EE 462 Development Macroeconomics (1 / 2012)

Lecture 4 Solow Growth Model with Human capital

Read Chapter 3 in Charles Jones

Neoclassical growth model: Solow with Human capital

- Gregory Mankiw, David Romer, และ David Weil (1992) "A Contribution to Empirics of Economic Growth" adds human capital into the Solow model and shows that it fits well with the data
- Human capital tries to capture the fact that different economics posses different levels of education and skills in labor. (not just numbers of workers or work hours)
- In MRW, economy accumulates human capital in the same way as physical capital by foregoing consumption.

- Given the Cobb-Douglas Production Function

$$Y=K^{\alpha}(AH)^{1-\alpha}$$

- Y is output

  K physical capital

  A labor-augmenting technology that grows exogenously at rate g.

  H human capital

  α output elasticity wrt K

- We follow Lucas (1988) in assuming that individuals spend time accumulating skills, much like a student going to school.

$$H=e^{\psi u}L$$

- u denotes the fraction of an individual's time spent learning skills

  ψ is a positive constant, increasing effective units of skilled labor H.

- Note that if u=0, then H=L. Without u, all labor is unskilled, L)
- We assume that u is constant and given exogenously

$$H = e^{\psi u} L$$

$$\ln H = \psi u \ln e + \ln L$$

$$\ln H = \psi u + \ln L$$

$$\frac{d \ln H}{du} = \psi \frac{du}{du} + \frac{d \ln L}{du}$$

$$\frac{1}{H} \frac{dH}{du} = \psi$$

- If u increases by one unit (year in schooling), suppose phi =0.1, thus H increase by 0.1 or 10 percent.
- This is intended to match the estimates of returns to schooling using the Mincerian regression on log wage with education as explanatory variable

- Physical capital is accumulated by investing

$$\mathring{K} = s_K Y - dK$$

- Y output

  s_K investment rate on K

  d depreciation rate (we change our notation)

- Our production function in terms of output per worker (we abuse our notation here)

$$Y = K^{\alpha}(AH)^{1-\alpha}$$

$$\frac{Y}{L} = \frac{K^{\alpha}(AH)^{1-\alpha}}{L}$$

$$\frac{Y}{L} = \frac{K^{\alpha}}{L^{\alpha}} A^{1-\alpha} \frac{H^{1-\alpha}}{L^{1-\alpha}}$$

$$y = k^{\alpha}(Ah)^{1-\alpha}$$

- Notice that $h = \frac{H}{L} = e^{\psi u}$ and is constant for u is assumed to be constant

- Per worker growth in output and physical capital in the balanced growth path

$$y = k^{\alpha}(Ah)^{1-\alpha}$$

$$\ln y = \alpha \ln k + (1-\alpha)\ln A + (1-\alpha)\ln h$$

$$\frac{\mathring{y}}{y} = \alpha \frac{\mathring{k}}{k} + (1-\alpha)\frac{\mathring{A}}{A} + (1-\alpha)\frac{\mathring{h}}{h}$$

$$g_y = \alpha g_k + (1-\alpha)g + (1-\alpha)0$$

$$g_y = \alpha g_y + (1-\alpha)g$$

$$(1-\alpha)g_y = (1-\alpha)g$$

$$g_y = g_k = g$$

- Along the balanced growth path, y and k will growth at the constant rate g or the rate of technological progress.
- h is constant by assuming as a constant fraction of labor. Also, H and L will grow at the same rate by this assumption in steady state.
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- To find state variables that are constant along a balanced growth path, we use

$$\tilde{y} = \frac{y}{Ah}$$

$$\tilde{k} = \frac{k}{Ah}$$

- Production function can be restated as

$$y = k^{\alpha}(Ah)^{1-\alpha}$$

$$\frac{y}{Ah} = k^{\alpha}(Ah)^{1-\alpha-1}$$

$$\tilde{y} = \tilde{k}^{\alpha}$$

- The capital accumulation equation can be written as

$$\tilde{k} = \frac{k}{Ah} = \frac{K}{L(Ah)}$$

$$\ln \tilde{k} = \ln K - \ln L - \ln A - \ln h$$

$$\frac{d\ln \tilde{k}}{dt} = \frac{d\ln K}{dt} - \frac{d\ln L}{dt} - \frac{d\ln A}{dt} - \frac{d\ln h}{dt}$$

$$\frac{\overset{\circ}{\tilde{k}}}{\tilde{k}} = \frac{\overset{\circ}{K}}{K} - \frac{\overset{\circ}{L}}{L} - \frac{\overset{\circ}{A}}{A} - \frac{\overset{\circ}{h}}{h}$$

$$\frac{\overset{\circ}{\tilde{k}}}{\tilde{k}} = \frac{s_K Y - dK}{K} - n - g$$

$$\frac{\overset{\circ}{\tilde{k}}}{\tilde{k}} = \frac{s_K Y}{K}\frac{L}{L}\frac{Ah}{Ah} - d - n - g$$

$$\frac{\overset{\circ}{\tilde{k}}}{\tilde{k}} = \frac{s_K \tilde{y}}{\tilde{k}} - (n + g + d)$$

$$\overset{\circ}{\tilde{k}} = s_K \tilde{y} - (n + g + d)\tilde{k}$$

- Thus, adding human capital does not change the basic flavor of the Solow Model
- At steady state $\mathring{\tilde{k}} = 0$ we have

$$0 = s_K \tilde{y} - (n + g + d)\tilde{k}$$

$$(n + d + g)\tilde{k} = s_K \tilde{y}$$

$$\tilde{k}^* = \left(\frac{s_K}{n + d + g} \right) \tilde{y}^*$$

- Put this into the production function and get

$$\tilde{y} = \tilde{k}^{\alpha}$$

$$\tilde{y}^{*} = \left(\frac{s_K}{n+g+d}\right)^{\alpha} \tilde{y}^{*\alpha}$$

$$\tilde{y}^{*1-\alpha} = \left(\frac{s_K}{n+g+d}\right)^{\alpha}$$

$$\tilde{y}^{*} = \left(\frac{s_K}{n+g+d}\right)^{\frac{\alpha}{1-\alpha}}$$

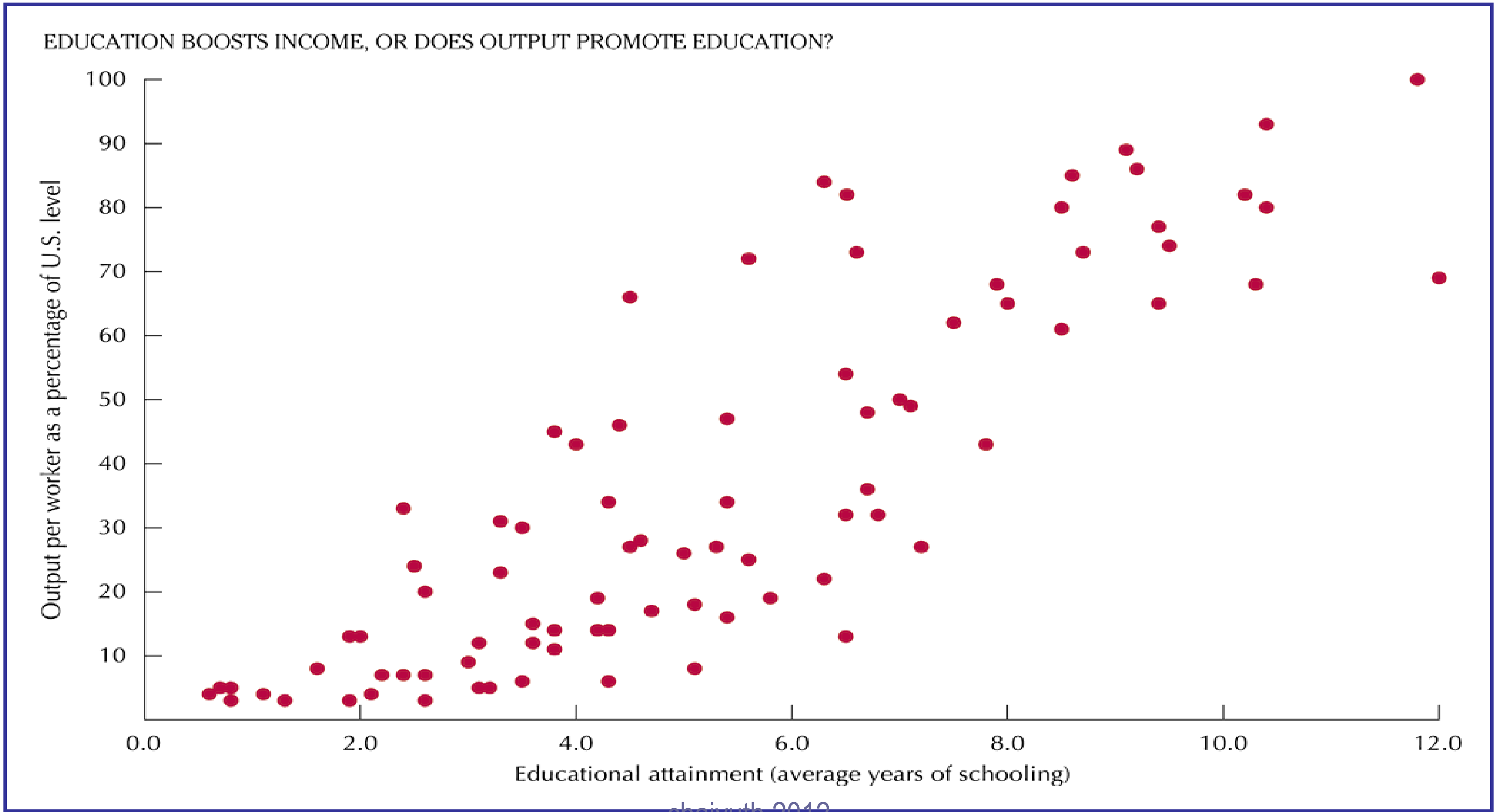
$$y^{*}(t) = \left(\frac{s_K}{n+g+d}\right)^{\frac{\alpha}{1-\alpha}} hA(t)$$

- The last equation explains high per capita income by
  (1) high investment rate in K (s_K)
  (2) more % of time for skill accumulation ($h=e^{\psi u}$)
  (3) low population growth (n)
  (4) high level of technology (A)
  (5) high rate of technical progress (g)
  (6) low depreciation rate of K (d)
- In steady state, per-capita output will grow at the rate of g as in the basic Solow model.

Human capital measures

- Unobservable or directly measured
- From education, work experiences, skill types
- Via proxy
- Highly correlated with human capital
  - Enrolment rates (gross and net, level of education)
  - Average years of schooling (how to weight, quality changes over time)
  - Adult literacy rate (underestimates, but available)

Output per Worker Relative to the United States and Educational Attainment Measured as Years of Schooling



remarks

- Although the neoclassical growth model emphasizes the importance of technology, it is left unmodeled or exogenously assumed.
- Differences in technologies across economies are unexplained.