# FN452: Asset management and portfolio analysis

# Risk, return, and capital allocation to risky assets

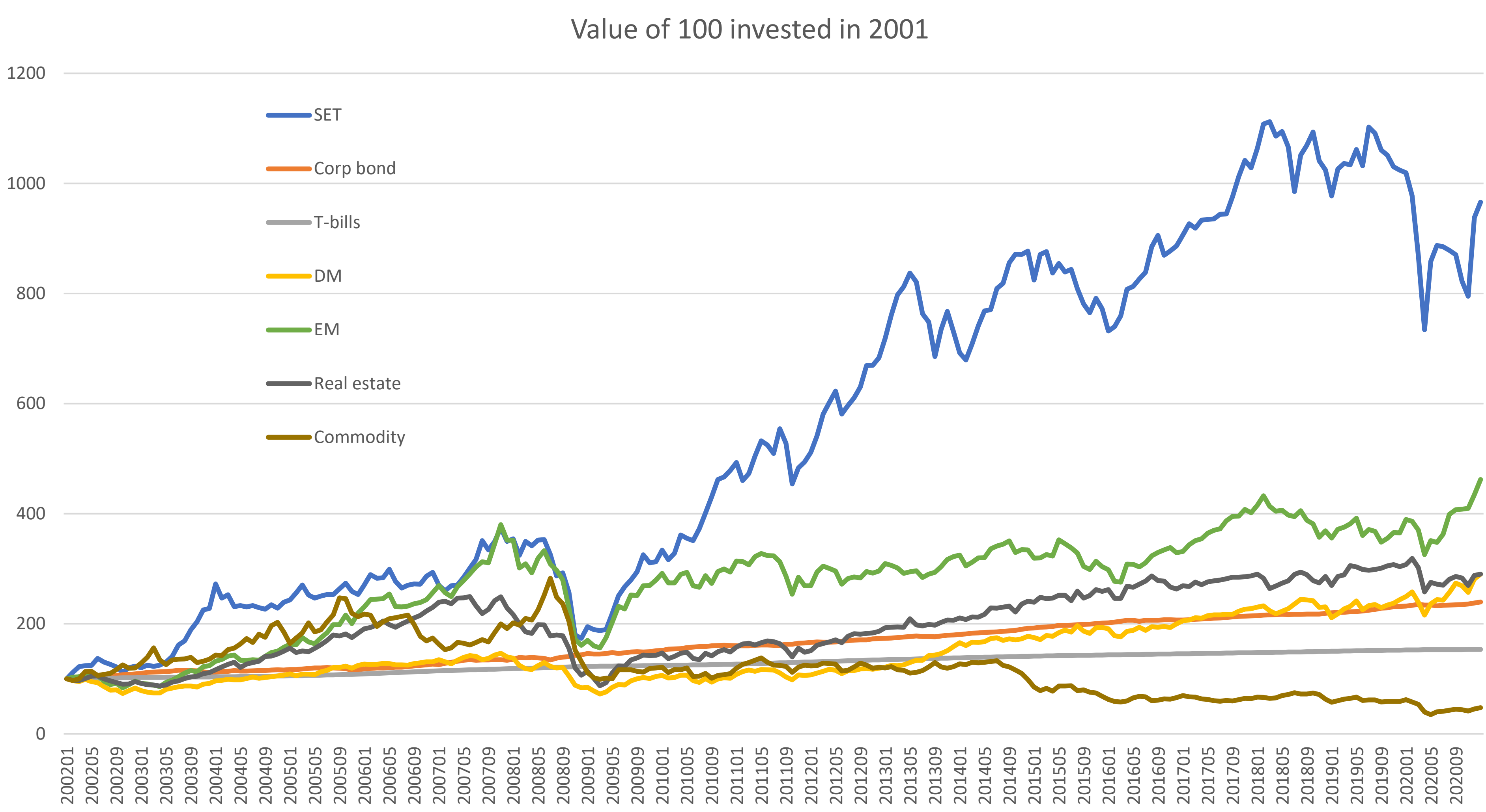
Sakkakom Maneenop

Thammasat Business School

# Content

- Learning from historical returns
- Risk and return of securities
- Portfolio risk and return
- Portfolios of two risky assets
- Portfolios of *N* risky assets
- Risk aversion

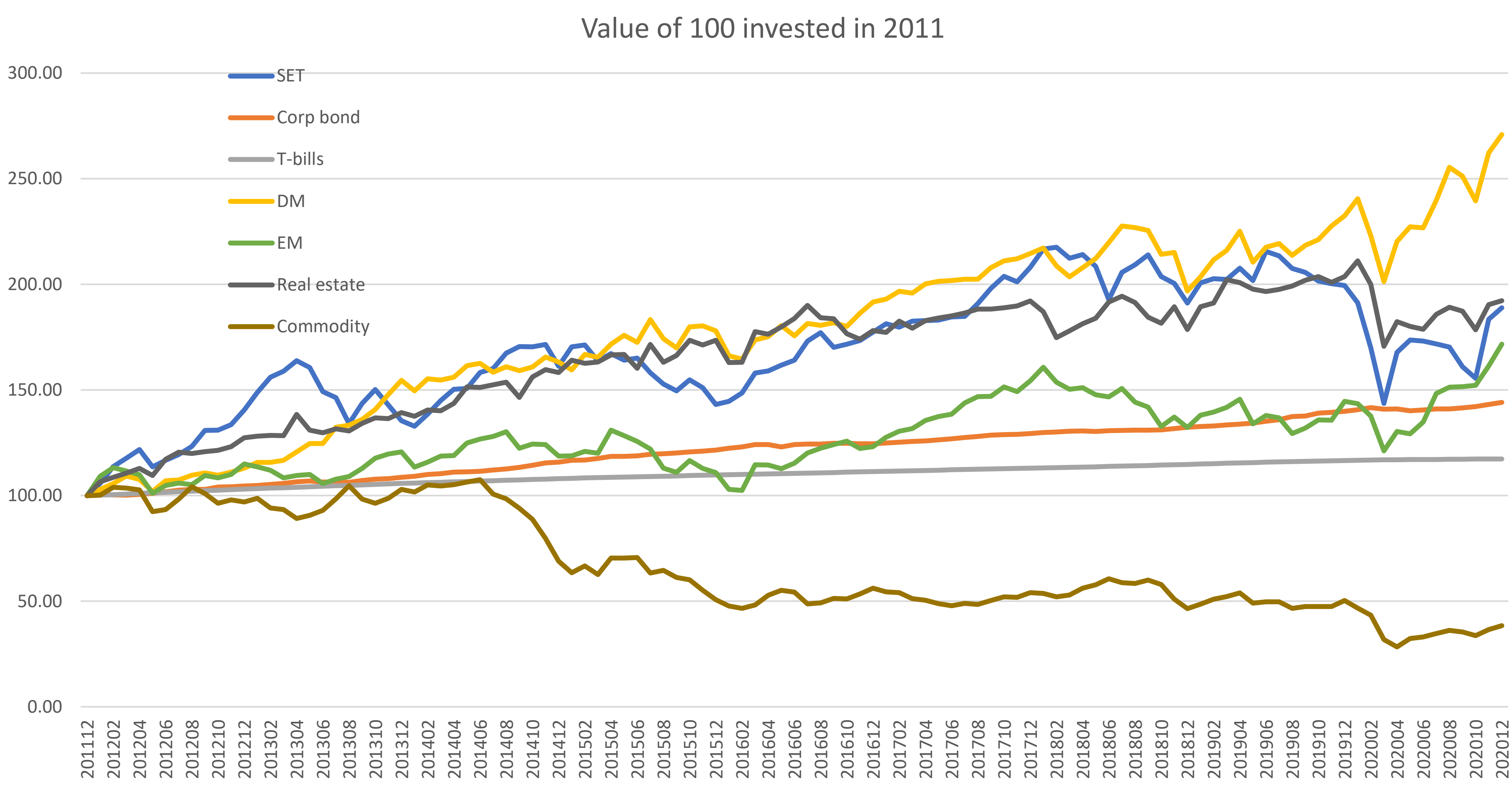
# Learning from historical returns




*Source: ThaiBMA, Eikon, MSCI*

# Learning from historical returns

THAMMASAT BUSINESS SCHOOL




*Source: ThaiBMA, Eikon, MSCI*

# Return of securities

- Equity securities
  - Dividend
  - Capital gain/loss
- Debt securities
  - Coupon interest
  - Capital gain/loss

# Risk and return of securities (Ex-ante)

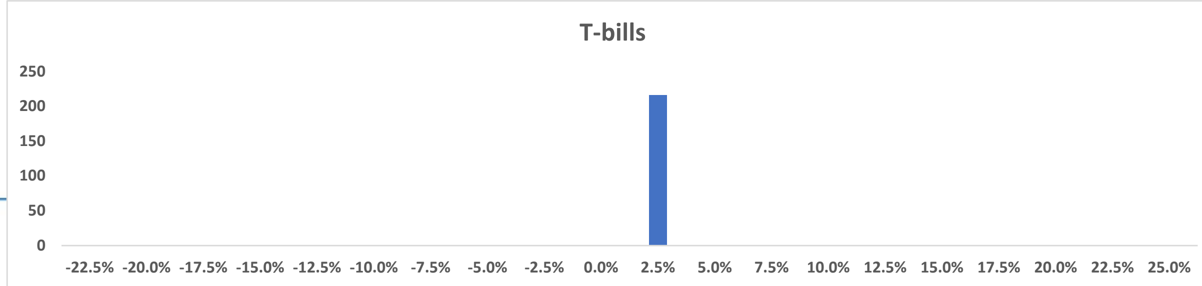
- Expected return

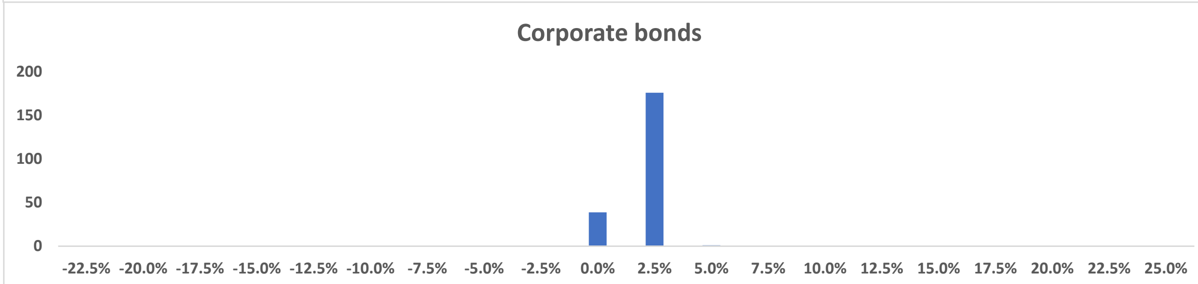
$$E[R] = \sum_R p_R \times R$$

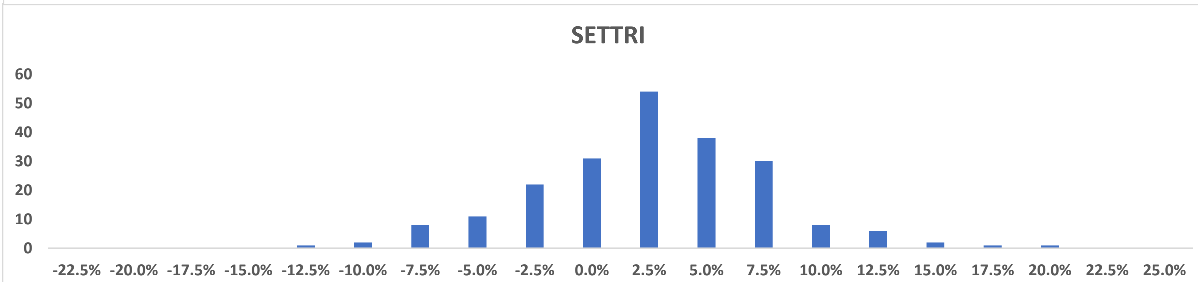
- Variance and standard deviation

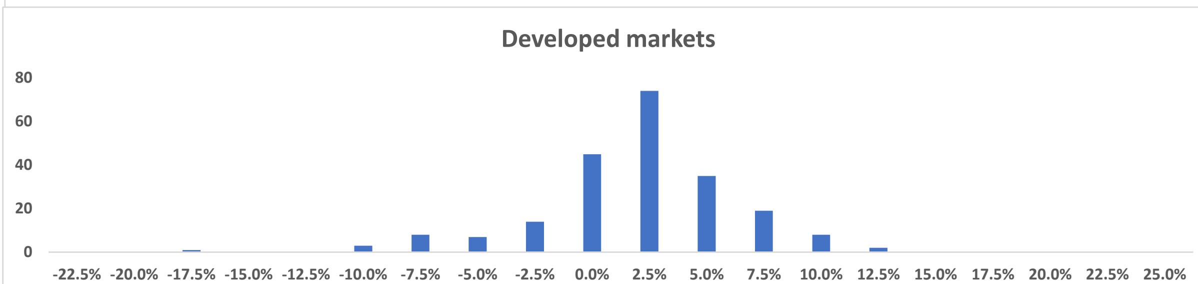
$$Var(R) = E\left[(R - E[R])^2\right] = \sum_R p_R \times (R - E[R])^2$$

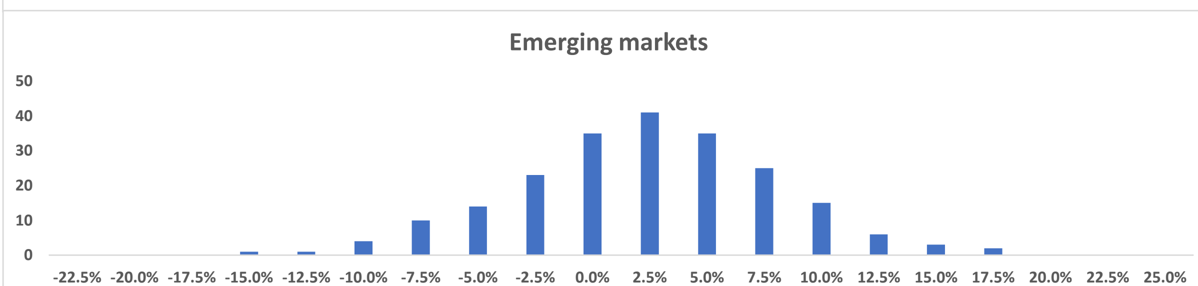
$$SD(R) = \sqrt{Var(R)}$$


T-bills
250
200
150
100
50
0
-22.5% -20.0% -17.5% -15.0% -12.5% -10.0% -7.5% -5.0% -2.5% 0.0% 2.5% 5.0% 7.5% 10.0% 12.5% 15.0% 17.5% 20.0% 22.5% 25.0%


Corporate bonds
200
150
100
50
0
-22.5% -20.0% -17.5% -15.0% -12.5% -10.0% -7.5% -5.0% -2.5% 0.0% 2.5% 5.0% 7.5% 10.0% 12.5% 15.0% 17.5% 20.0% 22.5% 25.0%


SETTRI
60
50
40
30
20
10
0
-22.5% -20.0% -17.5% -15.0% -12.5% -10.0% -7.5% -5.0% -2.5% 0.0% 2.5% 5.0% 7.5% 10.0% 12.5% 15.0% 17.5% 20.0% 22.5% 25.0%


Developed markets
80
60
40
20
0
-22.5% -20.0% -17.5% -15.0% -12.5% -10.0% -7.5% -5.0% -2.5% 0.0% 2.5% 5.0% 7.5% 10.0% 12.5% 15.0% 17.5% 20.0% 22.5% 25.0%


Emerging markets
50
40
30
20
10
0
-22.5% -20.0% -17.5% -15.0% -12.5% -10.0% -7.5% -5.0% -2.5% 0.0% 2.5% 5.0% 7.5% 10.0% 12.5% 15.0% 17.5% 20.0% 22.5% 25.0%

# Average rate of return (Ex-post)

- Arithmetic mean return

$$\bar{R} = \frac{1}{T}(R_1 + R_2 + \cdots + R_T) = \frac{1}{T}\sum_{t=1}^{T} R_T$$

- Geometric mean return

$$GM = [(1 + R_1)(1 + R_2) \ldots (1 + R_T)]^{1/T} - 1$$

# Risk of securities (Ex-post)

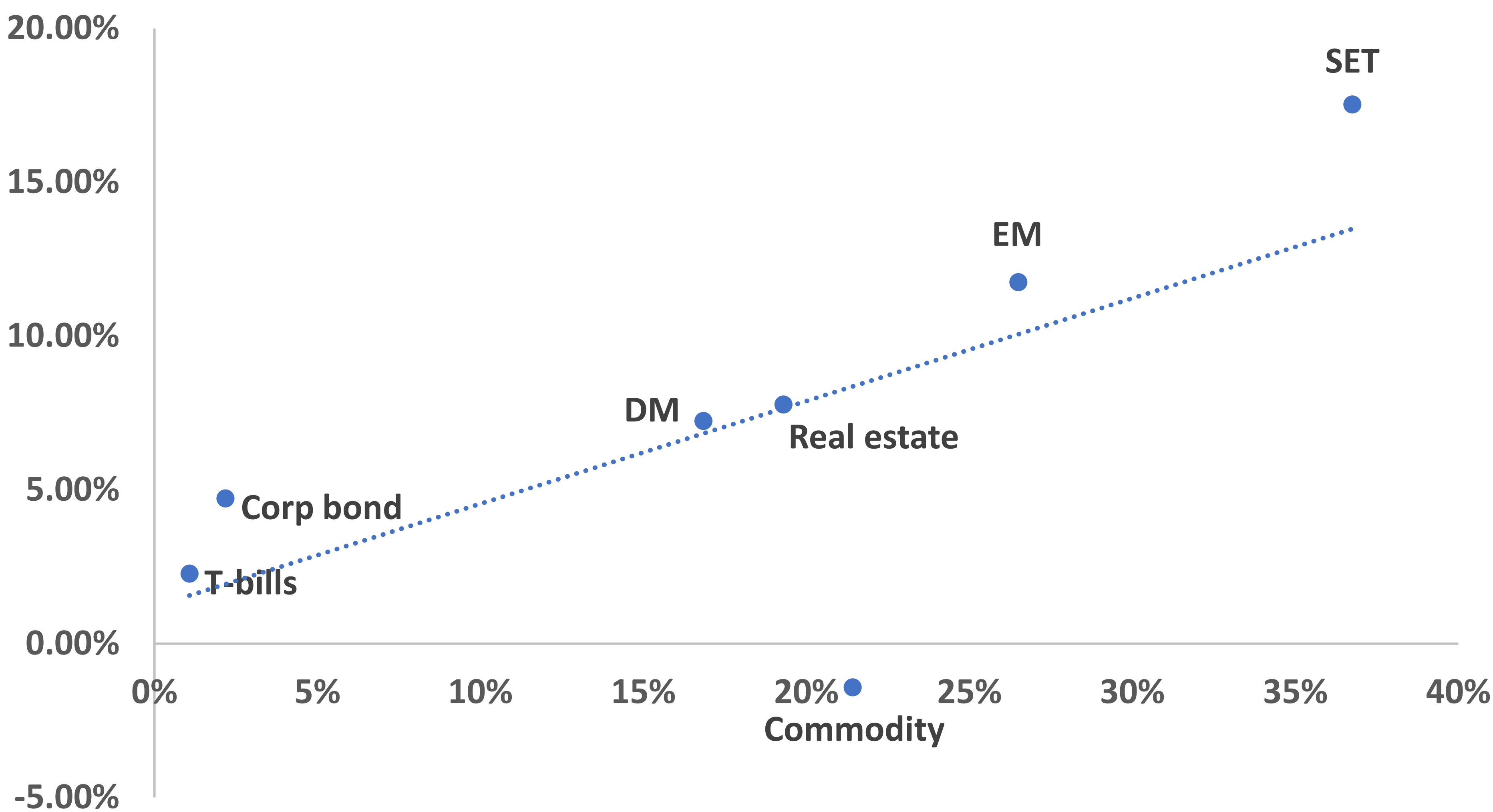
- Variance

$$Var(R) = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \bar{R}\right)^2$$

- Standard deviation

$$SD(R) = \sqrt{Var(R)}$$

# Risk and return of securities (Ex-post)




*Source: ThaiBMA, Eikon, MSCI*

# Portfolio risk and return

- Historical portfolio return

$$R_p = w_1 R_1 + w_2 R_2 + \cdots + w_n R_n = \sum_i w_i R_i$$

- Expected portfolio return

$$E[R_p] = E\left[\sum_i w_i R_i\right] = \sum_i E[w_i R_i] = \sum_i w_i E[R_i]$$

- Portfolio variance and standard deviation

$$Var(R) = \sigma^2 = E\left[(R - E[R])^2\right] = \sum_R p_R \times (R - E[R])^2$$

$$SD(R) = \sigma = \sqrt{Var(R)}$$

# Portfolio risk and return

- Covariance is the expected product of the deviations of two returns from their means

$$Cov(R_i, R_j) = \sigma_{i,j} = E\big[(R_i - E[R_i])(R_j - E[R_j])\big]$$
$$= \textstyle\sum_R p_R (R_i - E[R_i])(R_j - E[R_j])$$

- When estimating the covariance from historical data, we use the formula

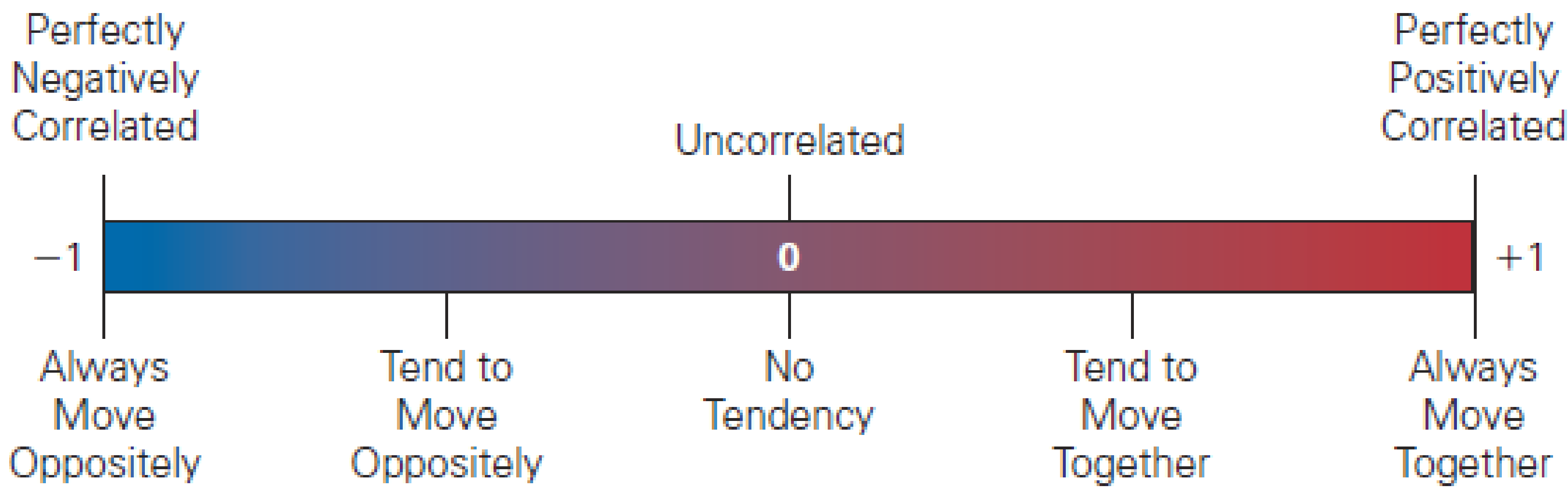
$$Cov(R_i, R_j) = \sigma_{i,j} = \frac{1}{T-1}\sum_{t=1}^{T}(R_{i,t} - \bar{R}_i)(Rj_{,t} - \bar{R}_j)$$

- While the sign of covariance is easy to interpret, its magnitude is not. In practice, correlation is thus used more often.

$$Corr(R_i, R_j) = \rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$$

# Portfolio risk and return

- Coefficient of correlation




*Source: Berk and DeMarzo (2019)*

# Portfolio risk and return

| Year | Stocks | Deviation | Dev squared | Corporate bonds | Deviation | Dev squared | Stock dev x Bond dev |
|---|---|---|---|---|---|---|---|
| 2002 | 20.27% | 2.74% | 0.08% | 9.61% | 4.89% | 0.24% | 0.13% |
| 2003 | 126.35% | 108.83% | 118.44% | 2.21% | -2.51% | 0.06% | -2.73% |
| 2004 | -10.64% | -28.17% | 7.93% | 4.04% | -0.68% | 0.00% | 0.19% |
| 2005 | 11.22% | -6.31% | 0.40% | 1.17% | -3.55% | 0.13% | 0.22% |
| 2006 | -0.26% | -17.78% | 3.16% | 6.58% | 1.85% | 0.03% | -0.33% |
| 2007 | 31.37% | 13.85% | 1.92% | 7.32% | 2.60% | 0.07% | 0.36% |
| 2008 | -45.10% | -62.62% | 39.21% | 8.05% | 3.33% | 0.11% | -2.09% |
| 2009 | 71.35% | 53.82% | 28.97% | 4.08% | -0.65% | 0.00% | -0.35% |
| 2010 | 47.80% | 30.28% | 9.17% | 5.26% | 0.53% | 0.00% | 0.16% |
| 2011 | 3.69% | -13.83% | 1.91% | 3.97% | -0.75% | 0.01% | 0.10% |
| 2012 | 40.53% | 23.01% | 5.29% | 4.56% | -0.16% | 0.00% | -0.04% |
| 2013 | -3.63% | -21.15% | 4.47% | 3.90% | -0.82% | 0.01% | 0.17% |
| 2014 | 19.12% | 1.60% | 0.03% | 6.68% | 1.96% | 0.04% | 0.03% |
| 2015 | -11.23% | -28.76% | 8.27% | 4.88% | 0.16% | 0.00% | -0.04% |
| 2016 | 23.85% | 6.32% | 0.40% | 2.51% | -2.21% | 0.05% | -0.14% |
| 2017 | 17.30% | -0.23% | 0.00% | 3.83% | -0.89% | 0.01% | 0.00% |
| 2018 | -8.08% | -25.61% | 6.56% | 2.29% | -2.43% | 0.06% | 0.62% |
| 2019 | 4.29% | -13.23% | 1.75% | 5.70% | 0.98% | 0.01% | -0.13% |
| 2020 | -5.24% | -22.77% | 5.18% | 3.08% | -1.64% | 0.03% | 0.37% |
| | **17.52%** | Sum | 243.14% | **4.72%** | Sum | 0.86% | -3.47% |
| | | Variance | 13.51% | | Variance | 0.05% | |
| | | **SD** | **36.75%** | | **SD** | **2.18%** | |
| | | | | | | Covariance | -0.19% |
| | | | | | | **Correlation** | **-24.05%** |

*Source: ThaiBMA, Datastream, Author's calculation*

# Portfolio risk and return

| | SET | DM | EM | Corp bonds | T-bills | Real estate | Commodity |
|---|---|---|---|---|---|---|---|
| SET | 1.0000 | | | | | | |
| Developed markets | 0.4152 | 1.0000 | | | | | |
| Emerging markets | 0.6704 | 0.7271 | 1.0000 | | | | |
| Corporate bonds | -0.2405 | -0.5921 | -0.4362 | 1.0000 | | | |
| T-bills | -0.2326 | -0.2555 | -0.1739 | 0.3633 | 1.0000 | | |
| Real estate | 0.4998 | 0.7627 | 0.7396 | -0.4167 | -0.0831 | 1.0000 | |
| Commodity | 0.4215 | 0.3146 | 0.5399 | -0.2014 | -0.0447 | 0.3538 | 1.0000 |

*Source: ThaiBMA, Datastream, Author's calculation*

# Portfolio risk and return

- For two risky assets,

$$
\begin{aligned}
Var(R_p) &= Cov(R_p, R_p) = Cov(w_1 R_1 + w_2 R_2, w_1 R_1 + w_2 R_2) \\
&= w_1 w_1 Cov(R_1, R_1) + w_1 w_2 Cov(R_1, R_2) + \\
&\quad w_2 w_1 Cov(R_2, R_1) + w_2 w_2 Cov(R_2, R_2) \\
&= w_1 w_1 \sigma_{1,1} + w_1 w_2 \sigma_{1,2} + w_2 w_1 \sigma_{2,1} + w_2 w_2 \sigma_{2,2} \\
&= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{1,2} \\
&= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{i,j} \sigma_1 \sigma_2
\end{aligned}
$$

# Portfolio risk and return

- Based on the previous stock and bond table, what is portfolio risk and return of 70/30 (stock/bond) portfolio?

# Portfolio risk and return

- For three risky assets,

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3)$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2$$
$$+2w_1w_2\sigma_{1,2} + 2w_1w_3\sigma_{1,3} + 2w_2w_3\sigma_{2,3}$$

- For *N* assets,

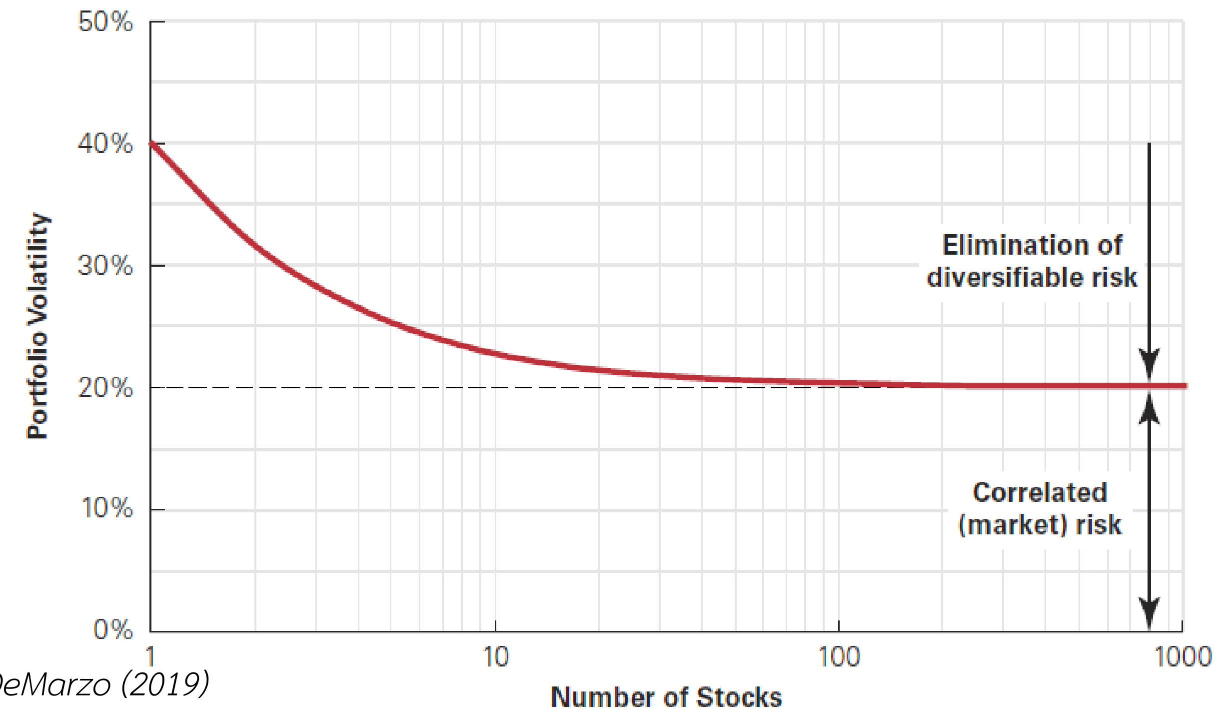
$$E[R_p] = E\left[\sum_i w_i R_i\right] = \sum_i E[w_i R_i] = \sum_i w_i E[R_i]$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{i,j}$$

# Portfolio risk and return

- To calculate the variance of an equal-weighted portfolio consisting of *N* stocks, we can use the following formula.

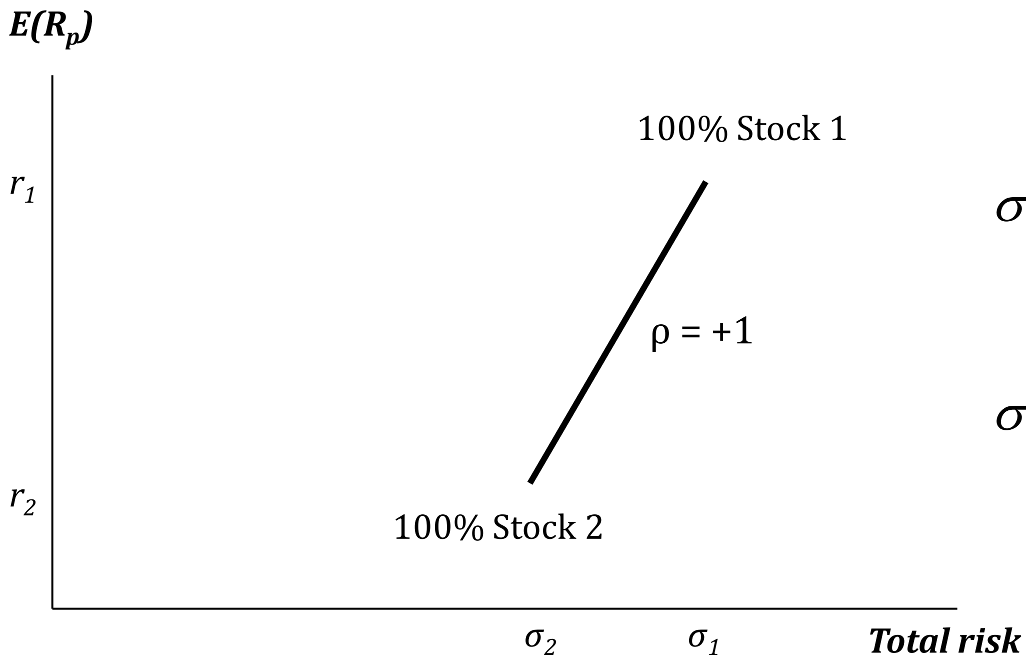
$$\sigma_p^2 = \frac{1}{N}(Average\ variance\ of\ the\ individual\ stocks) + \left(1 - \frac{1}{N}\right)(Average\ covariance\ btween\ the\ stocks)$$




*Source: Berk and DeMarzo (2019)*

# Portfolios of two risky assets

- When the correlation between two risky assets is perfectly positive (rho = 1), there are no gains from diversification or no risk reduction is possible.



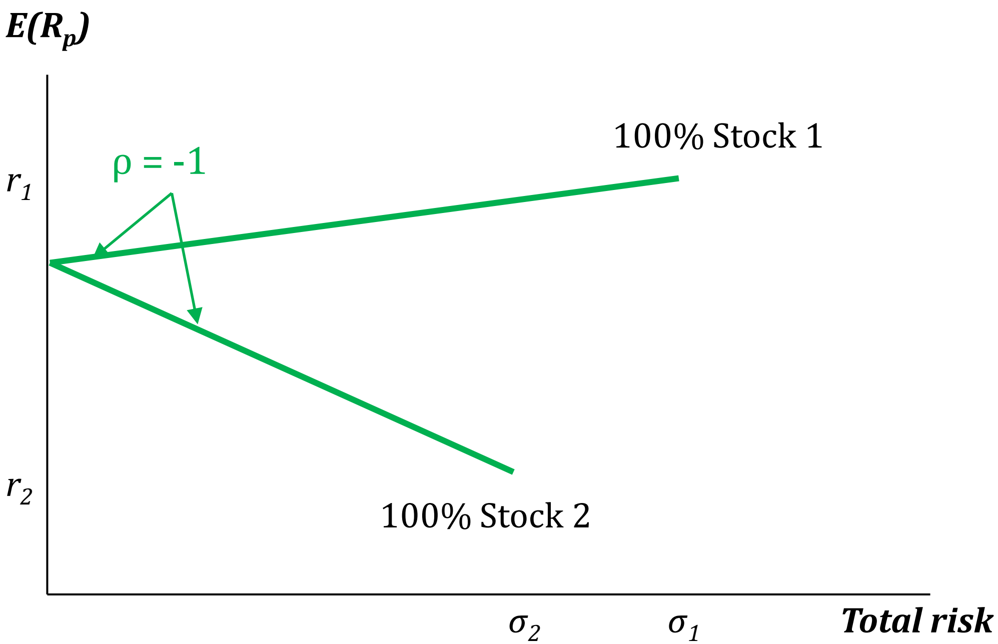

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2(1)$$

$$= \left(w_1\sigma_1 + w_2\sigma_2\right)^2$$

$$\sigma_p = w_1\sigma_1 + w_2\sigma_2$$

# Portfolios of two risky assets

- When the correlation between two risky assets is perfectly negative (rho = -1), risk can be totally eliminated via diversification (the standard deviation of the minimum variance portfolio is zero).

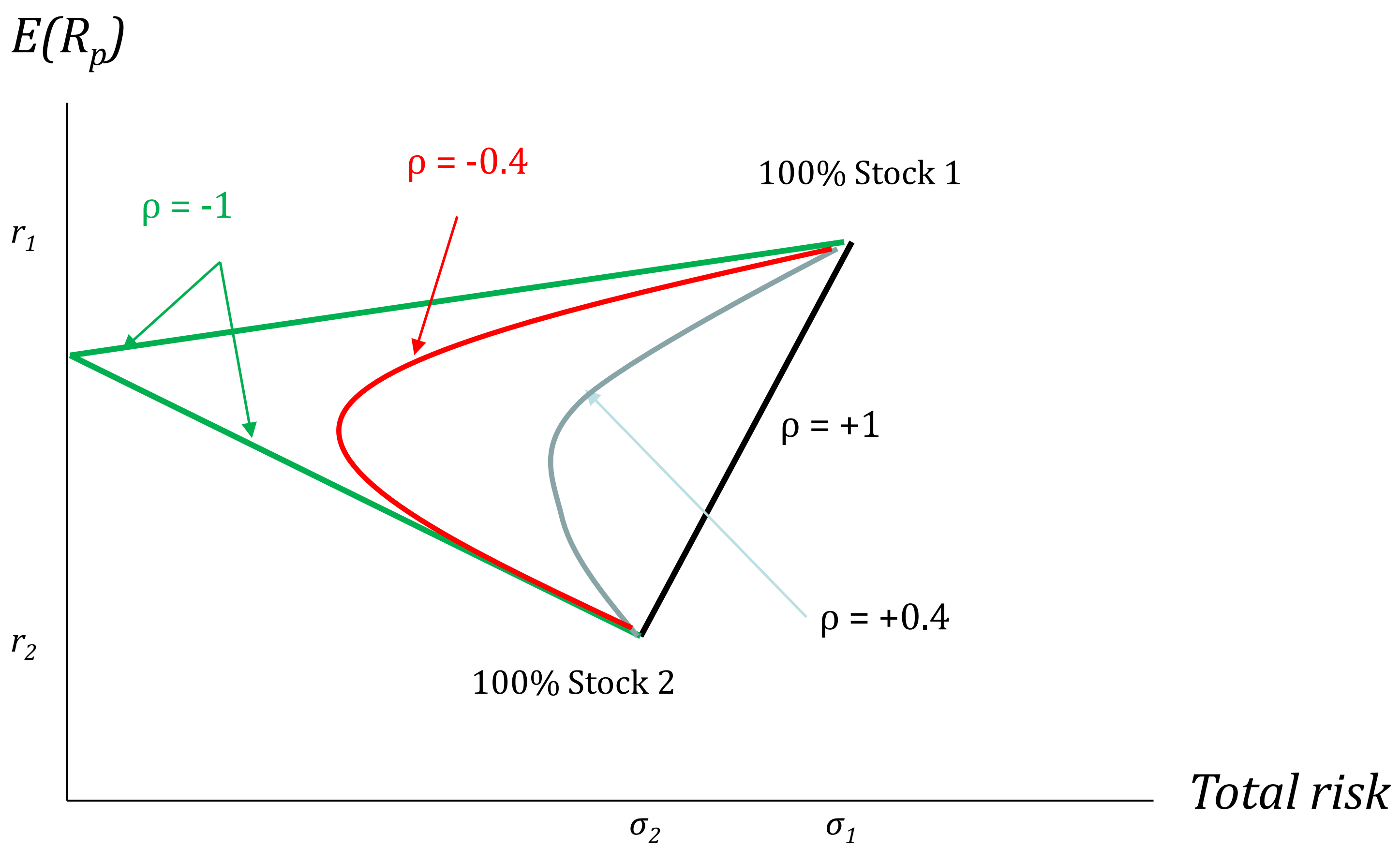



$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2(-1)$$

$$= \left(w_1\sigma_1 - w_2\sigma_2\right)^2$$

$$\sigma_p = \left|w_1\sigma_1 - w_2\sigma_2\right|$$

# Portfolios of two risky assets



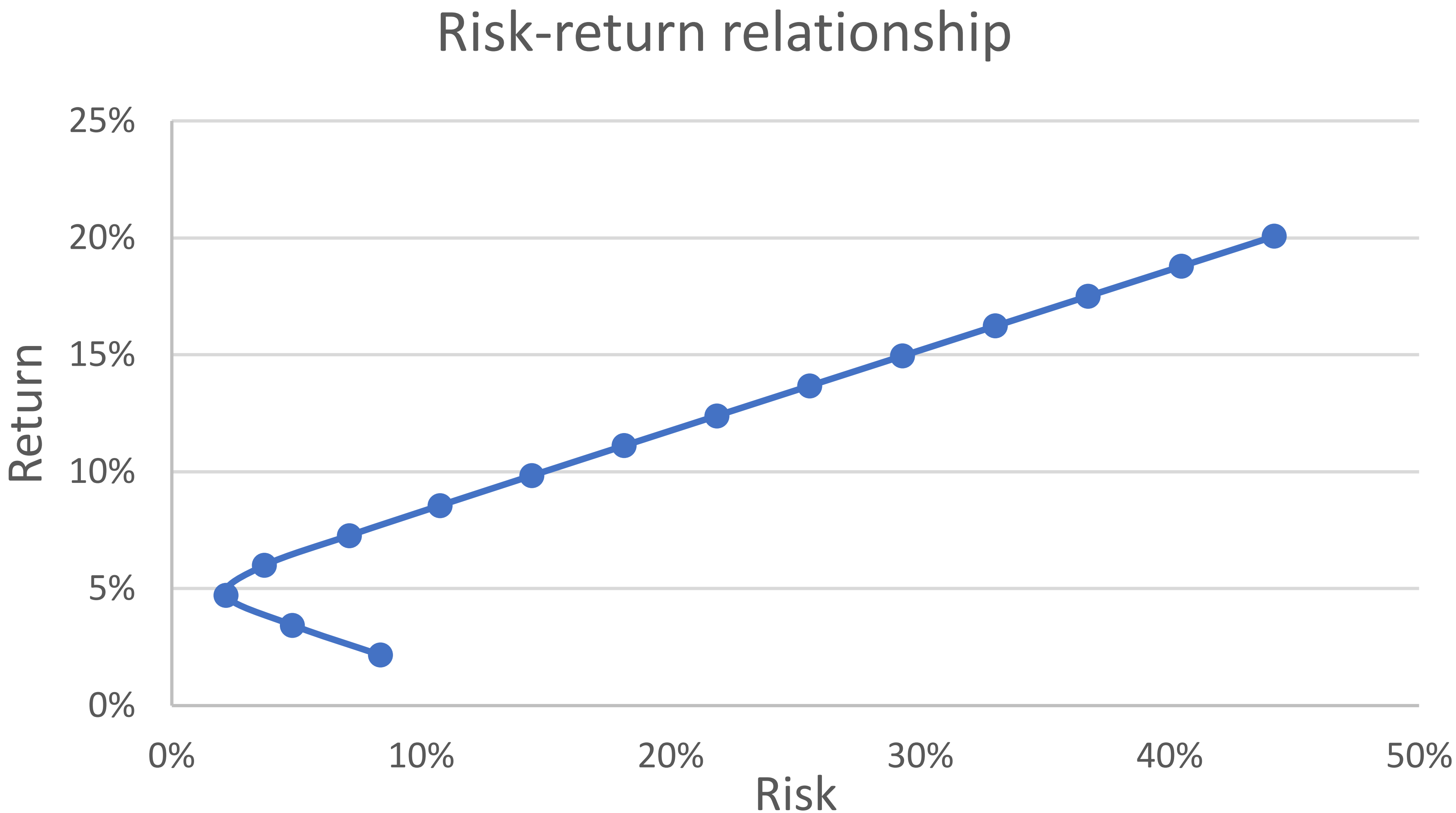
# Portfolios of two risky assets

| $E(r_S)$ | $E(r_B)$ | $\sigma_S$ | $\sigma_B$ | $\rho_{BS}$ |
|---|---|---|---|---|
| 17.52% | 4.72% | 36.75% | 2.18% | -24.05% |

| Portfolio Weights | | Expected Return, $E(r_P)$ | Std Dev |
|---|---|---|---|
| $w_S = 1 - w_B$ | $w_B$ | | |
| -20% | 120% | 2.16% | 8.37% |
| -10% | 110% | 3.44% | 4.85% |
| 0% | 100% | 4.72% | 2.18% |
| 10% | 90% | 6.00% | 3.73% |
| 20% | 80% | 7.28% | 7.13% |
| 30% | 70% | 8.56% | 10.76% |
| 40% | 60% | 9.84% | 14.44% |
| 50% | 50% | 11.12% | 18.15% |
| 60% | 40% | 12.40% | 21.86% |
| 70% | 30% | 13.68% | 25.58% |
| 80% | 20% | 14.96% | 29.30% |
| 90% | 10% | 16.24% | 33.03% |
| 100% | 0% | 17.52% | 36.75% |
| 110% | -10% | 18.80% | 40.48% |
| 120% | -20% | 20.08% | 44.21% |

*Source: ThaiBMA, Datastream, Author's calculation*

# Portfolios of two risky assets




*Source: ThaiBMA, Datastream, Author's calculation*

# Portfolios of two risky assets

- What is the minimum level to which portfolio SD can be held? For the parameter values stipulated in the previous table, the portfolio weights that solve this minimization problem turn out to be

$$w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}, w_1 + w_2 = 1$$

# Portfolios of two risky assets

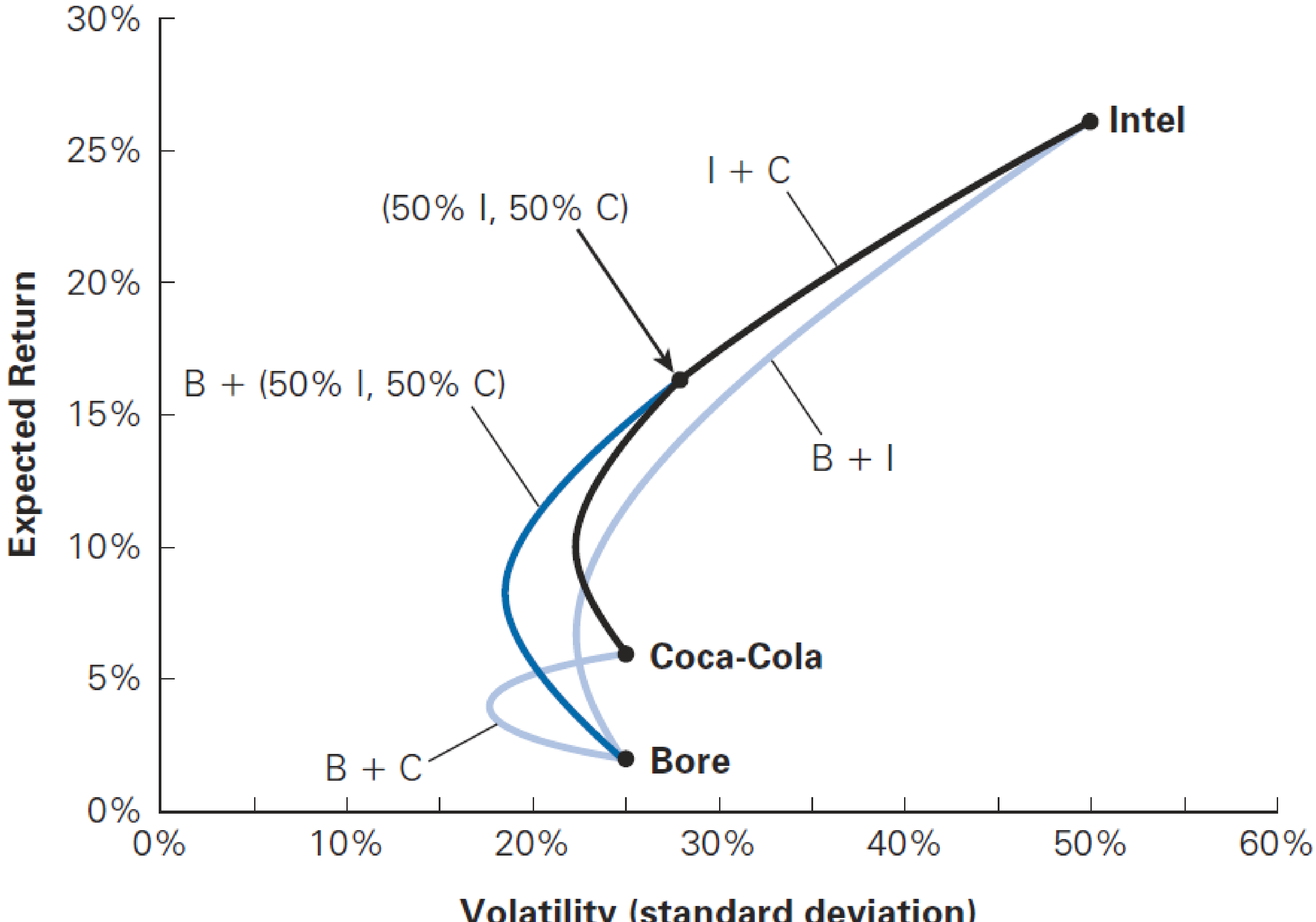
| $E(r_S)$ | $E(r_B)$ | $\sigma_S$ | $\sigma_B$ |
|---|---|---|---|
| 17.52% | 4.72% | 36.75% | 2.18% |

| Weight in stocks $w_S$ | Portoflio Expected return | Portfolio Standard Deviation for Given Correlation, ρ | | | | |
|---|---|---|---|---|---|---|
| | | -1 | 0 | 0.2 | 0.5 | 1 |
| 0% | 4.72% | 2.18% | 2.18% | 2.18% | 2.18% | 2.18% |
| 5% | 5.36% | 0.23% | 2.77% | 3.03% | 3.39% | 3.91% |
| 10% | 6.00% | 1.71% | 4.17% | 4.50% | 4.96% | 5.64% |
| 20% | 7.28% | 5.61% | 7.55% | 7.89% | 8.36% | 9.10% |
| 30% | 8.56% | 9.50% | 11.13% | 11.43% | 11.86% | 12.55% |
| 40% | 9.84% | 13.39% | 14.76% | 15.02% | 15.40% | 16.01% |
| 60% | 12.40% | 21.18% | 22.07% | 22.24% | 22.50% | 22.92% |
| 80% | 14.96% | 28.97% | 29.41% | 29.49% | 29.62% | 29.84% |
| 100% | 17.52% | 36.75% | 36.75% | 36.75% | 36.75% | 36.75% |
| **Minimum Variance Portfolio** | | | | | | |
| | Stock weight | 5.60% | 0.35% | -0.85% | -2.77% | -6.31% |
| | Expected return | 5.44% | 4.77% | 4.61% | 4.37% | 3.91% |
| | Standard deviation | 0.00% | 2.18% | 2.16% | 1.94% | 0.00% |

*Source: ThaiBMA, Datastream, Author's calculation*

# Portfolios of *N* risky assets

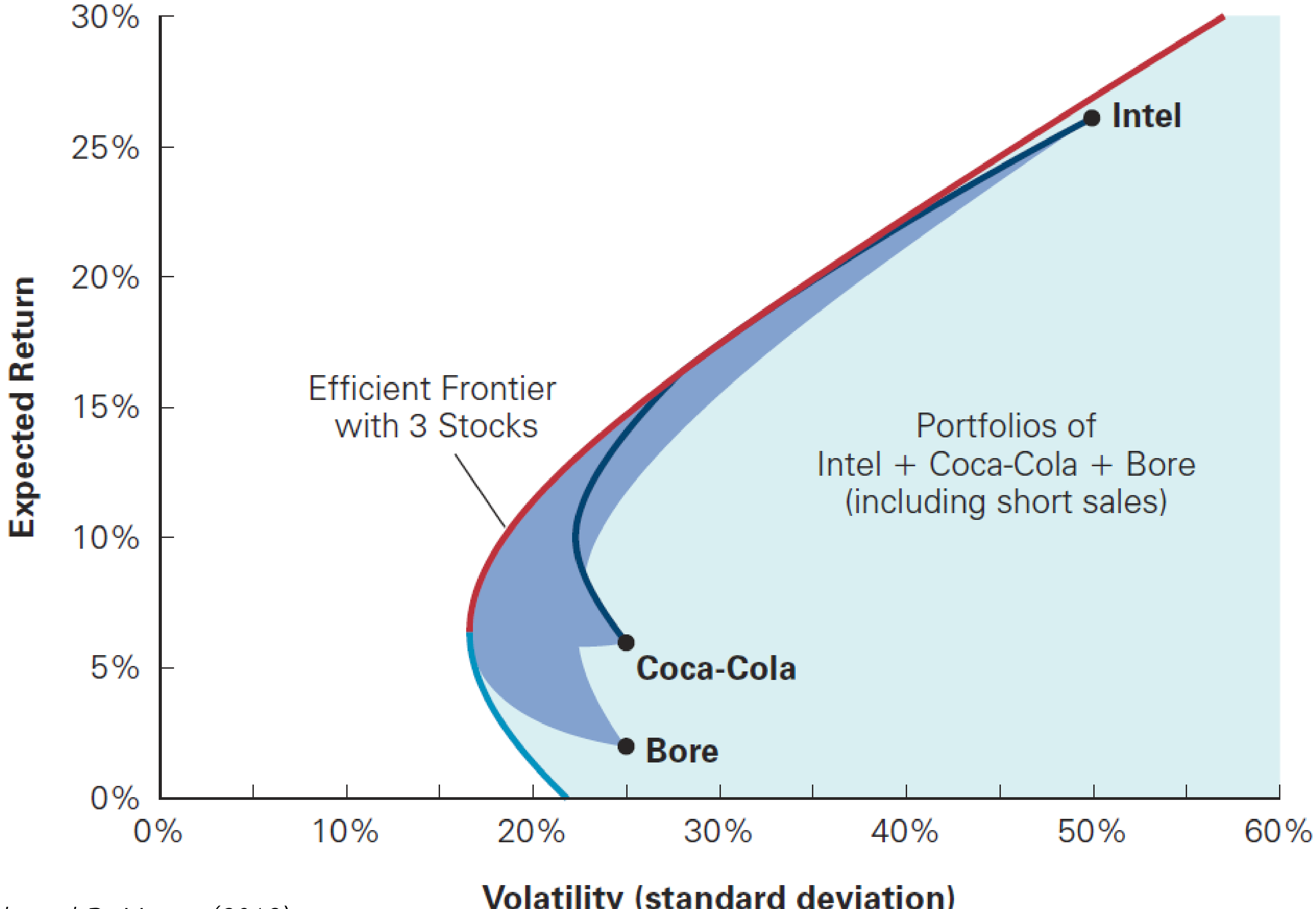
- For three risky assets,




*Source: Berk and DeMarzo (2019)*

# Portfolios of *N* risky assets

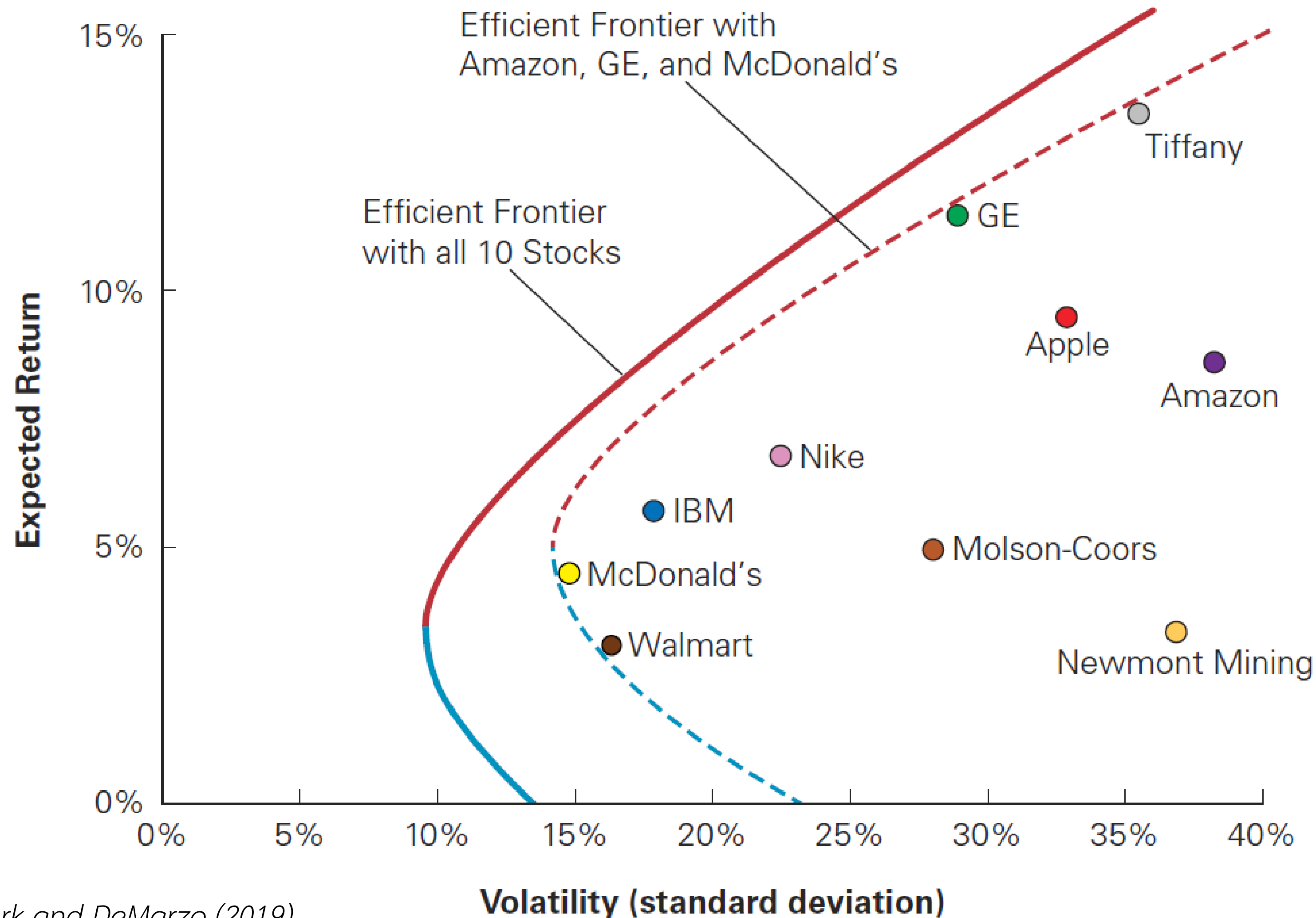
- For three risky assets,




*Source: Berk and DeMarzo (2019)*

# Portfolios of *N* risky assets

- For 10 risky assets,




*Source: Berk and DeMarzo (2019)*

# Risk aversion

THAMMASAT BUSINESS SCHOOL

How should investors allocate among risky assets?

- Investors with different levels of risk aversion will choose different positions in the risky asset
- Assume that each investor can assign utility score to competing investment portfolios on the basis of risk-return trade-off
- Portfolios with higher returns will receive higher utility score, and those with higher risk will receive lower utility score.

$$U = E(r) - \frac{1}{2}A\sigma^2$$

where $U$ is utility of investor, and

$A$ is risk-aversion factor (or coefficient of risk aversion)

# Risk aversion

| Investor Risk Aversion ($A$) | Utility Score of Portfolio $L$ [$E(r) = .07; \sigma = .05$] | Utility Score of Portfolio $M$ [$E(r) = .09; \sigma = .10$] | Utility Score of Portfolio $H$ [$E(r) = .13; \sigma = .20$] |
|---|---|---|---|
| 2.0 | $.07 - ½ \times 2 \times .05^2 = .0675$ | $.09 - ½ \times 2 \times .1^2 = .0800$ | **$.13 - ½ \times 2 \times .2^2 = .09$** |
| 3.5 | $.07 - ½ \times 3.5 \times .05^2 = .0656$ | **$.09 - ½ \times 3.5 \times .1^2 = .0725$** | $.13 - ½ \times 3.5 \times .2^2 = .06$ |
| 5.0 | $.07 - ½ \times 5 \times .05^2 = .0638$ | **$.09 - ½ \times 5 \times .1^2 = .0650$** | $.13 - ½ \times 5 \times .2^2 = .03$ |

**Table 6.2**

Utility scores of alternative portfolios for investors with varying degrees of risk aversion

*Source: Bodie et al. (2020)*

# Risk aversion

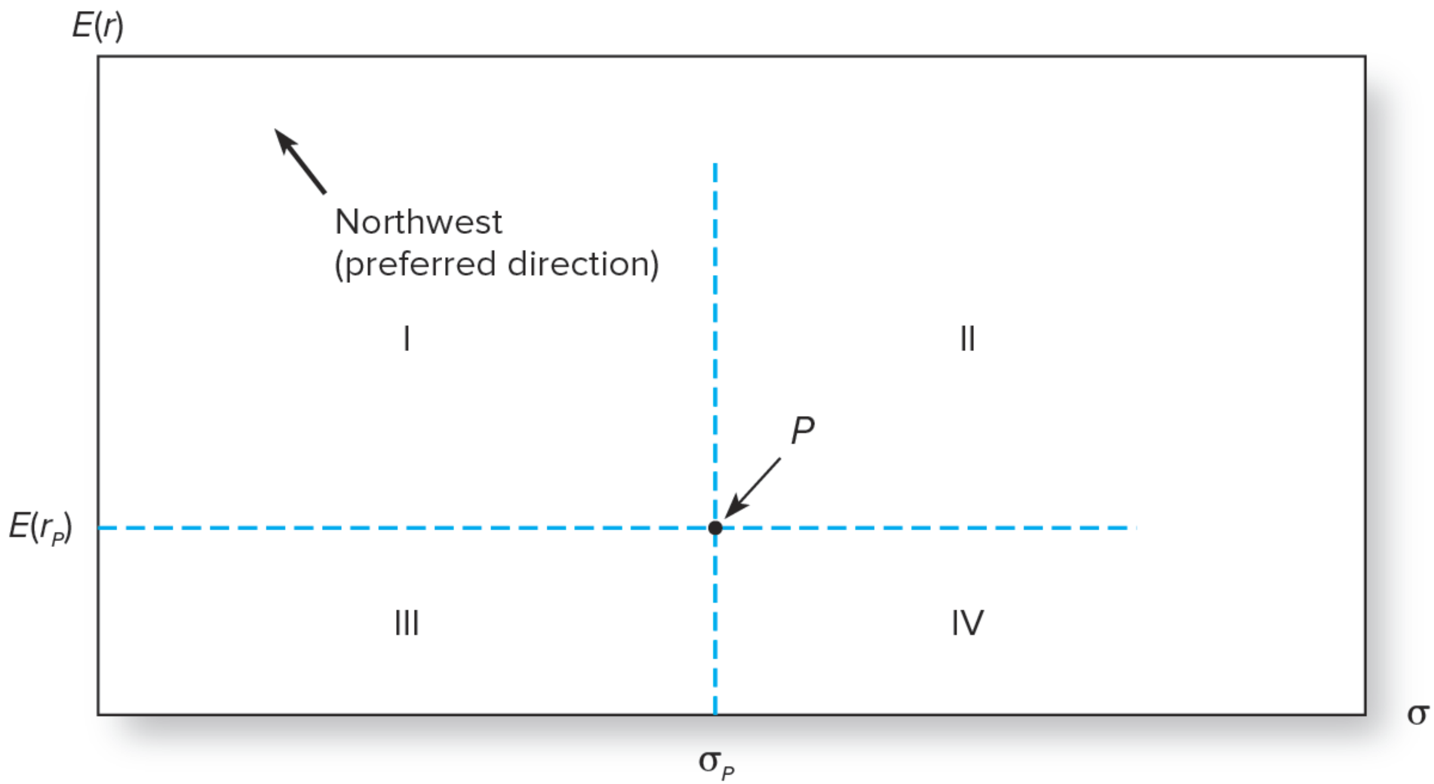



**Figure 6.1** The trade-off between risk and return of a potential investment portfolio, *P*

*Source: Bodie et al. (2020)*

# Risk aversion

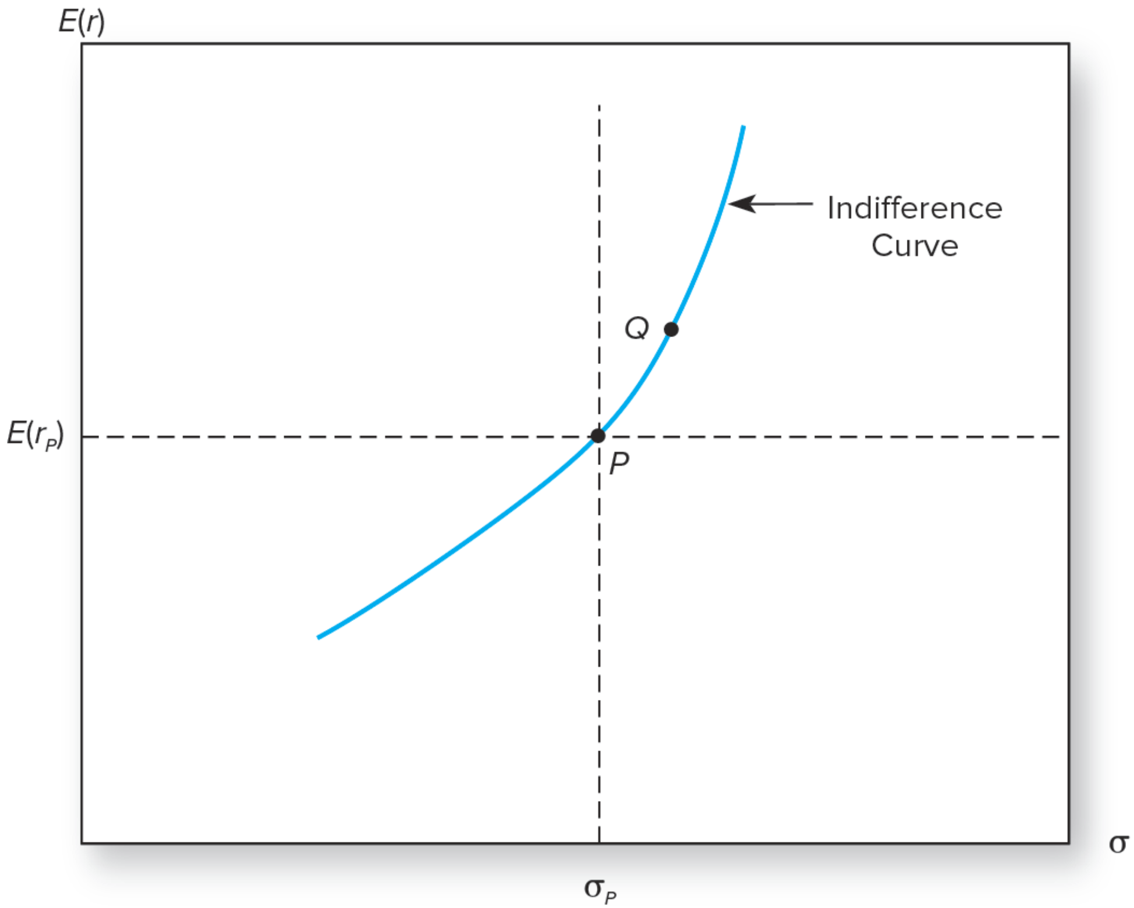



**Figure 6.2** The indifference curve

*Source: Bodie et al. (2020)*

# Risk aversion

| Expected Return, $E(r)$ | Standard Deviation, $\sigma$ | Utility = $E(r) - \frac{1}{2} A\sigma^2$ |
| --- | --- | --- |
| 0.10 | 0.200 | $0.10 - 0.5 \times 4 \times 0.04 = 0.02$ |
| 0.15 | 0.255 | $0.15 - 0.5 \times 4 \times 0.065 = 0.02$ |
| 0.20 | 0.300 | $0.20 - 0.5 \times 4 \times 0.09 = 0.02$ |
| 0.25 | 0.339 | $0.25 - 0.5 \times 4 \times 0.115 = 0.02$ |

*Source: Bodie et al. (2020)*

# Risk aversion and portfolio allocation