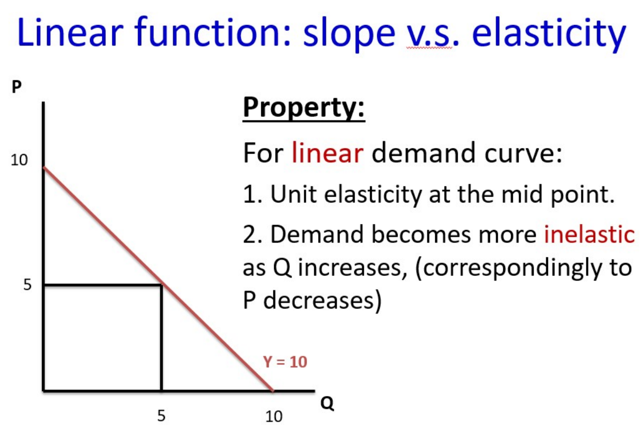
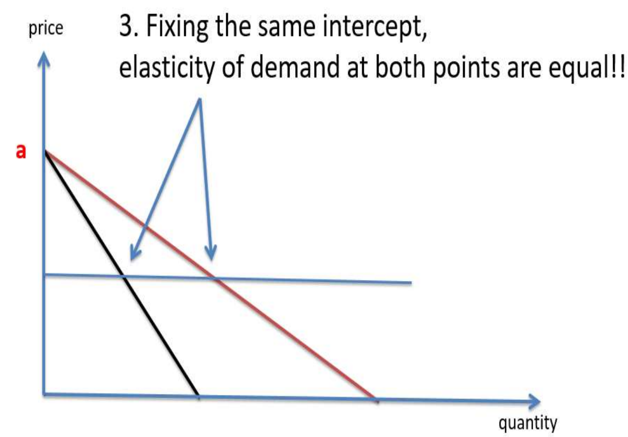




Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that
(i) elasticity of supply is always greater than 1 if c > 0,
(ii) elasticity of supply is always eqaul to 1 if c = 0,
(iii) elasticity of supply is always less to 1 if c < 0.

1 $P = a - bQ$ - — — — inverse demand eq.

$bQ = a - p$

$Q = \frac{a-p}{b}$ — — — demand equation.

$PED = \frac{\Delta Q}{\Delta p} \times \frac{P_0}{Q_0} = \frac{-1}{b} \times \frac{P_0}{Q_0}$

2 $p = c + dQ$

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

Revenue function : $TR(Q) = P(Q) \cdot Q$ — — — $P(Q) \cdot Q = (10 - Q) \cdot Q$

$= 10Q - Q$

when it is at maximum point, $\frac{dTR}{dQ} = 0$

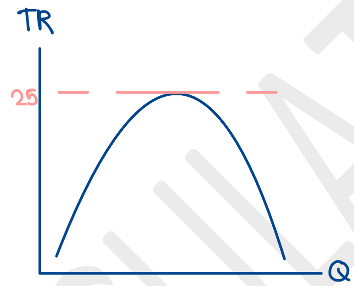
$0 = 10 - 2Q$

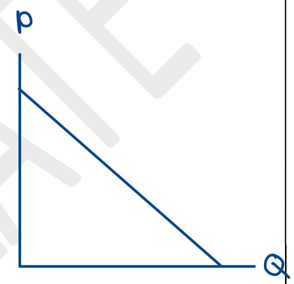
$2Q = 10$

$Q = 5$

At $Q = 5$, TR is maximized at $(10 - 5)(5)$

$= 25$





- What is the break-even output?

At Break-even, profit = 0 $\quad TR = TC$

$10Q - Q^2 = 4Q$

$Q^2 + 6Q = 0$

$Q^2 - 6Q = 0$

$Q(Q - 6) = 0$

$Q = 0, 6$

- What is the profit-maximizing level of output?

When the profit is maximized, $\frac{dy}{dx} = 0$

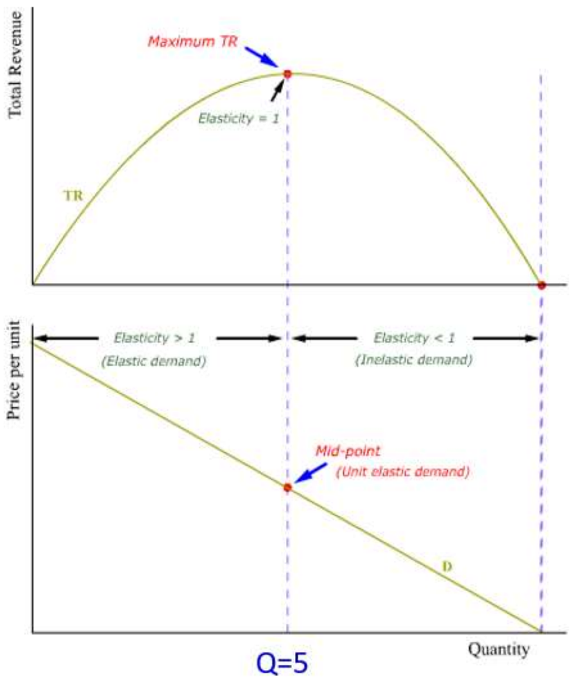
1 $MC = MR$

$\frac{d}{dQ} TC = \frac{d}{dQ} TR$

$10 - 2Q = 4$

$Q = 3$

Quadratic function: revenue function/break even analysis



$$\left|\frac{\%\Delta Q}{\%\Delta P}\right| : \begin{array}{lcc} >1 & ; & elastic \\ =1 & ; & unit\ elastic \\ <1 & ; & inelastic \end{array}$$

Revenue maximizing output:
unit elasticity

Profit-maximizing output:
under region with **elastic** demand

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Exrcise 2B. Consider a function that relates tax revenues R, in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?

(b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?

(c) What tax rate is consistent with the maximum tax revenue?

(a) $R = 350t - 500t^2$

$60 = 350t - 500t^2$

$500t^2 - 350t + 60 = 0$

$50t^2 - 35t + 6 = 0$

$t = 0.3$ or $t = 0.4$

∴ To hit the target, the tax rate would be 30% and 40%

(b) $R = 350t - 500t^2$

$61.25 = 350t - 500t^2$

$500t^2 - 350t + 61.25 = 0$

$t = 0.35$

∴ To hit the target, the tax rate would be 35%.

(c) When maximized, $\frac{dR}{dt} = 0$ $\quad \frac{dR}{dt} = 350 - 1000t$

$350 - 1000t = 0$

$350 = 1{,}000t$

$t = 0.35$

∴ the government revenue would be maximized at 35%.