Solution to HW #2

Problem 2 :

$$V^* = \Pi^* + \delta\Pi^* + \dots + \delta^\infty \Pi^* = \frac{\Pi^*}{1-\delta}$$

$$V^r = \Pi^r + \delta\Pi^r + \delta^2\Pi^c + \dots + \delta^\infty\Pi^c$$

$$= (1+\delta)\Pi^r + \frac{\delta^2\Pi^c}{1-\delta}$$

To sustain collusion

$$V^* > V^r$$

$$\frac{\Pi^*}{1-\delta} > (1+\delta)\Pi^r + \frac{\delta^2\Pi^c}{1-\delta}$$

$$\Pi^* > (1+\delta)(1-\delta)\Pi^r + \delta^2\Pi^c$$

$$\Pi^* > (1+\delta-\delta-\delta^2)\Pi^r + \delta^2\Pi^c$$

$$\Pi^* > \Pi^r - \delta^2\Pi^r + \delta^2\Pi^c$$

$$\delta^2(\Pi^r - \Pi^c) > \Pi^r - \Pi^*$$

$$\delta^2 > \frac{\Pi^r - \Pi^*}{\Pi^r - \Pi^c} \quad \star$$

① $\Pi^* = \frac{\Pi^{monopoly}}{2}$

$\Pi^{mono} = TR - TC$
$= PQ - MC \cdot Q$
$= (P - MC)Q$
$= (130 - Q - 10)Q$
$= 120Q - Q^2$

FOC: $\frac{\partial \Pi}{\partial Q} = 0 = 120 - 2Q$

$\boxed{Q = 60}$, $q^* = 30$

$\boxed{P = 130 - Q = 70}$

$\frac{\Pi^{mono}}{2} = \frac{(70-10)60}{2}$
$= 1{,}800$

② $\Pi^r = (130 - Q - 10)q^r$
$= (130 - (30 + q^r) - 10)q^r$
$= 90q^r - q^{r2}$

$\frac{\partial \Pi^r}{\partial q^r} = 0 = 90 - 2q^r \Rightarrow q^r = 45$

$P = 130 - Q$
$= 130 - (30+45) = 55$

$\Pi^r = (P - mc)q^r = (55-10)45$
$= 2{,}025$

③ $\Pi_i^c = (130 - Q - 10)q_i^c$
$= (130 - (q_j^c + q_i^c) - 10)q_i^c$
$= 120q_i^c - q_j^c q_i^c - (q_i^c)^2$

$\frac{\partial \Pi^c}{\partial q_i^c} = 0 = 120 - q_j^c - 2q_i^c$

$q_i^c = 60 - \frac{q_j^c}{2}$

by symmetric $q_j^c = 60 - \frac{q_i^c}{2}$

$\Rightarrow q_i^c = q_j^c = 40$

$\Rightarrow \Pi^c = (130 - (40+40) - 10)\,40$
$= 1{,}600$

$$\therefore \delta^2 > \frac{2{,}025 - 1{,}800}{2{,}025 - 1{,}600}$$

$$\delta^2 > 0.5294$$

$$\delta > 0.7276$$

$$\boxed{\text{Critical } \delta = 0.7276 \ \#}$$

Problem 3:

(a) from problem 2 (detection still takes 2 periods)

$$V^* = \frac{\pi^*}{1-\delta}$$

$$V^r = \pi^r + \delta\pi^r + \delta^2\pi^B + \ldots + \delta^\infty\pi^B$$

=0 if play a Bertrand game

$$= \pi^r + \delta\pi^r$$

for $V^* > V^r$

we have $\frac{\pi^*}{1-\delta} > \pi^r + \delta\pi^r$

$$\pi^* > (1-\delta)(1+\delta)\pi^r$$

$$\pi^* > (1-\delta^2)\pi^r$$

$$\pi^* > \pi^r - \delta^2\pi^r$$

$$\delta^2 > \frac{\pi^r - \pi^*}{\pi^r}$$

Thus, from the below calculation

$$\delta^2 > \frac{3599 - 1800}{3599}$$

$$> 0.49986$$

$$\delta > 0.707$$

$\boxed{\text{or critical } \delta = 0.707}$ (a)

from problem 2:

$\pi^* = 1{,}800$ ($P = 70$, $q^* = 30$ each)

π^r = profit which the cheating firm will get from undercutting price by a little bit. Then, gets to sell to all the demanders.

⇒ Here, You can assume that the cheater can set any price as long as it is lower than 70. However, under-cutting price by a lot would not be optimal for the cheater either. This is because it only needs to set price < 70 to sells to everyone in the market.

⇒ In this case, I assume that the cheater's price = 69

Thus, $\pi^r = (P - mc)q^r$ (where $P = 130 - Q$, $69 = 130 - Q$, $Q = 61$)

$$= (69-10)(61)$$

$$= 3{,}599$$

(b) Bertrand Competition is more likely to sustain collusion since Bertrand punishment is harsher!