FN241
Risk Management and Insurance
Insurance portfolio management

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Bond Pricing

- Bond value = Present value of coupon + Present par value

- Bond value = $\sum_{t=1}^{T} \frac{Coupon}{(1+r)^t} + \frac{Par\ value}{(1+r)^T}$
    - T = Maturity date
    - r = discount rate

Interest Rate Risk

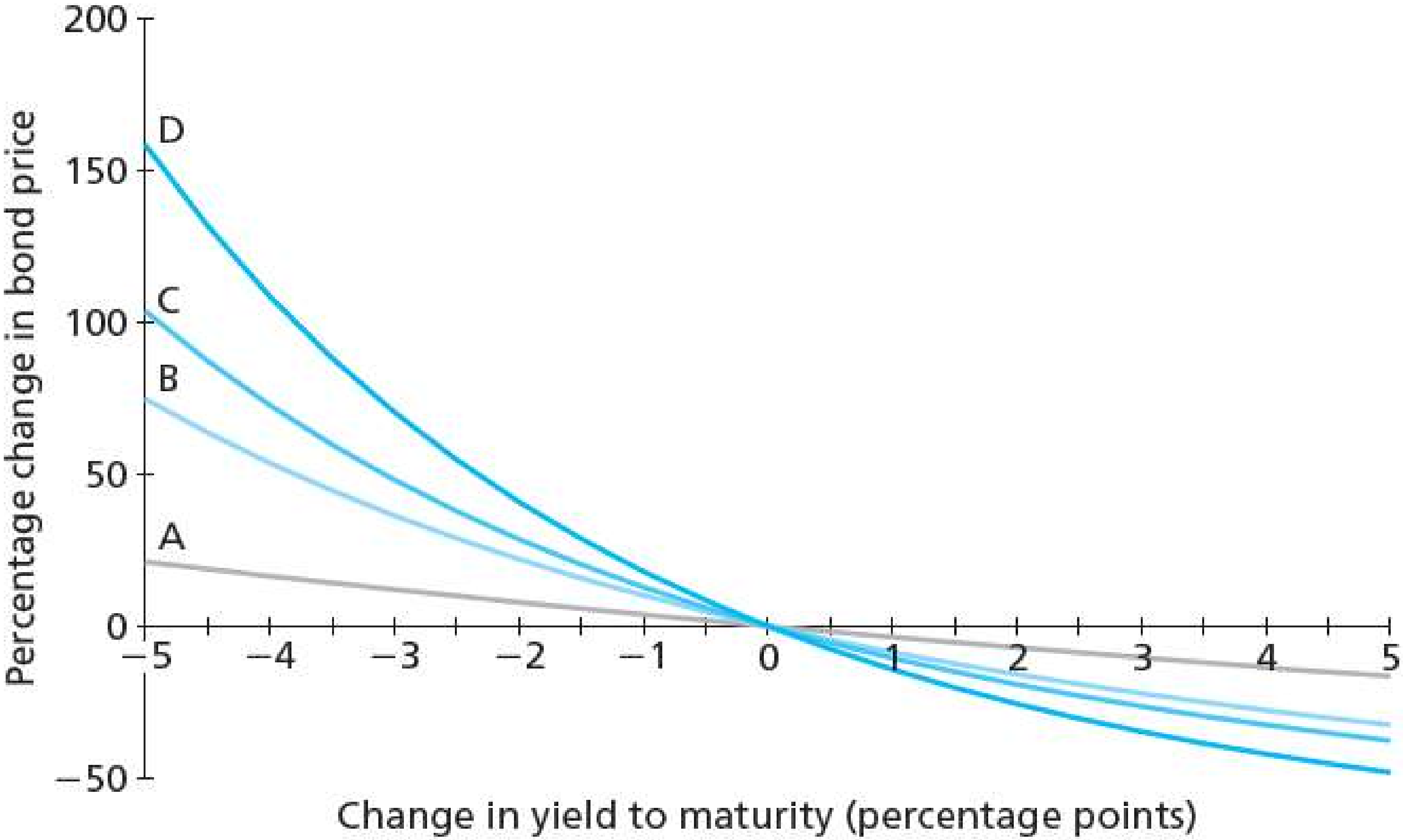
- There is an **inverse relationship between bond prices and yields** and that interest rates can fluctuate substantially
  - e.g. if the market rate rises to 9%, who would purchase an 8% coupon bond at par value?

- Interest rate sensitivity

  (1) Bond prices and yields are inversely related

  (2) Increase in bond’s YTM results in smaller price change than yield decrease of equal magnitude

Change in Bond Prices as a Function of Change in Yield to Maturity



Bond	Coupon	Maturity	Initial YTM
A	12%	5 years	10%
B	12	30	10
C	3	30	10
D	3	30	6

Source: BKM Figure 11.1

Annual Coupon Prices

Prices of 8% annual coupon bonds			
Bond's Yield to Maturity	***T* = 1 Year**	***T* = 10 Years**	***T* = 20 Years**
8%	1,000.00	1,000.00	1,000.00
9%	990.83	935.82	908.71
Percent change in price*	−0.92%	−6.42%	−9.13%

*Equals value of bond at a 9% yield to maturity minus value of bond at (the original) 8% yield, divided by the value at 8% yield.

- The shortest term bond falls in value by less than 1% while the 20-yr bond falls for 9%

Source: BKM Table 11.1

Zero-Coupon Bond Prices

Prices of zero-coupon bonds			
Bond's Yield to Maturity	***T* = 1 Year**	***T* = 10 Years**	***T* = 20 Years**
8%	925.93	463.19	214.55
9%	917.43	422.41	178.43
Percent change in price*	−0.92%	−8.80%	−16.84%

*Equals value of bond at a 9% yield to maturity minus value of bond at (the original) 8% yield, divided by the value at 8% yield.

- For maturities beyond 1 year, the price of **the zero-coupon bond falls by a greater proportional amount** than the price of the 8% coupon bond (in the previous slide)
- This suggests that a zero-coupon bond must represent a longer-term investment

Source: BKM Table 11.2

Interest Rate Risk

- Time to maturity (TTM) of the 2 bonds are not perfect measures of the long- or short-term nature of the bonds
  - Each coupon payment may be considered to have its own "maturity"
  - The **effective maturity** of the bond would be measured as **an average of the maturities of all the CFs**
- A high-coupon rate bond has a higher fraction of its value tied to coupons rather than payment of par value
  - That's why the price sensitivity falls when coupon rate is high
- Also, as YTM increases → **lower** effective maturity → sensitivity falls when YTM is high

Interest Rate Risk

- **Macaulay's duration**
  - A measure of the effective maturity of a bond
  - The weighted average of the times until each payment is received, with the weights proportional to the PV of the payment
  - Duration is shorter than maturity for all bonds **except** zero-coupon bonds
  - Duration is equal to maturity for zero-coupon bonds

$$D = \frac{\sum_{t=1}^{T} \frac{t \times CF_t}{(1+y)^t} + \frac{T \times Par}{(1+y)^T}}{P}$$

Calculation of Duration of 2 Bonds

Interest rate:	10%				
	Time until Payment (Years)	**Payment**	**Payment Discounted at 10%**	**Weight***	**Column (B) x Column (E)**
A. 8% coupon bond	1	80	72.727	0.0765	0.0765
	2	80	66.116	0.0696	0.1392
	3	1080	811.420	0.8539	2.5617
Sum:			950.263	1.0000	2.7774
B. Zero-coupon bond	1	0	0.000	0.0000	0.0000
	2	0	0.000	0.0000	0.0000
	3	1000	751.315	1.0000	3.0000
Sum:			751.315	1.0000	3.0000
*Weight = Present value of each payment (column D) divided by bond price					

Source: BKM Spreadsheet 11.1

Calculation of Duration of 2 Bonds

Interest rate:	0.1				
	Time until Payment (Years)	**Payment**	**Payment Discounted at 10%**	**Weight**	**Column (B) times Column (E)**
A. 8% coupon bond	1	80	=C6/(1+$B$1)^B6	=D6/D$9	=E6*B6
	2	80	=C7/(1+$B$1)^B7	=D7/D$9	=E7*B7
	3	1080	=C8/(1+$B$1)^B8	=D8/D$9	=E8*B8
Sum:			=SUM(D6:D8)	=D9/D$9	=SUM(F6:F8)
B. Zero-coupon	1	0	=C11/(1+$B$1)^B11	=D11/D$14	=E11*B11
	2	0	=C12/(1+$B$1)^B12	=D12/D$14	=E12*B12
	3	1000	=C13/(1+$B$1)^B13	=D13/D$14	=E13*B13
Sum:			=SUM(D11:D13)	=D14/D$14	=SUM(F11:F13)

Source: BKM Spreadsheet 11.1

Interest Rate Risk

- Recall the bond price equation:

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} + \frac{Par}{(1+y)^T}$$

- The 1st derivative of *P* wrt *y*:

$$\frac{\partial P}{\partial y} = -\frac{1}{(1+y)}\left[\sum_{t=1}^{T} \frac{t \times CF_t}{(1+y)^t} + \frac{T \times Par}{(1+y)^T}\right]$$

Multiply the RHS with *P/P*,

$$\frac{\partial P}{\partial y} = -\frac{1}{(1+y)}\left[\sum_{t=1}^{T} \frac{t \times CF_t}{(1+y)^t} + \frac{T \times Par}{(1+y)^T}\right]\frac{P}{P}$$

Interest Rate Risk

- Then we can write: $\frac{\partial P}{\partial y} = -\frac{1}{(1+y)} DP$

where $D = \frac{\sum_{t=1}^{T} \frac{t \times CF_t}{(1+y)^t} + \frac{T \times Par}{(1+y)^T}}{P}$

or $\frac{\partial P}{\partial y} = -D^* P$ where $D^* = \frac{D}{(1+y)}$

- The estimate proportional change in a bond's price is

$$\Delta P = -\frac{D \times \Delta y}{(1+y)} P \quad \text{or} \quad \Delta P = -D^* \Delta y P$$

D = Macaulay's duration and *D** = Modified duration

Interest Rate Risk – An Example

- A bond with maturity of 30 years has a coupon rate of 8% (paid annually) and a YTM of 9%. Its price is 897.26 Baht, and its (Macaulay's) duration is 11.37 years. What will happen to the bond price if the bond's YTM increases to 9.1%?

Interest Rate Risk – An Example

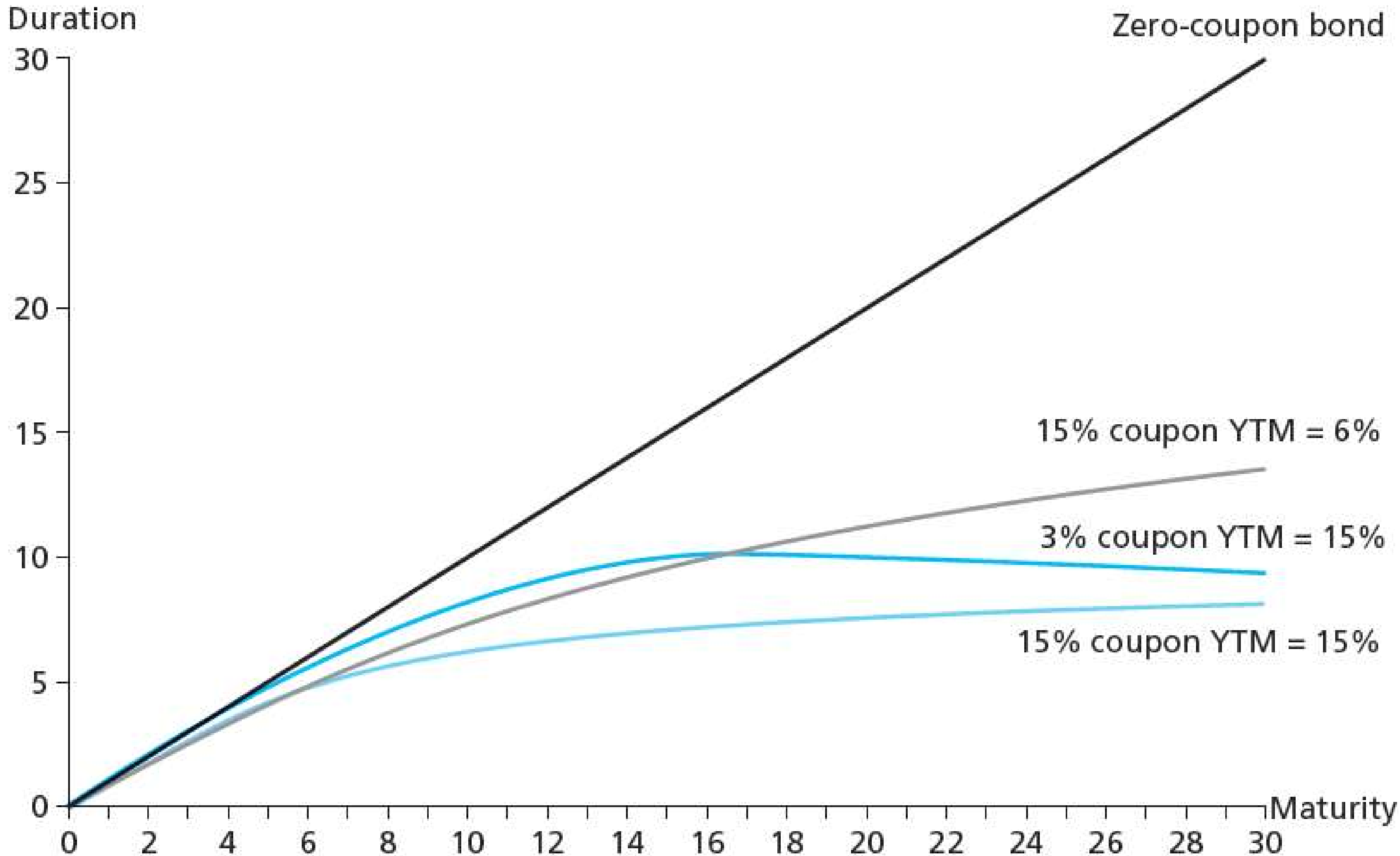
- Go back to Spreadsheet 11.1 and change the YTM from 10% to 10.1%
  - The new 8% bond price is $947.868 (difference of -2.395)
  - And the new zero-coupon bond price is $749.269 (difference of -2.046)

- If we use the estimate formula, we get

$$\Delta P = -\frac{D \times \Delta y}{(1+y)} P = -\frac{2.7774 \times (0.001)}{(1.10)} \times 950.263 = -2.399$$

which is very close to the real difference

Duration as Function of Maturity



Source: BKM Figure 11.2

Interest Rate Risk

- What Determines Duration?
  (1) Zero-coupon bond's duration is time to maturity
  (2) Duration is high when TTM increases
  (3) Duration is high when coupon rate is low
  (4) Duration is high when YTM is low
  (5) (Macaulay's) duration of a level perpetuity equals $\frac{1+y}{y}$

Annual Coupon Bond Duration

Durations of annual coupon bonds (initial bond yield = 6%)				
	Coupon Rates (% per year)			
Years to Maturity	**4%**	**6%**	**8%**	**10%**
1	1.000	1.000	1.000	1.000
5	4.611	4.465	4.342	4.237
10	8.281	7.802	7.445	7.169
20	13.216	12.158	11.495	11.041
Infinite (perpetuity)	17.667	17.667	17.667	17.667

Source: BKM Figure 11.3

Passive Bond Management

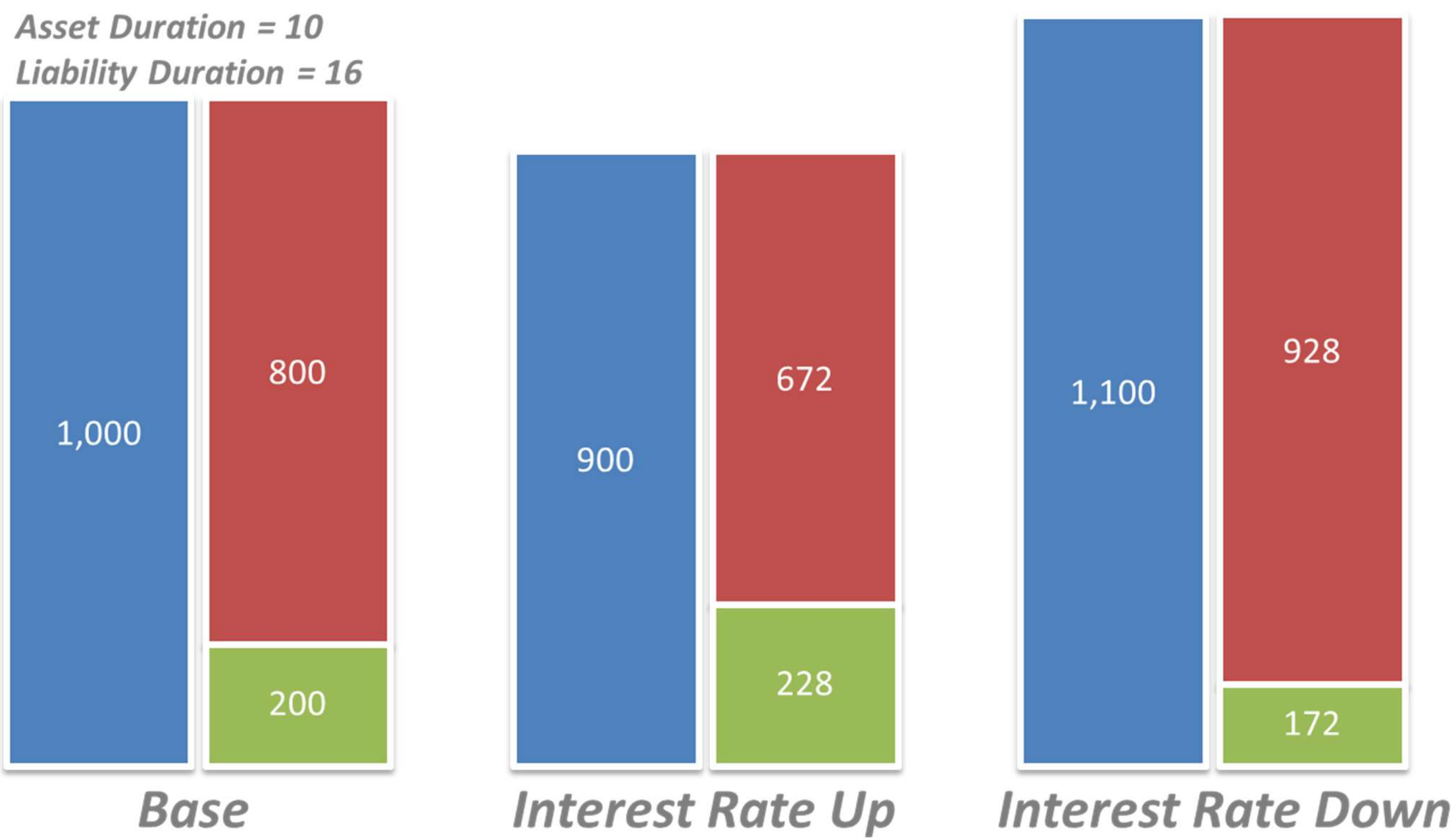
- **Immunization** - A strategy to shield net worth from interest rate movements
  - Widely used by pension funds, insurance companies, and banks
  - Immunize a portfolio by matching the interest rate exposure of assets and liabilities
    - Match the duration of the assets and liabilities
    - Price risk and reinvestment rate risk exactly cancel out
  - Result: Value of assets will track the value of liabilities whether rates rise or fall

Passive Bond Management

Balance sheet of a life insurance company

Assets	Liabilities
▪ Cash ▪ Loans ▪ Fixed income investment • Government bonds • Corporate bonds • Other bonds ▪ Equity investment ▪ Fixed assets ▪ Others	▪ Policyholders Reserve/ Technical Provision ▪ Unit linked / Separate account liabilities **Shareholders' Equity** ▪ Paid-up share capital ▪ Premium on share capital ▪ Retained Earnings

Passive Bond Management



Passive Bond Management

The situation:

- An insurance company issues a guaranteed investment contract to its customer for $10,000
- This product has a 5-yr maturity and guaranteed interest rate of 8%; the company, therefore, promises to pay

$$\$10{,}000 \times (1.08)^5 = \$14{,}693.28 \text{ in 5 years}$$

- Suppose that the company funds its obligation with $10,000 of 8% annual coupon bonds, selling at par value, with **6 years to maturity**

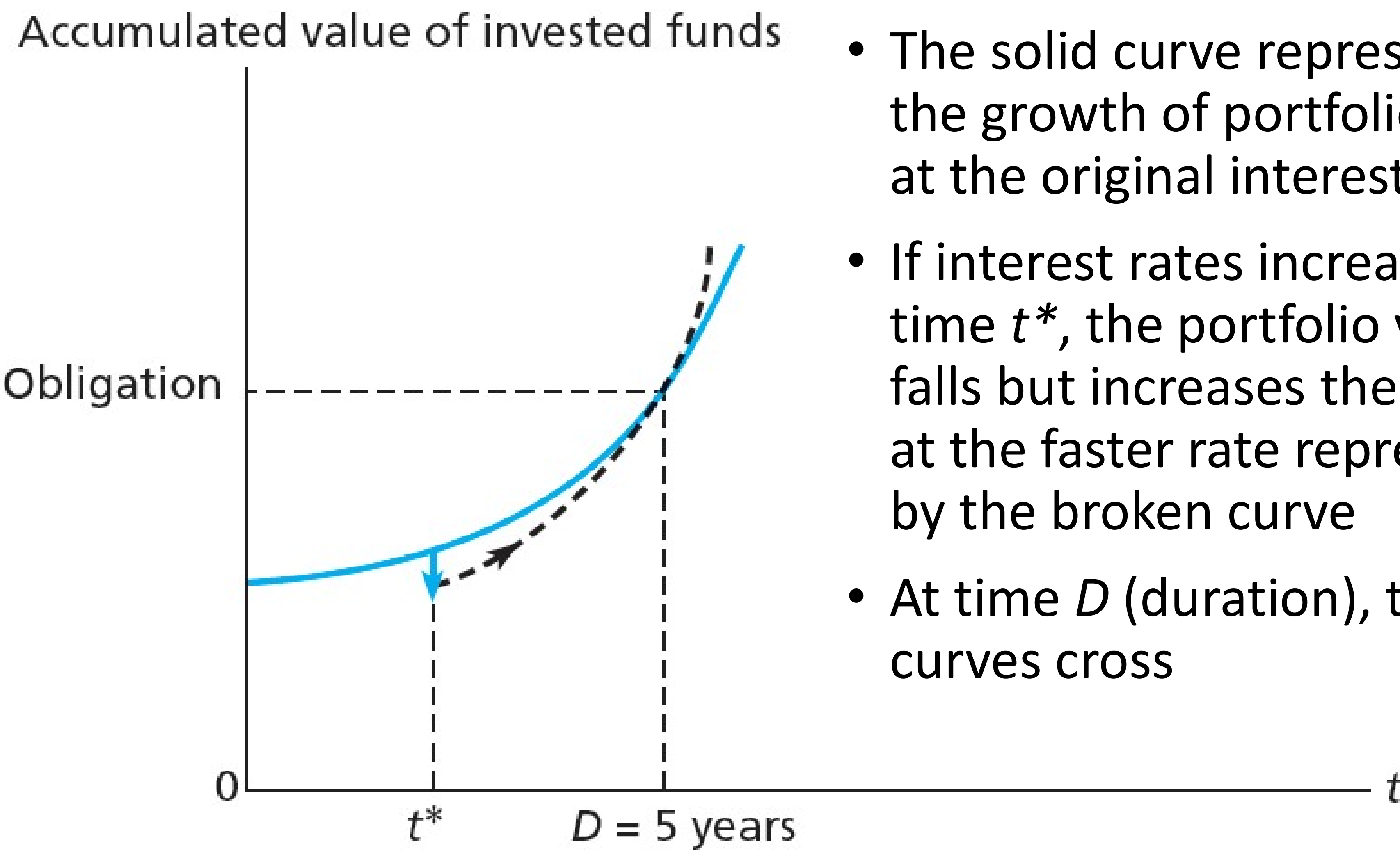
Terminal Value of Bond Portfolio after 5 Years

Payment Number	Years Remaining until Obligation	Accumulated Value of Invested Payment	
A. Rates remain at 8%			
1	4	$800 \times (1.08)^4 =$	1,088.39
2	3	$800 \times (1.08)^3 =$	1,007.77
3	2	$800 \times (1.08)^2 =$	933.12
4	1	$800 \times (1.08)^1 =$	864.00
5	0	$800 \times (1.08)^0 =$	800.00
Sale of bond	0	10,800/1.08 =	10,000.00
			14,693.28
B. Rates fall to 7%			
1	4	$800 \times (1.07)^4 =$	1,048.64
2	3	$800 \times (1.07)^3 =$	980.03
3	2	$800 \times (1.07)^2 =$	915.92
4	1	$800 \times (1.07)^1 =$	856.00
5	0	$800 \times (1.07)^0 =$	800.00
Sale of bond	0	10,800/1.07 =	10,093.46
			14,694.05
C. Rates increase to 9%			
1	4	$800 \times (1.09)^4 =$	1,129.27
2	3	$800 \times (1.09)^3 =$	1,036.02
3	2	$800 \times (1.09)^2 =$	950.48
4	1	$800 \times (1.09)^1 =$	872.00
5	0	$800 \times (1.09)^0 =$	800.00
Sale of bond	0	10,800/1.09 =	9,908.26
			14,696.02

Source: BKM Table 11.4

- Fixed-income investors face 2 types of interest rate risk: **price risk** and **reinvestment rate risk**
- If the portfolio duration is chosen appropriately, these two effects will cancel out
- In our example, the **duration** of the 6-yr bond is **5 years**

Growth of Invested Funds



- The solid curve represents the growth of portfolio value at the original interest rate
- If interest rates increase at time *t**, the portfolio value falls but increases thereafter at the faster rate represented by the broken curve
- At time *D* (duration), the curves cross

Source: BKM Figure 11.3

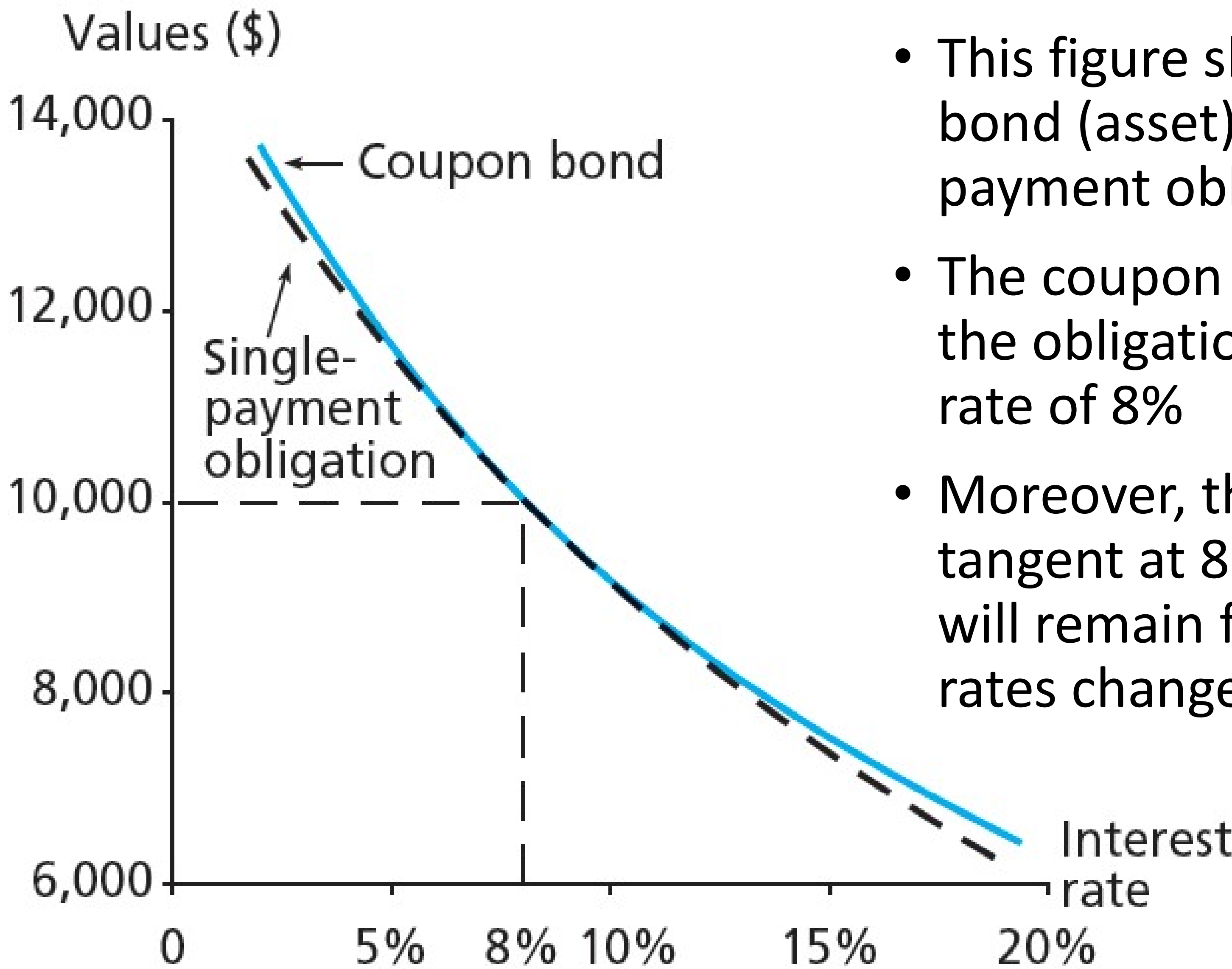
Market Value Balance Sheets

A. Interest rate = 8%			
Assets		**Liabilities**	
Bonds	$10,000	Obligation	$10,000
B. Interest rate = 7%			
Assets		**Liabilities**	
Bonds	$10,476.65	Obligation	$10,476.11
C. Interest rate = 9%			
Assets		**Liabilities**	
Bonds	$9,551.41	Obligation	$9,549.62

- Analyzed in terms of PV, assets equal liabilities even the interest rate fluctuates

Source: BKM Table 11.5

Immunization



- This figure shows the PV of the bond (asset) and the single payment obligation (liability)
- The coupon bond fully funds the obligation at an interest rate of 8%
- Moreover, the PV curves are tangent at 8%, so the obligation will remain fully funded even if rates change by a small amount

Source: BKM Figure 11.4

Passive Bond Management

- As interest rates and asset durations continually change, managers must adjust the portfolio to realign its duration with the duration of the obligation
- In fact, even if interest rates do not change, asset durations will change solely because of the passage of time
- So we need to **rebalance** immunized portfolios
- However, rebalancing involves transaction costs; in practice, managers strike a compromise between the desire for perfect immunization and the need to control trading costs

Passive Bond Management – An Example

The situation:

- An insurance company must make a payment of 19,487 Baht in 7 years. The market interest rate is 10%, so the PV of obligation is 10,000 Baht.
- The portfolio manager wishes to fund the obligation using 3-yr zero-coupon bonds and perpetuities paying annual coupons
- How can the manager immunize the obligation?

Source: BKM Example 11.2

1. Calculate the duration of the liability = 7 years
2. Calculate the (Macaulay's) duration of the asset portfolio. The portfolio duration is the weighted average of duration of each component asset

   Asset duration = w x 3 years + (1-w) x 11 years

3. Equate the asset duration to liability duration (7 years), we obtain

   7 years = w x 3 years + (1-w) x 11 years

   we can solve that w = 0.5

4. This means that the manager must purchase 5,000 Baht of the zero-coupon bond and 5,000 Baht of the perpetuity. And the face value of the zero-coupon bond will be $5{,}000 \times 1.10^3 = 6{,}655$ Baht

Source: BKM Example 11.2

Passive Bond Management – An example

The situation:

- Now suppose that 1 year has passed and the interest rate remains at 10%. The portfolio manager needs to reexamine her position
- Is the position still fully funded? Is it still immunized? If not, what actions are required?

Source: BKM Example 11.3

Passive Bond Management – An Example

- Now the PV of the obligation will grow to 11,000
- The zero-coupon bond value grows to 5,500 and the perpetuity has paid its 500 Baht of coupon and remains worth 5,000 Baht
- So, the obligation is **still fully funded**
- But the portfolio weight must be changed! The obligation's duration is now 6 years

6 years = w x 2 years + (1-w) x 11 years

we can solve that w = 5/9

- This means that the manager must purchase $11{,}000 \times \frac{5}{9} = 6{,}111.11$ Baht of the zero-coupon bond and the rest in the perpetuity

Source: BKM Example 11.3

Passive Bond Management

- Cash Flow Matching and Dedication
  - Cash flow matching
    - Matching cash flows from fixed-income portfolio with those of obligation
  - Dedication strategy
    - Multi-period cash flow matching
    - The manager selects bonds with total CFs that match a series of obligations → Once the CFs are matched, there is no need for rebalancing
    - However, sometimes, CF matching is impossible, e.g. hundred years of CFs are not sold in the market

Life insurance objectives

Risk

- Having the ability to pay death benefits when due is a critical concern
- life insurers need to maintain a reserve as a cushion against substantial losses of portfolio value or investment income
- Worldwide the movement is towards risk-based capital, which requires the company to have more capital (and less financial leverage) the riskier the assets in the portfolio

Life insurance objectives

Risk

- **Valuation risk** and ALM will figure prominently in any discussion of risk with interest rate risk will be the prime issue
  - Any mismatch between duration of assets and of liabilities will make the surplus highly volatile
  - The result is the duration of assets will be closely tied to the duration of liabilities
- **Reinvestment risk** will be important for some products
  - The company must invest the premium and build sufficient value to pay off at maturity

Life insurance objectives

Risk

- **Cash flow volatility**
  - Reinvesting interest on cash flow coming in is a major component of return over long periods
  - Most companies seek investments that offer minimum cash flow volatility
- **Credit risk**
  - Credit analysis is required to gauge potential losses of investment income
  - Controlling credit risk is a major concern for life insurance companies and is often managed through a broadly diversified portfolio

Life insurance objectives

Return

- to earn a **net interest spread**, a return higher than the actuarial assumption
- The investments are heavily fixed-income oriented with an exception

Life insurance constraints

- **Disintermediation risk** occurs during periods of high interest rates when policyholders are more likely to withdraw cash value causing increased demand for liquidity from the portfolio
- This leads to shorter durations, higher liquidity reserves, and closer ALM matching
- **Asset marketability risk** has also become a larger consideration. Life insurers held relatively large portions of the portfolio in illiquid assets.
- Time horizon and eligible investments by asset class

Non-life insurance objectives

Risk

- The non-life business is both cyclical and erratic in profitability and cash flow
- **Long tail** - A claim could be filed and take years to process before payout
- **Inflation risk** - Less certain and higher payoffs on claims
- Non-life is hard to predict in both amount and timing
- Non-life business risk can be very concentrated geographically or with regard to specific events

Non-life insurance objectives

Return

- The investment portfolio seeks to smooth profitability and provide for unpredictable liquidity needs
- Setting a long-term portfolio return objective is difficult due to competitive product pricing and volatile profitability