

Tutorial - Matrix

1. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 7 & 0 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, compute

(i) $3\mathbf{A}^T + \mathbf{D}$

Ans: $\begin{bmatrix} 4 & 2 & 20 \\ 7 & -3 & 2 \end{bmatrix}$

(ii) $(\mathbf{B} - \mathbf{C})^T$

Ans: $\begin{bmatrix} 0 & 3 \\ 3 & -3 \end{bmatrix}$

(iii) $2\mathbf{B}^T - 3\mathbf{C}^T$

Ans: $\begin{bmatrix} -1 & 5 \\ 6 & -8 \end{bmatrix}$

(iv) $2\mathbf{B} + \mathbf{B}^T$

Ans: $\begin{bmatrix} 3 & 10 \\ 11 & -3 \end{bmatrix}$

(v) $\mathbf{C}^T - \mathbf{D}$

Ans: undefined

(vi) $(\mathbf{D} - 2\mathbf{A}^T)^T$

Ans: $\begin{bmatrix} -1 & -3 \\ 2 & 2 \\ -15 & 2 \end{bmatrix}$

2. If $\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 0 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}$ $\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$\mathbf{O} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, compute

(i) \mathbf{A}^2

Ans: undefined

(ii) $\mathbf{A}^T \mathbf{A}$

Ans: $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

(iii) \mathbf{B}^3

Ans: $\begin{bmatrix} 0 & 0 & -4 \\ 2 & -1 & -2 \\ 0 & 0 & 8 \end{bmatrix}$

(iv) $\mathbf{A}(\mathbf{B}^T)^2 \mathbf{C}$

Ans: $\begin{bmatrix} -6 & 3 \\ -4 & 5 \end{bmatrix}$

(v) $(\mathbf{AC})^2$

Ans: $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

(vi) $\mathbf{A}^T(2\mathbf{C}^T)$

Ans: $\begin{bmatrix} 2 & 4 & 0 \\ -2 & -6 & 2 \\ 0 & -2 & 2 \end{bmatrix}$

(vii) $(\mathbf{BA}^T)^T$

Ans: $\begin{bmatrix} 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}$

(viii) $(2\mathbf{B})^T$

Ans: $\begin{bmatrix} 0 & 4 & 0 \\ 0 & -2 & 0 \\ -2 & 0 & 4 \end{bmatrix}$

(ix) $(2\mathbf{I})^2 - 2\mathbf{I}^2$

Ans: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(x) $\mathbf{A}(\mathbf{I} - \mathbf{O})$

Ans: $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(xi) $\mathbf{I}^T \mathbf{O}$

Ans: \mathbf{O}

(xii) $(\mathbf{AB})(\mathbf{AB})^T$

Ans: $\begin{bmatrix} 6 & -7 \\ -7 & 9 \end{bmatrix}$

(xiii) $\mathbf{B}^2 - 3\mathbf{B} + 2\mathbf{I}$

Ans: $\begin{bmatrix} 2 & 0 & 1 \\ -8 & 6 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

3. If **A** is 2 x 3, **B** is 3 x 1, **C** is 2 x 5, **D** is 4 x 3, **E** is 3 x 2, and **F** is 2 x 3, determine the size of the following matrices:

AE, DE, EC, DB, FB, BC, EA, E(AE), E(FB) and (F + A)B

4. If **A** is a 4x4 matrix such that $\underline{\mathbf{A}} \begin{bmatrix} 0 \\ 6 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ and $\underline{\mathbf{A}} \begin{bmatrix} 3 \\ 6 \\ 6 \\ 15 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 6 \\ 3 \end{bmatrix}$, what is the product $\underline{\mathbf{A}} \begin{bmatrix} 1 \\ 11 \\ 5 \\ 11 \end{bmatrix}$?

(Hints: you can obtain $\underline{\mathbf{A}} \begin{bmatrix} 1 \\ 11 \\ 5 \\ 11 \end{bmatrix}$ without finding the matrix **A**)

Ans: $\begin{bmatrix} -1 \\ 0 \\ 2 \\ -2 \end{bmatrix}$

5. By using the row operations, solve the linear system

$$\begin{aligned} 2x + 3y &= -1 \\ 2x + y &= 5 \\ x + y &= 1 \end{aligned}$$

Ans: $x = 4$ and $y = -3$

6. By using the row operations, solve the linear system

$$\begin{aligned} x + 2y + z - 6 &= 0 \\ 3z + y - 3 &= 0 \\ x + y + 2z - 1 &= 0 \end{aligned}$$

Ans: No solution (inconsistent)

7. By using the row operations, solve the linear system

$$\begin{aligned} 2x_1 + 3x_2 + 2x_3 + 6x_4 &= 10 \\ x_2 + 2x_3 + x_4 &= 2 \\ 3x_1 - 3x_3 + 6x_4 &= 9 \end{aligned}$$

Ans: Many solutions that satisfied $\begin{cases} x_1 + \frac{5}{2}x_4 = 4 \\ x_2 = 0 \\ x_3 + \frac{1}{2}x_4 = 1 \end{cases}$

8. By using the row operations, solve the linear system

$$\begin{aligned} x_1 + x_2 - x_3 + x_4 + x_5 &= 0 \\ x_1 + x_2 + x_3 - x_4 + x_5 &= 0 \\ x_1 - x_2 - x_3 + x_4 - x_5 &= 0 \\ x_1 + x_2 - x_3 - x_4 - x_5 &= 0 \end{aligned}$$

Ans: Many solutions that satisfied $x_1 = 0, x_2 = -x_5, x_3 = -x_5, x_4 = -x_5$

9. By using the row operations, solve the linear system

$$x_1 + x_2 + x_3 - x_4 = 0$$

$$x_1 - x_2 - x_3 + x_4 = 0$$

$$x_1 + x_2 - x_3 - x_4 = 0$$

$$x_1 + x_2 - x_3 + x_4 = 0$$

Ans: Many solutions that satisfied $x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0$

10. the following system of linear equations over the real numbers, where x_1, x_2, x_3 are variables also b_1, b_2, b_3 and C are real constants.

$$x_1 + 5x_2 + 6x_3 = b_1$$

$$2x_1 + 4x_2 + 6x_3 = b_2$$

$$2x_1 + 6x_2 + Cx_3 = b_3$$

- Write down the matrix equation ($\mathbf{Ax} = \mathbf{b}$) and the augmented matrix that represents the above system of linear equations.
- By performing row operations, obtain the upper triangle form of \mathbf{A} .
- If $b_1 = 0, b_2 = 0, b_3 = 0$, is it possible that this linear system will have no solution?
- If $b_1 = 6, b_2 = 6, b_3 = 8$, is it possible that this linear system will have infinitely many solutions? If possible, give an example of a value of C .
- Solve the given linear system with your specified value of C in (d).
- If $b_1 = 6, b_2 = 6, b_3 = 8$, is it possible that this linear system to have a unique solution? If possible, give an example of a value of C .
- Solve the given linear system with your specified value of C in (f).

11. Find the inverse of

(i) $\begin{bmatrix} 6 & 1 \\ 7 & 1 \end{bmatrix}$

Ans: $\begin{bmatrix} -1 & 1 \\ 7 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

Ans: non-singular/not invertable

(iii) $\begin{bmatrix} -5 & 4 & -3 \\ 10 & -7 & 6 \\ 8 & -6 & 5 \end{bmatrix}$

Ans: $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Ans: non-singular/not invertable

12. Compute the determinant of **A**, **B** and **C**

$$|\mathbf{A}| = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 17 & 21 \\ 2 & 7 & 1 \end{vmatrix}$$

$$|\mathbf{B}| = \begin{vmatrix} 1 & -2 & 3 & 0 \\ 0 & 4 & 5 & -6 \\ 1 & 0 & 2 & 3 \\ 4 & 0 & 2 & 3 \end{vmatrix}$$

$$|\mathbf{C}| = \begin{vmatrix} 1 & 2 & 3 & 0 & 0 \\ 1 & 0 & 4 & 2 & 3 \\ 2 & 0 & 1 & 4 & 5 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & 2 & 1 & 2 & 3 \end{vmatrix}$$

Ans: 16, 270, 16

13. Given $\underline{\mathbf{A}} = \begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$; $\underline{\mathbf{B}} = \begin{bmatrix} 4 & 0 & 0 & 3 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 \end{bmatrix}$ and

$$\underline{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 11 & 1/6 & 0 & 0 \\ -5 & 1/3 & 1/9 & 0 \\ 4 & 5/2 & 7/9 & 2/3 \end{bmatrix}$$

(a) If $\underline{\mathbf{A}} = \underline{\mathbf{B}}\underline{\mathbf{D}}^3\underline{\mathbf{C}}$, find the determinant of $\underline{\mathbf{D}}$.

Ans: $\mathbf{D} = \frac{3}{2}(1 - t^2)$

(b) Find the solution to $\underline{\mathbf{B}}^T \underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.

Ans: $\begin{bmatrix} \frac{1}{4} \\ \frac{1}{11} \\ \frac{4}{1} \\ \frac{1}{2} \end{bmatrix}$