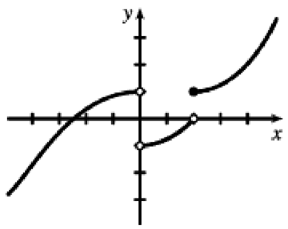
2. As x approaches 1 from the left, $f(x)$ approaches 3; and as x approaches 1 from the right, $f(x)$ approaches 7. No, the limit does not exist because the left- and right-hand limits are different.

3. (a) $\lim_{x\to -3} f(x) = \infty$ means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to -3 (but not equal to -3).

(b) $\lim_{x\to 4^+} f(x) = -\infty$ means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to 4 through values larger than 4.

16. $\lim_{x\to 0^-} f(x) = 1$, $\lim_{x\to 0^+} f(x) = -1$, $\lim_{x\to 2^-} f(x) = 0$,

$\lim_{x\to 2^+} f(x) = 1$, $f(2) = 1$, $f(0)$ is undefined



30. $\lim_{x\to -3^-} \dfrac{x+2}{x+3} = \infty$ since the numerator is negative and the denominator approaches 0 from the negative side as $x \to -3^-$.

32. $\lim_{x\to 0} \dfrac{x-1}{x^2(x+2)} = -\infty$ since $x^2 \to 0$ as $x \to 0$ and $\dfrac{x-1}{x^2(x+2)} < 0$ for $0 < x < 1$ and for $-2 < x < 0$.

18.
$$\lim_{h\to 0} \frac{(2+h)^3 - 8}{h} = \lim_{h\to 0} \frac{(8 + 12h + 6h^2 + h^3) - 8}{h} = \lim_{h\to 0} \frac{12h + 6h^2 + h^3}{h}$$
$$= \lim_{h\to 0} (12 + 6h + h^2) = 12 + 0 + 0 = 12$$

20. We use the difference of squares in the numerator and the difference of cubes in the denominator.

$$\lim_{t\to 1} \frac{t^4 - 1}{t^3 - 1} = \lim_{t\to 1} \frac{(t^2-1)(t^2+1)}{(t-1)(t^2+t+1)} = \lim_{t\to 1} \frac{(t-1)(t+1)(t^2+1)}{(t-1)(t^2+t+1)} = \lim_{t\to 1} \frac{(t+1)(t^2+1)}{t^2+t+1} = \frac{2(2)}{3} = \frac{4}{3}$$

22.
$$\lim_{u\to 2} \frac{\sqrt{4u+1} - 3}{u-2} = \lim_{u\to 2} \frac{\sqrt{4u+1} - 3}{u-2} \cdot \frac{\sqrt{4u+1} + 3}{\sqrt{4u+1} + 3} = \lim_{u\to 2} \frac{\left(\sqrt{4u+1}\right)^2 - 3^2}{(u-2)\left(\sqrt{4u+1} + 3\right)}$$
$$= \lim_{u\to 2} \frac{4u + 1 - 9}{(u-2)\left(\sqrt{4u+1} + 3\right)} = \lim_{u\to 2} \frac{4(u-2)}{(u-2)\left(\sqrt{4u+1} + 3\right)}$$
$$= \lim_{u\to 2} \frac{4}{\sqrt{4u+1} + 3} = \frac{4}{\sqrt{9} + 3} = \frac{2}{3}$$

26.
$$\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right) = \lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t(t+1)}\right) = \lim_{t\to 0} \frac{t+1-1}{t(t+1)} = \lim_{t\to 0} \frac{1}{t+1} = \frac{1}{0+1} = 1$$

44. Since $|x| = -x$ for $x < 0$, we have $\lim\limits_{x\to-2} \dfrac{2-|x|}{2+x} = \lim\limits_{x\to-2} \dfrac{2-(-x)}{2+x} = \lim\limits_{x\to-2} \dfrac{2+x}{2+x} = \lim\limits_{x\to-2} 1 = 1.$

46. $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$

At $x = 2$: $\lim\limits_{x\to2^-} f(x) = \lim\limits_{x\to2^-} \dfrac{x^2-4}{x-2} = \lim\limits_{x\to2^-} \dfrac{(x+2)(x-2)}{x-2} = \lim\limits_{x\to2^-} (x+2) = 2+2 = 4$

$\lim\limits_{x\to2^+} f(x) = \lim\limits_{x\to2^+} (ax^2 - bx + 3) = 4a - 2b + 3$

We must have $4a - 2b + 3 = 4$, or $\mathbf{4a - 2b = 1}$ **(1)**.

At $x = 3$: $\lim\limits_{x\to3^-} f(x) = \lim\limits_{x\to3^-} (ax^2 - bx + 3) = 9a - 3b + 3$

$\lim\limits_{x\to3^+} f(x) = \lim\limits_{x\to3^+} (2x - a + b) = 6 - a + b$

We must have $9a - 3b + 3 = 6 - a + b$, or $\mathbf{10a - 4b = 3}$ **(2)**.

Now solve the system of equations by adding -2 times equation (1) to equation (2).

$$\begin{aligned} -8a + 4b &= -2 \\ 10a - 4b &= 3 \\ \hline 2a \phantom{{}-4b} &= 1 \end{aligned}$$

So $a = \frac{1}{2}$. Substituting $\frac{1}{2}$ for a in (1) gives us $-2b = -1$, so $b = \frac{1}{2}$ as well. Thus, for f to be continuous on $(-\infty, \infty)$, $a = b = \frac{1}{2}$.

12. $\lim\limits_{y\to\infty} \dfrac{2-3y^2}{5y^2+4y} = \lim\limits_{y\to\infty} \dfrac{(2-3y^2)/y^2}{(5y^2+4y)/y^2} = \dfrac{\lim\limits_{y\to\infty}(2/y^2 - 3)}{\lim\limits_{y\to\infty}(5 + 4/y)} = \dfrac{2\lim\limits_{y\to\infty}(1/y^2) - \lim\limits_{y\to\infty} 3}{\lim\limits_{y\to\infty} 5 + 4\lim\limits_{y\to\infty}(1/y)} = \dfrac{2(0) - 3}{5 + 4(0)} = -\dfrac{3}{5}$

16. $\lim\limits_{x\to\infty} \dfrac{x^2}{\sqrt{x^4+1}} = \lim\limits_{x\to\infty} \dfrac{x^2/x^2}{\sqrt{x^4+1}/x^2} = \lim\limits_{x\to\infty} \dfrac{1}{\sqrt{(x^4+1)/x^4}}$ [since $x^2 = \sqrt{x^4}$ for $x > 0$]

$= \lim\limits_{x\to\infty} \dfrac{1}{\sqrt{1 + 1/x^4}} = \dfrac{1}{\sqrt{1+0}} = 1$

18. $\lim\limits_{x\to-\infty} \dfrac{\sqrt{9x^6 - x}}{x^3+1} = \lim\limits_{x\to-\infty} \dfrac{\sqrt{9x^6-x}/x^3}{(x^3+1)/x^3} = \dfrac{\lim\limits_{x\to-\infty} -\sqrt{(9x^6-x)/x^6}}{\lim\limits_{x\to-\infty}(1 + 1/x^3)}$ [since $x^3 = -\sqrt{x^6}$ for $x < 0$]

$= \dfrac{\lim\limits_{x\to-\infty} -\sqrt{9 - 1/x^5}}{\lim\limits_{x\to-\infty} 1 + \lim\limits_{x\to-\infty}(1/x^3)} = \dfrac{-\sqrt{\lim\limits_{x\to-\infty} 9 - \lim\limits_{x\to-\infty}(1/x^5)}}{1+0} = -\sqrt{9-0} = -3$

20. $\lim\limits_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right) = \lim\limits_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)\left[\dfrac{x-\sqrt{x^2+2x}}{x-\sqrt{x^2+2x}}\right] = \lim\limits_{x\to-\infty}\dfrac{x^2-(x^2+2x)}{x-\sqrt{x^2+2x}}$

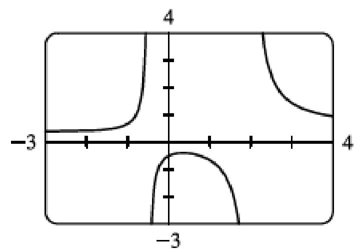
$$= \lim_{x\to-\infty}\frac{-2x}{x-\sqrt{x^2+2x}} = \lim_{x\to-\infty}\frac{-2}{1+\sqrt{1+2/x}} = \frac{-2}{1+\sqrt{1+2(0)}} = -1$$

Note: In dividing numerator and denominator by x, we used the fact that for $x<0$, $x=-\sqrt{x^2}$.

34. $\lim\limits_{x\to\infty}\dfrac{x^2+1}{2x^2-3x-2} = \lim\limits_{x\to\infty}\dfrac{\dfrac{x^2+1}{x^2}}{\dfrac{2x^2-3x-2}{x^2}} = \lim\limits_{x\to\infty}\dfrac{1+\dfrac{1}{x^2}}{2-\dfrac{3}{x}-\dfrac{2}{x^2}} = \dfrac{\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x^2}\right)}{\lim\limits_{x\to\infty}\left(2-\dfrac{3}{x}-\dfrac{2}{x^2}\right)}$

$$= \frac{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}\dfrac{1}{x^2}}{\lim\limits_{x\to\infty}2-\lim\limits_{x\to\infty}\dfrac{3}{x}-\lim\limits_{x\to\infty}\dfrac{2}{x^2}} = \frac{1+0}{2-0-0} = \frac{1}{2},$$ so $y=\frac{1}{2}$ is a horizontal asymptote.

$y=f(x)=\dfrac{x^2+1}{2x^2-3x-2}=\dfrac{x^2+1}{(2x+1)(x-2)}$, so $\lim\limits_{x\to(-1/2)^-}f(x)=\infty$ because as $x\to(-1/2)^-$ the numerator is positive while the denominator approaches 0 through positive values. Similarly, $\lim\limits_{x\to(-1/2)^+}f(x)=-\infty$, $\lim\limits_{x\to2^-}f(x)=-\infty$, and $\lim\limits_{x\to2^+}f(x)=\infty$. Thus, $x=-\frac{1}{2}$ and $x=2$ are vertical asymptotes. The graph confirms our work.



36. $\lim\limits_{x\to\infty}\dfrac{1+x^4}{x^2-x^4} = \lim\limits_{x\to\infty}\dfrac{\dfrac{1+x^4}{x^4}}{\dfrac{x^2-x^4}{x^4}} = \lim\limits_{x\to\infty}\dfrac{\dfrac{1}{x^4}+1}{\dfrac{1}{x^2}-1} = \dfrac{\lim\limits_{x\to\infty}\left(\dfrac{1}{x^4}+1\right)}{\lim\limits_{x\to\infty}\left(\dfrac{1}{x^2}-1\right)} = \dfrac{\lim\limits_{x\to\infty}\dfrac{1}{x^4}+\lim\limits_{x\to\infty}1}{\lim\limits_{x\to\infty}\dfrac{1}{x^2}-\lim\limits_{x\to\infty}1}$

$$= \frac{0+1}{0-1} = -1,$$ so $y=-1$ is a horizontal asymptote.

$y=f(x)=\dfrac{1+x^4}{x^2-x^4}=\dfrac{1+x^4}{x^2(1-x^2)}=\dfrac{1+x^4}{x^2(1+x)(1-x)}$. The denominator is zero when $x=0$, -1, and 1, but the numerator is nonzero, so $x=0$, $x=-1$, and $x=1$ are vertical asymptotes. Notice that as $x\to0$, the numerator and denominator are both positive, so $\lim\limits_{x\to0}f(x)=\infty$. The graph confirms our work.