Chapter 6 : A Real Intertemporal Model with Investment (Part 1)

EE312

Macroeconomics, Stephen Wiliamson, Chapter 11

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Outline

- Real Intertemporal Model (Introduciton) (Part 1)
- The representative consumer: (Part 1)
  - Consumer's opitmal decisions
  - budget constraint
  - labour supply
- The representative firm : (Part 1)
  - profit maximization
  - labour demand
  - firm's investment decision
- Government Sector (Part 1)
- Competitive Equilibrium (Part 2)

- **The real model (no money)** with three sectors as the basis for the analysis of short-term economic fluctuations.
  - **Representative consumers** (consumption, labor supply and saving);
  - **Representative firms** (production, labor demand and investment);
  - **Government** (spending, taxes and borrowing).

Two-maket model

- **The labor market:** the firm's demand and the consumer's supply of labor.
  $\Rightarrow$ The real wage rate.
- **The output market:** the firm's supply and the consumer's demand for output.
  $\Rightarrow$The real interest rate. Analysis of real macroeconomic shocks.
- **Changes in government spending, capital stock, total factor productivity.**

Investment

- Expenditure on plants, equipment and new housing.
  - Goods currently produced for future production of goods and services.
  - Increases in future productive capacity.
  - **The consumer's tradeoff** between current and future consumptions (saving).
  - **The firm's tradeoff** between current profits and higher future capital stock (and future profits).

2. Consumer's optimal decisions

- Work-leisure in current and future periods.
- Consumption-savings in the current period.
  - h = total time available;
  - w and w' = current and future real wages;
  - r = the real interest rate;
  - T and T' = current and future lump-sum taxes;
  - C and C' = current and future consumptions;
  - ℓ and ℓ' = current and future leisure time;
  - S^p = private savings.

Current budget constraint

- The consumer is a price-taker (w, w', r and T are given).
- $w(h-\ell)$ = real-wage income;
- π = dividend income from the firm;
- T = lump-sum taxes paid to the government.
- Then, disposable income is:

$$C + S^P = w(h-\ell) + \pi - T$$

Future budget constraint

- The consumer still receives real wages, dividend income, and pays future taxes.
- Receives the principal and interest on savings.
- No bequests; all wealth is consumed.

$$C' = w'(h - \ell') + \pi' - T' + (1 + r)S^p$$

Lifetime budget constraint

$$C + \frac{C'}{1+r} = w(h-\ell) + \pi - T + \frac{w'(h-\ell') + \pi' - T'}{1+r}$$

- The PV of lifetime consumption equals the PV of lifetime disposable income.
- Decision on the optimal bundles of C, C', ℓ and ℓ' subject to the lifetime budget constraint.

Current period optimal condition

$$MRS_{\ell,C} = w$$

- The consumer chooses the optimal bundle of current leisure and consumption:
  - The marginal rate of substitution of current leisure for current consumption is equal to the real wage.
  - w is the relative price of leisure in terms of consumption goods.

Future period optimal condition

$$MRS_{\ell',C'} = w'$$

- The consumer chooses the optimal bundle of future leisure and future consumption:
- The marginal rate of substitution of future leisure for future consumption is equal to the future real wage.

Intertemporal optimal condition

$$MRS_{C,C'} = 1 + r$$

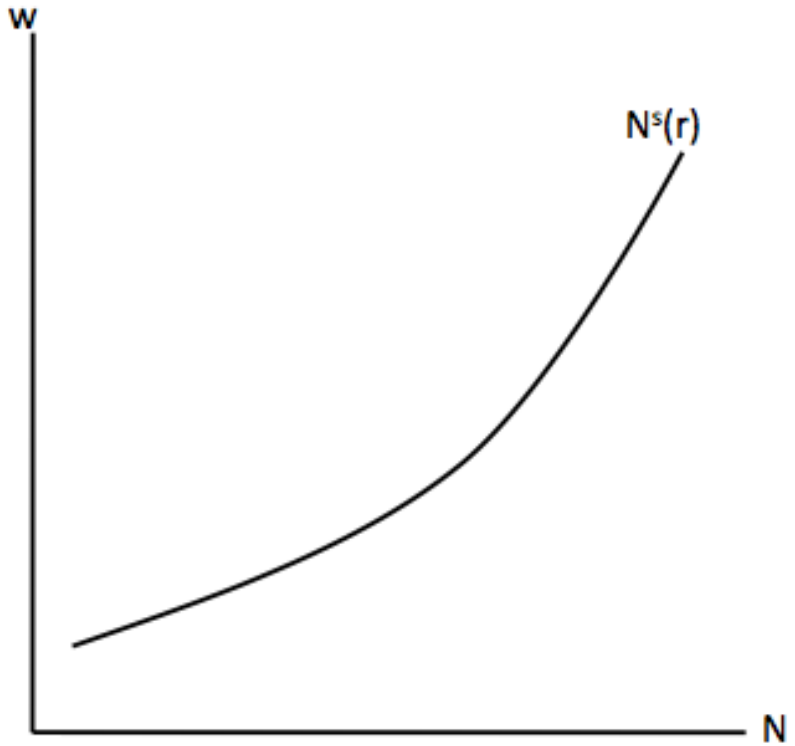
- The consumer chooses the optimal bundle of current and future consumption (savings):
- The marginal rate of substitution of current consumption for future consumption is equal to the real interest rate.
- (1 + r) is the relative price of current consumption in terms of future consumption.

Current labor supply

- The consumer provides labor supply to the firm through work-leisure decision.
- Factors which determine current labor supply:
  - The current real wage;
  - The real interest rate;
  - Lifetime wealth.

Current labor supply curve

- Current labor supply increases with the real wage, given r (assuming the dominant substitution effect).

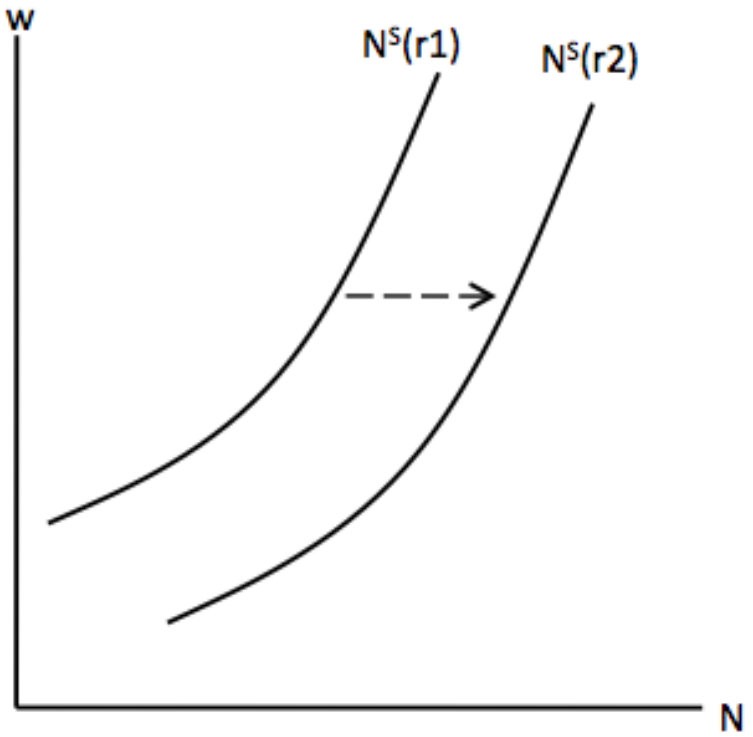


An increase in the real interest rate

- Current labor supply increases as the real interest rate increases.
- $\frac{w(1+r)}{w'}$ is the relative price of current leisure in terms of future leisure.
- Given w and w', a higher r means the higher price of current leisure in terms of future leisure.
- Less current leisure, and more current supply of labor, assuming the dominant substitution effect.

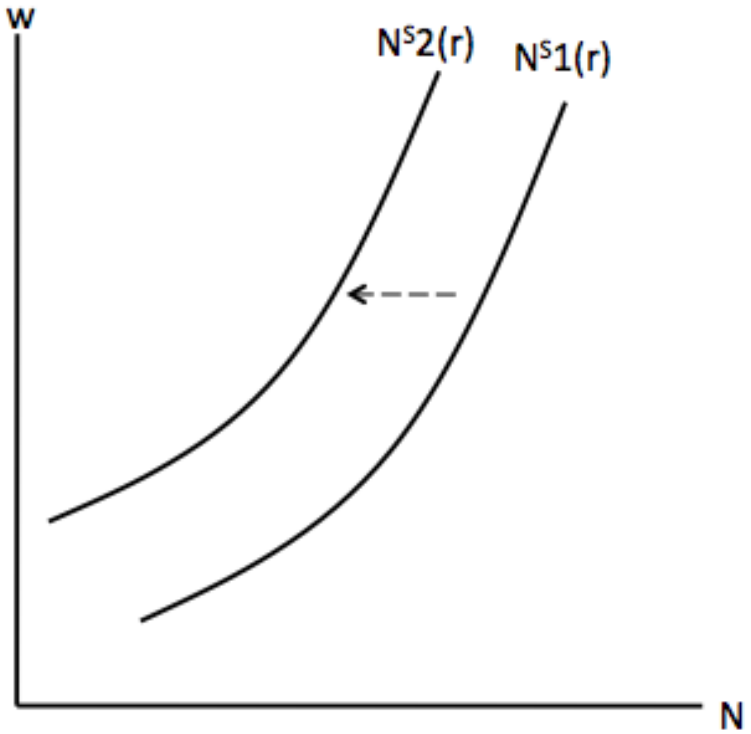
Labor supply increases with r

- Given w, labor supply increases with the rising real interest rate (r2 > r1), assuming the dominant substitution effect.



An increase in lifetime wealth

- Current leisure increases and current labor supply decreases with rising lifetime wealth.
  - Current and future consumption also increase.

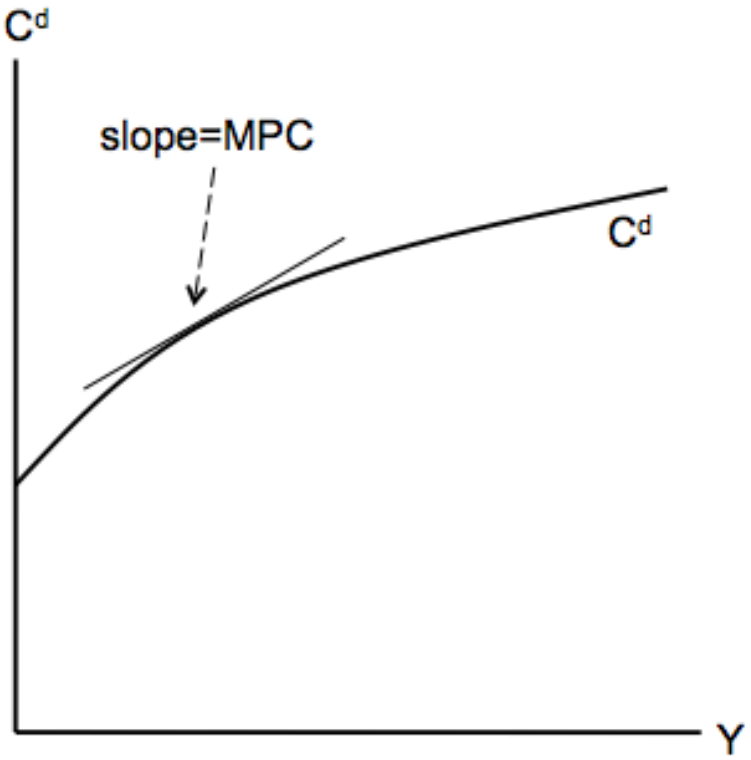


Demand for current consumption goods

- The individual demand for current consumption goods (C^d) is a function of current income (Y), given r.
- The marginal propensity to consume (MPC) < 1 .
- A higher real interest rate (r) causes the demand to fall, assuming:
- The substitution effect dominates the income effect; the consumer is a lender.

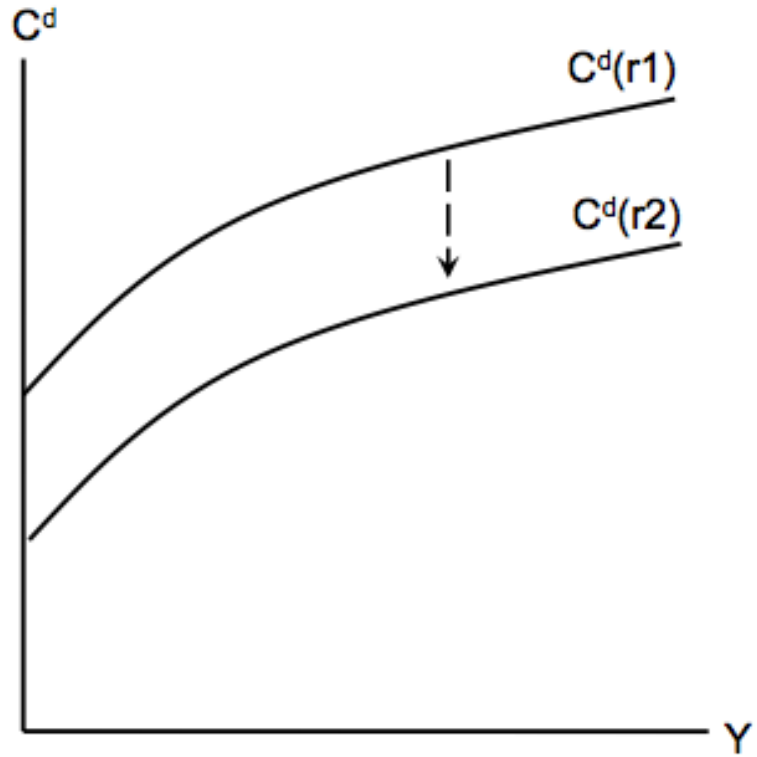
Demand for current consumption

- $C^d = f(Y, r)$
- $MPC = \dfrac{\partial C^d}{\partial Y} < 1$;
- $\dfrac{\partial C^d}{\partial r} < 0; \dfrac{\partial C^d}{\partial we} > 0$



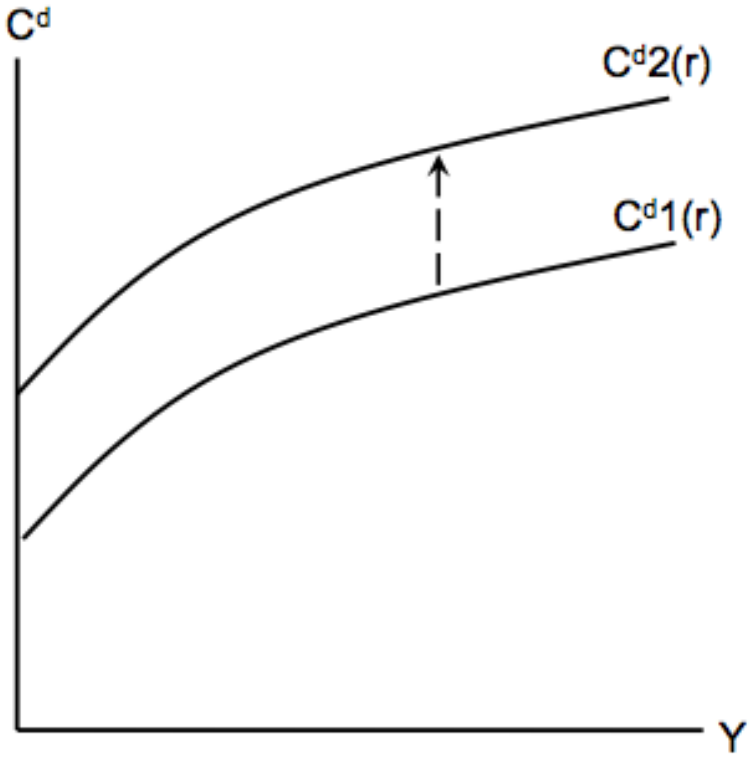
A higher r reduces C^d

- r2 > r1.
- The consumer reduces current consumption, assuming stronger substitution effect and a lender.



An increase in we raises C^d

- An increase in lifetime wealth raises current consumption.



3. The Representative firm

- Optimal decisions on:
  - Maximized present value of profits;
  - The level of current labor inputs.
  - **Optimal investment level**: tradeoff between current profits and future capital stock (and future profits).

Current production function

$$Y = zF(K, N)$$

- Y = current output;
- z = current total factor productivity;
- K = current capital stock;
- N = current labour input.
- And the future production function:

$$Y' = z'F(K', N')$$

Change in capital stock

- The firm's investment is foregone current profits (consumption) for future profits:
  - d = the rate of depreciation;
  - I = current investment.
- Future capital stock is current capital stock net of depreciation plus investment.

$$K' = (1 - d)K + I$$

The firm's current profits

- Maximization of the present value of current and future profits.
- π = current profits;
- I = current investment = foregone current consumption goods.

$$\pi = Y - wN - I$$

The firm's future profits

- The leftover capital stock in the future period can be sold off as junk value.
- $(1-d)K'$ = capital stock remaining as junk at the end of the future period.

$$\pi' = Y' - w'N' + (1-d)K'$$

Maximized PV of profits

- The firm maximizes the PV of profits.
  - The same as maximized PV of dividend income for the consumer.
  - V = max. present value of profits through optimal choice on N, N' and I:

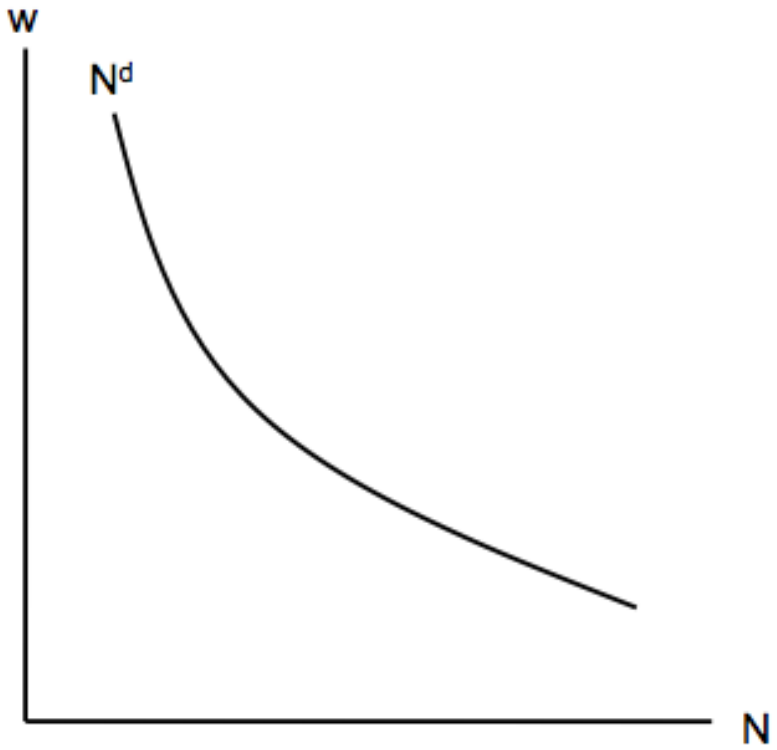
$$V = \pi + \frac{\pi'}{1 + r}$$

Current labor demand

- The firm's choice of current labor demand (N^d) affects only current profits (π).
- The firm hires current labor until the current marginal product of labor equals the current real wage (MPN= w).
- Thus the firm's MP_N curve is also the firm's current labor demand curve.
- An increase in current z or K raises MP_N and current labor demand.

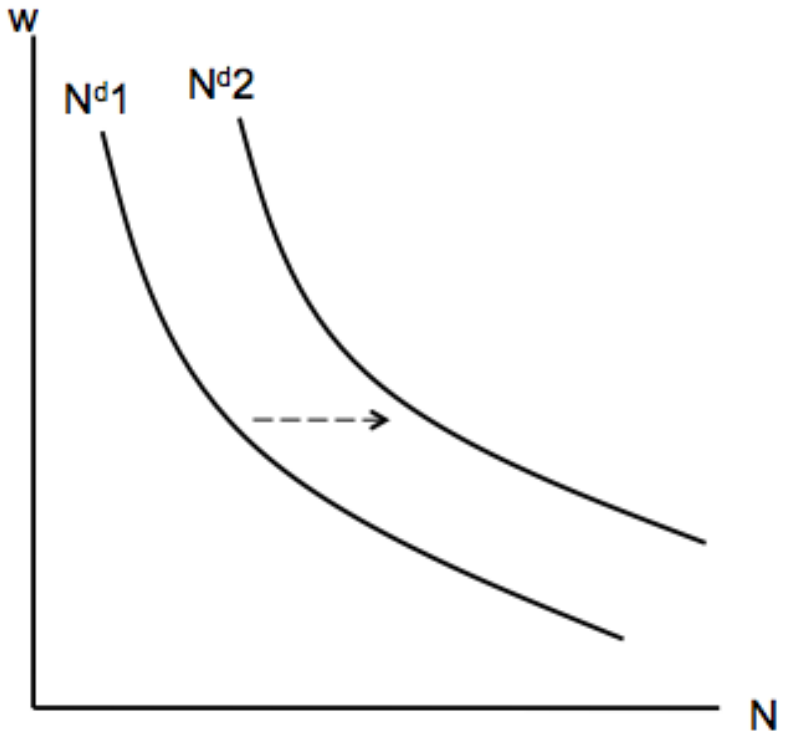
Current labor demand curve

- The current labor demand: MP_N = w.
- MP_N is falling as the labor input increases.



Labor demand with rising z or K

- An increase in current z or K shifts the current labor demand curve to the right.



The firm's investment decision

- The firm invests to the point where the marginal benefit from investment equals marginal cost.
- MC(I) = marginal cost of investment = PV of profits (V) given up for one unit of capital.
- One unit of investment reduces current π (and V) by one unit.

$$MC(I) = 1$$

- MB(I) = marginal benefit of investment = additional units of V (PV of profits) received from one extra unit of current investment.
- MPK' = additional output from one extra unit of K' .
- Quantity of capital left from depreciation at the end of the future period (1 - d) for liquidation.
- I's future profits is $(MP'_K + 1 - d) = \pi'$.

$$MB(I) = \frac{MP'_K + 1 - d}{1 + r}.$$

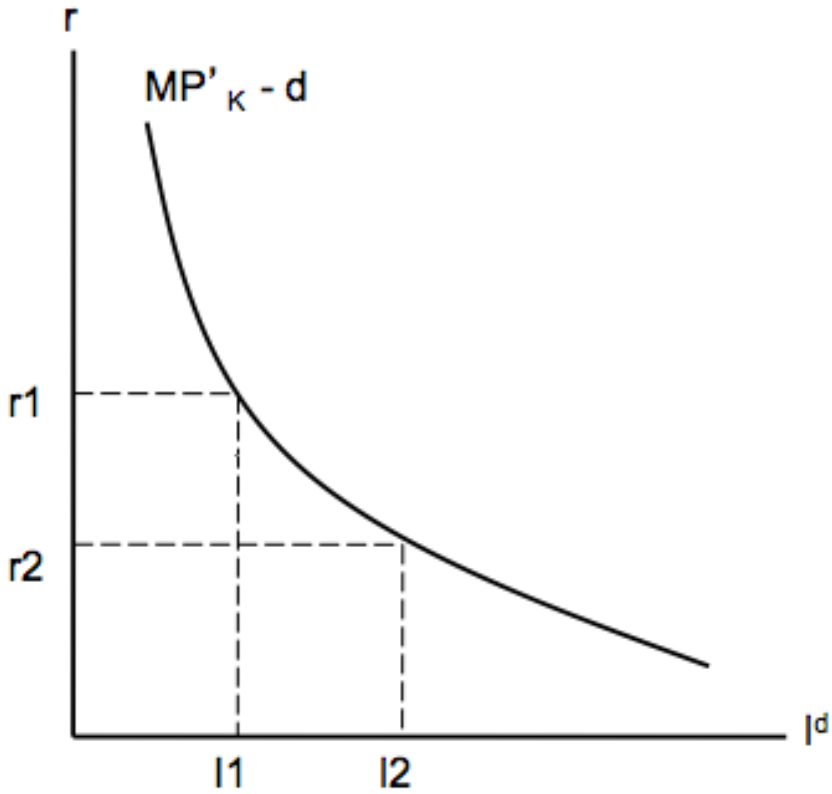
Optimal investment decision

$$\frac{MP'_K + 1 - d}{1 + r} = 1.$$

$$MP'_K - d = r.$$

- The firm invests until the net future marginal product of capital equals the real interest rate.
- **r = the opportunity cost of more capital** = the rate of return on the alternative asset (bonds) otherwise earned by the consumer who owns the firm.

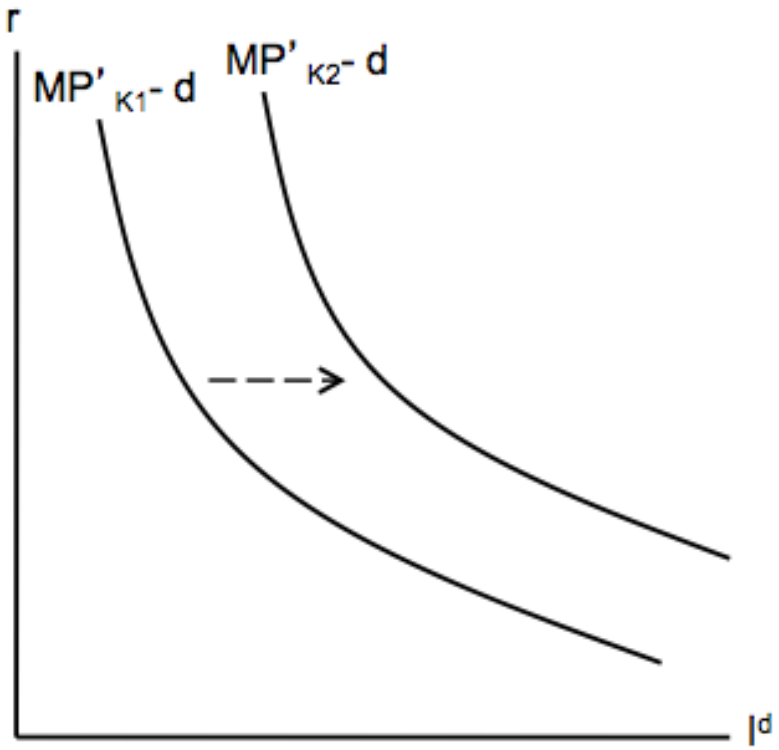
- $I^d = (MP'_K - d)$ gives the level of I required for the net MP'K to be equal to the real interest rate, given K.



Changes in z' and K

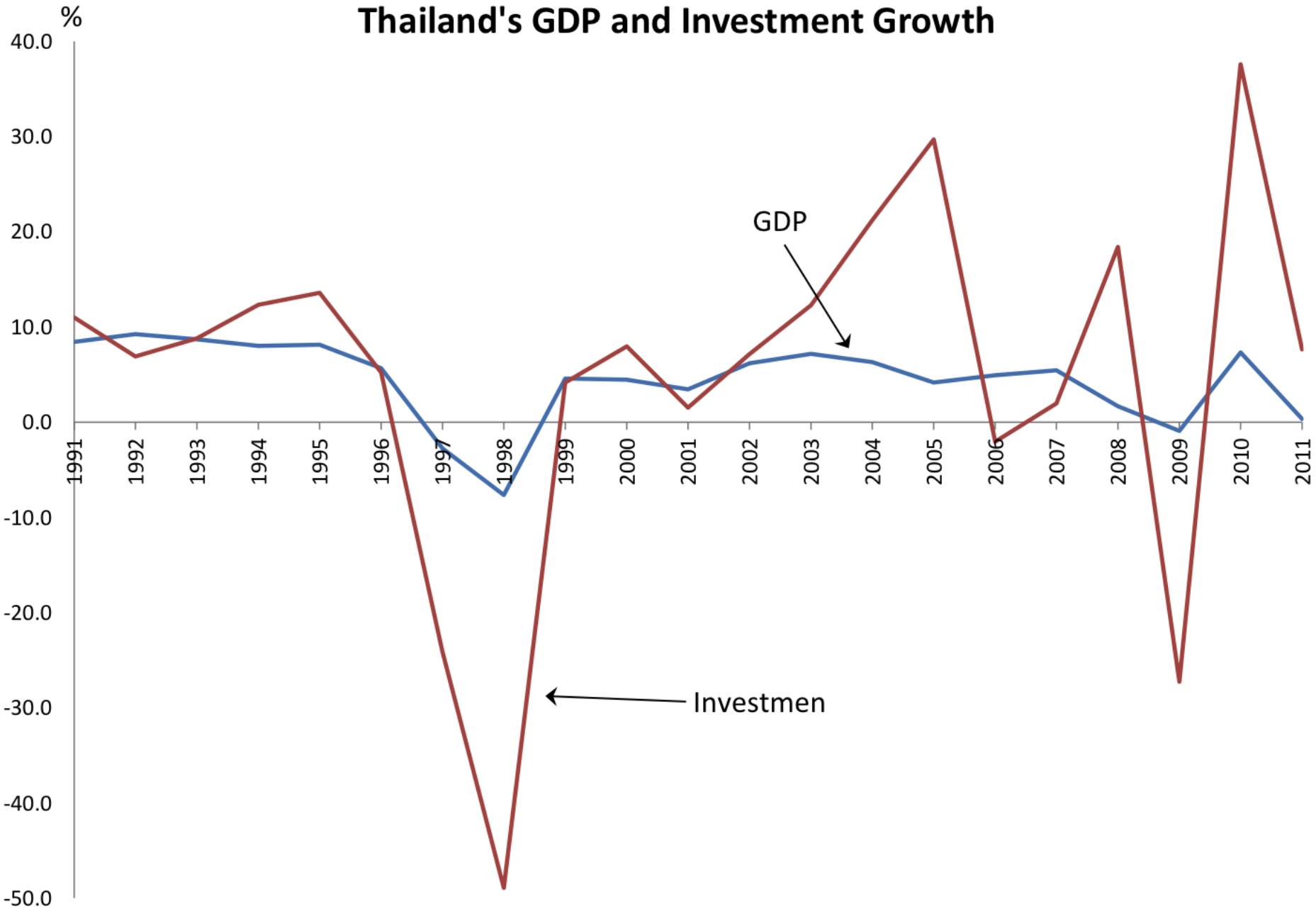
- Factors affecting future marginal product of capital shift the optimal investment curve.
- Higher **future total factor productivity** (z') increases future MP'_K and current optimal investment.
- The optimal investment curve shifts to the right.
- Higher **current capital stock** results in larger future net capital stock and lower MP'_K .
- The optimal investment curve shift to the left.

- A higher z' or a lower K increases MP'_K. The optimal investment curve shifts to the right.



Volatile investment and GDP

- Aggregate consumption is less variable than income due to consumption smoothing.
- Investment is much more volatile — short-run economic fluctuations.
  - Investment responds to perceived marginal rates of return to investment.
  - Changes in the real interest rate cause movements along the investment curve.
  - Changes in future total factor productivity shift the investment curve.


Thailand's GDP and Investment Growth
%
40.0
30.0
20.0
10.0
0.0
-10.0
-20.0
-30.0
-40.0
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
GDP
Investmen

Government sector

- Government purchases of consumption goods (G and G') are exogenously determined.
- Government financing:
  - Current lump-sum taxes and bond sale;
  - Future lump-sum taxes and payments of the principal and interest.

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}.$$

Competitive equilibrium

- The labor market:
  - The consumer supplies labor service.
  - The firm demands labor service.
  - The real wage and the level of employment.
- The output market:
  - The consumer, the firm and government purchase output.
  - The firm supplies the goods.
  - The real interest rate and the level of aggregate output.