Question 0: (required for all)

0.1) Given that $Z = \frac{x^3 - y^3}{x^2 y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

0.2) Given that $Z = \frac{x - y}{x + y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

0.3) If z = $2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

0.5) Given that $ln\,(x + y + z) + xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

0.1)

$$\frac{\partial Z}{\partial x} = \frac{x^2y^2 \frac{\partial (x^3 - y^3)}{\partial x} - (x^3 - y^3)\frac{\partial (x^2y^2)}{\partial x}}{(x^2y^2)^2}$$

$$= \frac{x^2y^2(3x^2) - (x^3 - y^3)(2xy^2)}{(x^2y^2)^2}$$

$$= \frac{3x^4y^2 - 2x^4y^2 + 2xy^5}{x^4y^4}$$

$$= \frac{xy^2(3x^3 - 2x^3 + 2y^3)}{x^4y^4}$$

$$= \frac{x^3 + 2y^3}{x^3y^2}$$

$$\frac{\partial Z}{\partial y} = \frac{x^2y^2\frac{\partial (x^3 - y^3)}{\partial y} - (x^3 - y^3)\frac{\partial (x^2y^2)}{\partial y}}{(x^2y^2)^2}$$

$$= \frac{x^2y^2(-3y^2) - (x^3-y^3)(2x^2y)}{(x^2y^2)^2}$$

$$= \frac{-3x^2y^4 - 2x^5y + 2x^2y^4}{x^4y^4}$$

$$= \frac{x^2y(-3y^3 - 2x^3 + 2y^3)}{x^4y^4}$$

$$= \frac{-2x^3 - y^3}{x^2y^3}$$

$$x\frac{\partial z}{\partial x} = x\left(\frac{x^3 + 2y^3}{x^3y^2}\right)$$

$$= \frac{x^3 + 2y^3}{x^2y^2}$$

$$y\frac{\partial z}{\partial y} = y\left(\frac{-2x^3 + 5y^3}{x^2y^3}\right)$$

$$= \frac{-2x^3 - y^3}{x^2y^2}$$

$$\therefore \quad x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{x^3 + 2y^3}{x^2y^2} + \left(\frac{-2x^3 - y^3}{x^2y^2}\right)$$

$$= \frac{y^3 - x^3}{x^2y^2} = -z$$

0.2)

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$$

$$= 0$$

$$\frac{\partial z}{\partial x} = \frac{(x+y)\frac{\partial (x-y)}{\partial x} - (x-y)\frac{\partial (x+y)}{\partial x}}{(x+y)^2}$$

$$= \frac{(x+y)(1) - (x-y)}{(x+y)^2}$$

$$= \frac{2y}{(x+y)^2}$$

If both x and y = 1

So $\frac{\partial z}{\partial x} = \frac{2(1)}{(2)^2}$

$$= \frac{1}{2}$$

$$\therefore \frac{2y}{(x+y)^2} > 0$$

if X increase, Z will increase

$$\frac{\partial z}{\partial y} = \frac{(x+y)\frac{\partial(x-y)}{\partial y} - (x-y)\frac{\partial(x+y)}{\partial y}}{(x+y)^2}$$

$$= \frac{(x+y)(-1) - (x-y)(1)}{(x+y)^2}$$

$$= \frac{-2x}{(x+y)^2}$$

If both x and y = 1

So $\frac{\partial z}{\partial y} = \frac{-2(1)}{((1)+(1))^2}$

$$= \frac{-1}{2}$$

$\therefore \frac{-2x}{(x+y)^2} < 0$

if y increase, z will decrease

$$dz = \frac{2y}{(x+y)^2} \cdot dx + \frac{(-2x)}{(x+y)^2} \cdot dy$$

$$= \frac{1}{2} \cdot dx - \frac{1}{2} \cdot dy$$

$\therefore$ When x increase 1 unit

z will increase 0.5 unit

When y decrease 1 unit

z will increase 0.5 unit

0.3) $\frac{\partial z}{\partial x} = 4xy + 3y$

$= 14$

$\frac{\partial z}{\partial y} = 2x^2 + 3x + 2y$

$= 9$

$\frac{\partial x}{\partial s} = 2r$

$= 2$

$\frac{\partial x}{\partial r} = 2r + 2s$

$= 2$

$\frac{\partial y}{\partial s} = -4$

$\frac{\partial y}{\partial r} = 2$

$$dz = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial r}\,dr + \frac{\partial x}{\partial s}\,ds\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial r}\,dr + \frac{\partial y}{\partial s}\,ds\right)$$

$$\therefore \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial s}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial s}\right)$$

$$= (4xy + 3y)(2r) + (2x^2 + 3x + 2y)(-4)$$

$$= 14(2) + 9(-4)$$

$$= -8$$

$$\therefore \frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial r}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial r}\right)$$

$$= (4xy + 3y)(2r + 2s) + (2x^2 + 3x + 2y)(2)$$

$$= 14(2) + 9(2)$$

$$= 46$$

if $r = 1$, $s = 0$

$$x = r^2 + 2rs$$

$$= 1$$

$$y = 2r - 4s$$

$$= 2$$

Q.4)

$$2x^2 + 3y^2 + 2z^2 = 16$$

$$F(x,y,z) = 2x^2 + 3y^2 + 2z^2 - 16$$

$$= 0$$

$$\frac{dz}{dy} = \frac{-F_y}{F_z}$$

$$= \frac{-\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$

$$= \frac{-6y}{4z}$$

$$= \frac{-3y}{2z}$$

given $x = 1$, $y = 2$, $z = -1$

$$\frac{dz}{dy} = \frac{-3(2)}{2(-1)}$$

$$= 3$$

0.5 $\ln(x+\ \ +z) + xyz = ze^{x+y+z}$

$$F(x,y,z) = \ln(x+y+z) + xyz - ze^{x+y+z}$$

$$\frac{dz}{dx} = \frac{-F_x}{F_z}$$

$$= \frac{-1}{1-e}$$

Given $x=0$, $y=1$, $z=0$

$$F_x = \frac{\partial F}{\partial x}$$

$$= \frac{1}{(x+y+z)}(1) + yz - ze^{x+y+z}$$

$$= \frac{1}{(x+y+z)} + yz - ze^{x+y+z}$$

$$= 1$$

$$F_z = \frac{\partial F}{\partial z}$$

$$= \frac{1}{(x+y+z)}(1) + xy - \left(\frac{z\,d(e^{x+y+z})}{dz} + e^{x+y+z}\frac{d(z)}{dz}\right)$$

$$= \frac{1}{(x+y+z)} + xy - ze^{(x+y+z)} - e^{(x+y+z)}$$

$$= 1 - e$$

1.1) from patial derivative respect to P_y

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5 I^2$$

$$\frac{dQ_x}{dP_y} \longrightarrow Q_x = 25 P_y^{-\frac{3}{2}}$$

$$= \frac{25}{\sqrt{P_y^3}}$$

if $P_y = 1$, $Q_x = 25$

$P_y = 2$, $Q_x = 8.834$

So when P_y increase , Q_x will decrease

1.2 $$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5 I^2$$

from patial derivative respect to I

$$\frac{dQ_x}{dI} = Q_x = 1 I$$

When income rise quantity of x is rise , x isn't inferior good.

1.3 $Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2$

Assume $P_x = 10$
$P_y = 25$
$I = 10$

$$Q_x = 100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(10)^2$$
$$= 100 - 40 - 10 + 50$$
$$= 100$$

1.4 $Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2$

Assume $P_x = 10$
$P_y = 25$
$I = 10$

$$Q_x = 100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(10)^2$$
$$= 100 - 40 - 10 + 50$$
$$= 100$$

Assume $P_x = 20$
$P_y = 25$
$I = 10$

$$Q_x = 100 - 4(20) - \frac{50}{\sqrt{25}} + 0.5(10^2)$$
$$= 100 - 80 - 10 + 50$$
$$= 60$$

1.5 $$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5\, I^2$$

Assume $P_x = 10$
$P_y = 25$
$I = 10$

$$Q_x = 100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5\,(10)^2$$

$$= 100 - 40 - 10 + 50$$

$$= 100$$

Assume $P_x = 10$
$P_y = 100$
$I = 10$

$$Q_x = 100 - 4(10) - \frac{50}{\sqrt{100}} + 0.5\,(10)^2$$

$$= 100 - 40 - 5 + 50$$

$$= 105$$

Check cross price elasticity

$$\frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{105 - 100}{100 - 25} \cdot \frac{25}{100}$$

$$= 0.0167$$

1.6 $Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2$

Assume $P_x = 10$
$P_y = 25$
$I = 10$

$Q_x = 100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(10)^2$

$= 100 - 40 - 10 + 50$

$= 100$

Assume $P_x = 10$
$P_y = 25$
$I = 20$

$Q_x = 100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(20)^2$

$= 100 - 40 - 10 + 200$

$= 250$

Check income elasticity, $\frac{\Delta Q}{\Delta I} \cdot \frac{I}{Q} = \frac{250-100}{20-10} \cdot \frac{10}{100} = 1.5$

because value of income elasticity of demand is 1.5

X is luxurious product

3.1 $u(x,y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$

marginal utility of x

$\frac{du}{dx}$ (y held constant) $= \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

Marginal utility of y

$\frac{du}{dx}$ (x held constant) $= \frac{1}{2} y^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{y}}$

3.2 Law of diminishing marginal utility

is marginal utility decrease with

each unit obtain by consumer by the way

this utility function not satisfy to

Law of diminishing because in formula

if we increase x or y for every unit

it can't make value of total utility negative

3.3 no because marginal utility of good x is $\frac{1}{2\sqrt{x}}$, the marginal utility of good x doesn't depend on good y

3.4 $u(x,y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$

consumer buy 1 unit of x
2 unit of y

$$= 1\left(\frac{1}{2\sqrt{x}}\right) + 2\left(\frac{1}{2\sqrt{y}}\right)$$

$$= \frac{2\sqrt{x} + \sqrt{y}}{2\sqrt{x}\sqrt{y}}$$

3.5 $u(x,y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$

$$\frac{\partial u}{\partial x} = \frac{1}{2\sqrt{x}}$$

$$\frac{\partial u}{\partial y} = \frac{1}{2\sqrt{y}}$$

$$du = \left(\frac{1}{2\sqrt{x}}\right)dx + \left(\frac{1}{2\sqrt{y}}\right)dy$$

$$= \left(\frac{1}{2\sqrt{x}}\right)3 + \left(\frac{1}{2\sqrt{y}}\right)(-1)$$

$$= \frac{6\sqrt{y} - 2\sqrt{x}}{4\sqrt{x}\sqrt{y}}$$

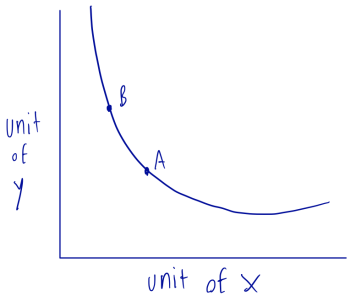
$$= \frac{6\sqrt{y} - 2\sqrt{x}}{4\sqrt{x}\sqrt{y}} - \frac{2\sqrt{x} + \sqrt{y}}{2\sqrt{x}\sqrt{y}}$$

$$= \frac{6\sqrt{y} - 2\sqrt{x}}{4\sqrt{x}\sqrt{y}} - \left(\frac{4\sqrt{x} + 2\sqrt{y}}{4\sqrt{x}\sqrt{y}}\right)$$

$$= \frac{4\sqrt{y} - 6\sqrt{x}}{4\sqrt{x}\sqrt{y}}$$

$$= \frac{2\sqrt{y} - 3\sqrt{x}}{2\sqrt{x} - \sqrt{y}}$$

3.6



MRS = marginal rate of subsitution

from point A and B when X decrease, Y will increase

$$MRS_{xy} = \frac{dy}{dx} = \frac{MUx}{MUy}$$

$$MU_x = \frac{1}{2\sqrt{x}}$$

$$MU_y = \frac{1}{2\sqrt{y}}$$

$$\frac{MU_x}{MUy} = \frac{2\sqrt{y}}{2\sqrt{x}} = \frac{\sqrt{y}}{\sqrt{x}}$$