CASE #1: Simple Economy

Approach #1 → $DAE = Y$ Income of HH = Expenditure of HH

$\cancel{C} + I = \cancel{C} + S$

Approach #2 → $I = S$ Leakage

Injection

Approach #1: $Y = DAE$

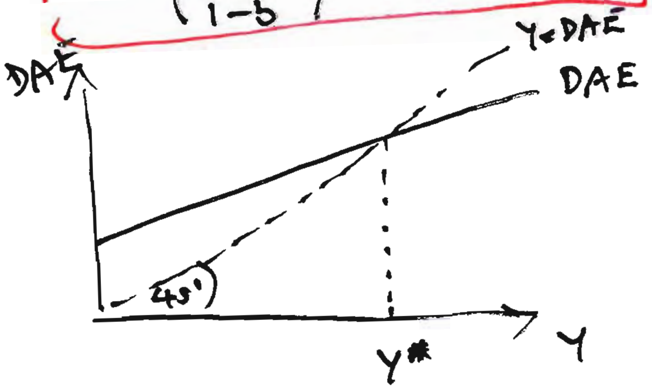
$DAE = C + I$; $I = I_a$

$= C_a + bY + I_a$

$Y = DAE = C_a + bY + I_a$

$(1-b)Y = C_a + I_a$

$$\boxed{Y^* = \left(\frac{1}{1-b}\right)(C_a + I_a)}$$



$Y^* = Y$ at Equilibrium

Approach #2: Leakage = Injection

$I = S$

$S = Y - C$

$= Y - (C_a + bY)$

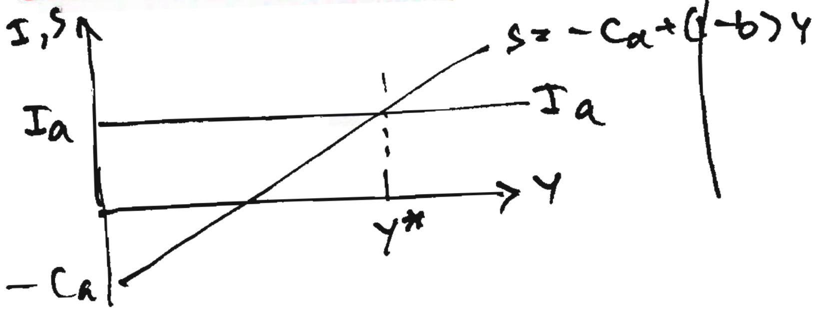
$= Y - C_a - bY$

$= -C_a + (1-b)Y$

$I = I_a$

$I_a = -C_a + (1-b)Y$

$$\boxed{Y^* = \left(\frac{1}{1-b}\right)(I_a + C_a)}$$



Autonomous Spending Multiplier

$$\frac{\Delta Y}{\Delta C_a} = \frac{1}{1-b}$$

$$\frac{\Delta Y}{\Delta I_a} = \frac{1}{1-b}$$

CASE#2 : Closed Economy with Govt

Approach #1 → DAE = Y → Total Income of HH

Approach #2: $\cancel{C} + I + G = \cancel{C} + S + T$ → Total Expenditure of HH

↳ $\underbrace{I+G}_{\text{Injection}} = \underbrace{S+T}_{\text{Leakage}}$

Approach #1:

$C = C_a + bY^d$

$= C_a + b(Y - T_a)$

$= C_a + bY - bT_a$

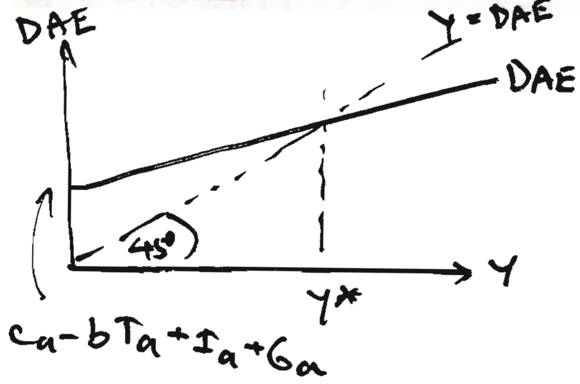
$I = I_a$; $G = G_a$

$DAE = C + I + G = Y$

$Y = C_a + bY - bT_a + I_a + G_a$

$(1-b)Y = C_a - bT_a + I_a + G_a$

$$\boxed{Y^* = \left(\frac{1}{1-b}\right)(C_a - bT_a + I_a + G_a)}$$



Approach #2: $I + G = S + T$

$S = Y^d - C = Y^d - (C_a + bY^d)$

$= Y^d - C_a - bY^d$

$= (1-b)Y^d - C_a = (1-b)(Y - T_a) - C_a$

$$\boxed{S = -C_a + (1-b)Y - (1-b)T_a}$$

$I = I_a$; $G = G_a$; $T = T_a$

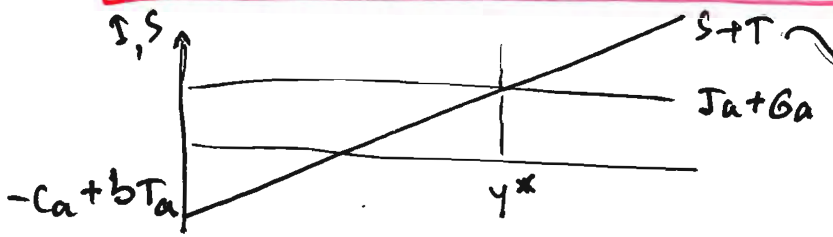
$I_a + G_a = -C_a + (1-b)Y - (1-b)T_a + T_a$

$-(1-b)Y = -C_a \underbrace{-(1-b)T_a}_{-\cancel{T_a} + bT_a} + \cancel{T_a} - I_a - G_a$

$-(1-b)Y = -C_a + bT_a - I_a - G_a$

$Y = \left(\frac{-1}{1-b}\right)(-C_a + bT_a - I_a - G_a)$

$$\boxed{Y^* = \left(\frac{1}{1-b}\right)(C_a - bT_a + I_a + G_a)}$$



$S + T = -C_a + bT_a + (1-b)Y$

Autonomous Spending Multiplier

$\frac{\Delta Y}{\Delta C_a} = \frac{1}{1-b}$

$\frac{\Delta Y}{I_a} = \frac{1}{1-b}$

$\frac{\Delta Y}{\Delta G_a} = \frac{1}{1-b}$

$\frac{\Delta Y}{\Delta T_a} = \frac{-b}{1-b}$

CASE#3: OPEN ECONOMY

Approach#1 $\longrightarrow DAE = Y$

$C + I + G + X - M = C + S + T$

Approach #2 $\longrightarrow \cancel{C} + I + G + X = \cancel{C} + S + T + M$

Injection ↲ ↳ Leakage

Approach#1 $DAE = Y = C + I + G + (X - M)$

$Y = C_a + bY^d + I_a + G_a + X_a - M_a - mY$

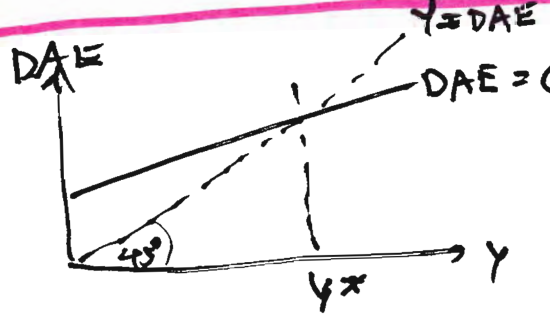
$= C_a + b(Y - T_a) + I_a + G_a + X_a - M_a - mY$

$Y = C_a + bY - bT_a + I_a + G_a + X_a - M_a - mY$

$Y + mY - bY = C_a - bT_a + I_a + G_a + X_a - M_a$

$Y(1 + m - b) = C_a - bT_a + I_a + G_a + X_a - M_a$

$$\boxed{Y^* = \left(\frac{1}{1+m-b}\right)(C_a - bT_a + I_a + G_a + X_a - M_a)}$$



Approach#2: Injection = Leakage

$I + G + X = S + T + M$

$I = I_a$; $G = G_a$; $X = X_a$; $T = T_a$

$M = M_a + mY$; $S = -C_a - (1-b)T_a + (1-b)Y$

$I + G + X = S + T + M$

$I_a + G_a + X_a = -C_a - (1-b)T_a + (1-b)Y + T_a + M_a + mY$

$-(1-b)Y - mY = -C_a - (1-b)T_a + T_a + M_a - I_a - G_a - X_a$

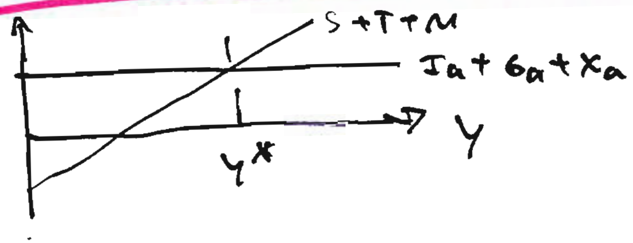
$Y(-1 + b - m) = -C_a - (1-b)T_a + T_a + M_a - I_a - G_a - X_a$

$\cancel{-T_a} + bT_a + \cancel{T_a}$

$Y = \left(\frac{1}{-1+b-m}\right)(-C_a + bT_a + M_a - I_a - G_a - X_a)$

Multiply both sides with $\frac{(-1)}{(-1)}$

$$\boxed{Y^* = \left(\frac{1}{1-b+m}\right)(C_a - bT_a - M_a + I_a + G_a + X_a)}$$



Autonomous Spending Multiplier

$\frac{\Delta Y}{\Delta C_a} = \frac{1}{1-b+m}$

$\frac{\Delta Y}{I_a} = \frac{1}{1-b+m}$

$\frac{\Delta Y}{\Delta G_a} = \frac{1}{1-b+m}$

$\frac{\Delta Y}{\Delta M_a} = \frac{-1}{1-b+m}$

$\frac{\Delta Y}{\Delta T_a} = \frac{-b}{1-b+m}$