frac{\Delta Y}{Y}}{\Delta X}$$

RELATIVE CHANGE IN Y

ABSOLUTE CHANGE IN Y

$$100 \cdot \beta_2 = \frac{\frac{\Delta Y}{Y} \cdot 100}{\Delta X}$$

PERCENTAGE CHANGE IN Y

ΔY = ABSOLUTE CHANGE IN Y

$\frac{\Delta Y}{Y}$ = RELATIVE CHANGE IN Y

$\frac{\Delta Y}{Y} \times 100$ = PERCENTAGE CHANGE IN Y

GIVES YOU "GROWTH RATE IN Y FOR AN ABSOLUTE CHANGE IN X"!