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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i)$ $= 5a + b\sum_{i=1}^{5} x_i = 5a + b(x_1 + \dots x_5)$

   b. $\sum_{y=0}^{5} f(x+y)$ $= f(x) + f(x+1) + f(x+2) + \dots f(x+5)$

   c. $\sum_{i=1}^{10} i^2$ $= 1^2 + 2^2 + \dots + 10^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(11)(21)}{6} = 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$ $= (2(1)+2) + (2(1)+3) + (2(2)+2) + (2(2)+3) = 23.$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b

   $\sum f(x) = 1 = 0.5b + b + \dots 0.25b = 8b$ so $b = 1/8$

   b. Find the answer for $P(X \le 2)$

   $P(X \le 2) = 1 - P(X=3) - P(X=4) = 1 - \frac{0.5}{8} - \frac{0.25}{8} = \frac{8-0.75}{8} = \frac{7.25}{8} = 0.90625$

   c. Find the answer for $P(-2 \le X \le 3)$

   $P(-2 \le X \le 3) = 1 - P(X=4) = 1 - \frac{0.25}{8} = \frac{7.75}{8} = 0.96875$

   d. Find the answer for $P(X \ge 1)$

   $P(X \ge 1) = 1 - P(X=-2) - P(X=-1) - P(X=0)$

   $= 1 - 3.75b = \frac{8-3.75}{8} = 0.53125.$

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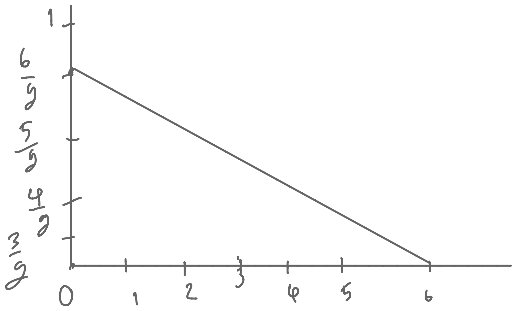
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$
$= 0$ other wise

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$\int_1^3 f(x)\,dx = -\frac{1}{9}\frac{x^2}{2} + \frac{6}{9}x \Big|_1^3$$

$$= \left(-\frac{1}{2} + 2\right) - \left(-\frac{1}{18} + \frac{12}{18}\right) = \frac{3}{4} - \frac{11}{18} = \frac{27-22}{36}$$

$$= \frac{5}{36} \#$$

c. Find the answer for $P(X \ge 2)$

$$\int_2^\infty f(x)\,dx = \int_2^3 f(x)\,dx + \int_3^\infty f(x)\,dx = \int_2^3 f(x)\,dx + \int_3^\infty 0\,dx)$$

$$= -\frac{1}{9}\frac{x^2}{2} + \frac{6}{9}x \Big|_2^3 = -\frac{1}{2} + 2 - \left(-\frac{2}{9} + \frac{12}{9}\right) = \frac{3}{2} - \frac{10}{9} = \frac{27-20}{18} = \frac{7}{18} \#$$

d. Find the expected value of X

$$\int_{-\infty}^{\infty} x f(x)\,dx = \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx = -\frac{1}{9}\frac{x^3}{3} + \frac{6x^2}{18}\Big|_0^3 = \left|-1+3-0\right| = 2 \#$$

$$\int_{-\infty}^{0} x(0)\,dx = 0, \quad \int_3^\infty x(0)\,dx = 0$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0. $1/2$

a. Construct the joint probability distribution function (PDF) table of X and Y

Y \ X	1	2	3	4	5	6	
1	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.5
0	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.5
	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666	1

b. Find the marginal probability distribution function (PDF) of X

$P(X=1,2,3,4,5,6) = 0.1666$

c. Find the marginal probability distribution function (PDF) of Y

$P(Y=1) = 0.5, \quad P(Y=0) = 0.5$

d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X=x_i|Y=1) = \frac{P(X=x_i, Y=1)}{P(Y=1)}$$

e. Find the expected value of X given Y is equal to 1

$$E(X=x_i|Y=1) = \sum_{i=1}^{N} x_i P(X=x_i|Y=1) = \sum_{i=1}^{N} \frac{x_i P(X=x_i, Y=1)}{P(Y=1)} = \frac{1(0.0833)+2(0.08333)+\ldots+6(0.08333)}{0.5}$$

$$= \frac{0.083(21)}{0.5} = 3.48$$

f. Find the variance of X given Y is equal to 1

$$Var(X) = E(X^2) - \mu^2$$

$$= \frac{\sum (x_i)^2}{n} - \mu^2 = \frac{(0.08333)(1^2+\ldots+6^2)}{6} - (0.3491)^2 = 91(0.0833) - 0.12152101$$

$$\approx 7.46$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$E(xy) = \sum_{i=1}^{n} x_i y_i \overbrace{P(x_i, y_i)}^{\text{joint prob.}}$

$Cov(x_1, x_2) = E(x_1 x_2) - \mu_{x_1}\mu_{x_2} = E(x_1 x_2) - \mu^2$

$\sigma^2 = E(x_i^2) - \mu^2$

so $\frac{\sigma^2}{4} = Cov(x_1, x_2)$

$\frac{E(x_i^2) - \mu^2}{4} = E(x_1 x_2) - \mu^2$

$E(x_i^2) - \mu^2 = 4E(x_1 x_2) - 4\mu^2$

$3\mu^2 = 4E(x_1 x_2) - E(x_i^2)$

$\mu^2 = \frac{4E(x_1 x_2) - E(x_i^2)}{3}$

$\mu = \sqrt{\frac{4E(x_1 x_2) - E(x_i^2)}{3}}$

$E(\bar{x}) = \frac{1}{3}E(x_1 + x_2 + x_3) = \mu = \sqrt{\frac{4E(x_1 x_2) - E(x_i^2)}{3}}$

$Var(\bar{x}) = \frac{1}{3^2} Var(x_1 + x_2 + x_3)$

$= \frac{\sigma^2}{3} = \frac{E(x_i^2) - \mu^2}{3}$

substitute $\mu = \sqrt{\frac{4E(x_1 x_2) - E(x_i^2)}{3}}$

so $Var(\bar{x}) = E(x_i^2) - \frac{4E(x_1 x_2) + E(x_i^2)}{3}$

$= \frac{4E(x_i^2) - 4E(x_1 x_2)}{3}$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$\boxed{E(X_1) = \mu}$

$E(\bar{x}) = E\left(\frac{1}{4}(x_1 + \dots x_4)\right) = \frac{E(x_1 + \dots x_4)}{4} = \frac{4\mu}{4} = \mu.$

$Var(\bar{x}) = Var\left(\frac{1}{4}(x_1 + \dots + x_4)\right) = \frac{1}{4^2}\left[Var(x_1) + \dots Var(x_4)\right] = \frac{4\sigma^2}{16} = \frac{\sigma^2}{4}$

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$E(\tilde{X}) = E\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right)$$

$$= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4)$$

$$= \frac{\mu}{8} + \frac{\mu}{4} + \frac{\mu}{8} + \frac{\mu}{2} = \frac{\mu + 2\mu + \mu + 4\mu}{8} = \frac{8\mu}{8} = \mu.$$

So $E(\tilde{X}) = \mu \Rightarrow$ it's unbiased estimator of μ.

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

lower variance is said to more efficient than another estimators with higher variance.

$$\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{4}, \quad \mathrm{Var}(\tilde{X}) = \mathrm{Var}\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right)$$

$$= \mathrm{Var}\left(\frac{X_1}{8}\right) + \mathrm{Var}\left(\frac{X_2}{4}\right) + \mathrm{Var}\left(\frac{X_3}{8}\right) + \mathrm{Var}\left(\frac{X_4}{2}\right)$$

$$= \frac{1}{64}\mathrm{Var}(X_1) + \frac{1}{16}\mathrm{Var}(X_2) + \frac{1}{64}\mathrm{Var}(X_3) + \frac{1}{4}\mathrm{Var}(X_4)$$

$$= \frac{\sigma^2}{64} + \frac{4\sigma^2}{64} + \frac{\sigma^2}{64} + \frac{16\sigma^2}{64}$$

$$= \frac{22\sigma^2}{64} = \frac{11\sigma^2}{32} = \frac{\sigma^2}{23}$$

$$\boxed{\text{finally } \mathrm{Var}(\bar{X}) < \mathrm{Var}(\tilde{X}) \text{ therefore, } \bar{X} \text{ is the better estimator for } \mu.}$$