Numerical Solution MA217/2013 Final

1. (a) No, it is not possible to solve using matrix because the system is non-linear system.

   $\lambda = 1 \; f(x, y, z) = f(1,3,5) = 70$;

   $\lambda = -1 \; f(x, y, z) = f(-1,-3,-5) = -70$; ➔ absolute minimum

   (b) case 1: $x^2 + y^2 + z^2 = 35$ produces one critical point $\mu = 1$

   $f(x, y, z) = f(1,3,5) = 70$; $\mu = -1$ doesn't satisfy the complimentary and slackness condition.

   Case 2: $x^2 + y^2 + z^2 < 35$, produces no critical point.

   Hence, the absolute maximum is $f(1,3,5) = 70$

2. Free variable is x_4. $\quad \underline{\mathbf{x}} = \begin{bmatrix} 7 \\ 5 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \end{bmatrix} C; \quad C \in \Re$

3. Note that there is collection in the question (b) $\underline{\mathbf{B}} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$ should be $\underline{\mathbf{b}} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$.

   (a) $\underline{\mathbf{A}} \approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & a-3 \end{bmatrix}$

   If $a = 3$; $\underline{\mathbf{A}} \approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. 2 pivots and 3 unknowns

   1. It is not possible to have a unique solution because number of pivots (2) < number of unknowns (3).
   2. It is possible to has no solution or infinite number of solutions depending on $\underline{\mathbf{b}}$ if after the row operations it produces inconsistency or consistency system respectively.

   (b) If $a = -1$; $\underline{\mathbf{A}} \approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -4 \end{bmatrix}$. After the same row operations as (a), $\underline{\mathbf{b}} \approx \begin{bmatrix} 9 \\ 2 \\ -12 \end{bmatrix}$.

   Hence, $\underline{\mathbf{x}} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

4. (a) $\underline{\mathbf{b}} = \underline{0}$, $\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{0}$. This means that the system is always consistent since [0 0 … 0|*non-zero*] will not happen.

   (1) Hence, it is not possible to have no solution.
   (2) It is possible to have unique solution. Since the system is always consistent and if number of pivots (rank of matrix) = number of unknowns.
   (3) It is possible to have infinite solutions. Since the system is always consistent and if number of pivots (rank of matrix) < number of unknowns.

(b) $r \le n$; $\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{\mathbf{b}}$

(1) It is possible to have an unique solution if

(i) $r = m$ (no zero row) and $r=n$ or

(ii) [0 0 … 0|0] i.e. the system is consistent, $r<m$ and $r=n$.

(2) It is possible to have infinite number of solutions if

(i) $r = m$ (no zero row) and $r<n$ or

(ii) [0 0 … 0|0] i.e. the system is consistent, $r<m$ and $r<n$.

(4) It is possible to have no solution if [0 0 … 0|*non-zero*] i.e. the system is inconsistent and $r<m$

5. (a) $|\underline{\mathbf{A}}| = abcd$. Hence, for $\underline{\mathbf{A}}^{-1}$ to exist, $|\underline{\mathbf{A}}| \neq abcd \neq 0$.

Thus, $a \neq 0$, $b \neq 0$, $c \neq 0$ and $d \neq 0$.

(b) $\underline{\mathbf{A}}^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 & 0 \\ 0 & \frac{1}{b} & 0 & 0 \\ 0 & 0 & \frac{1}{c} & 0 \\ 0 & 0 & 0 & \frac{1}{d} \end{bmatrix}$, and $\underline{\mathbf{A}} \cdot \underline{\mathbf{A}}^{-1} = \underline{\mathbf{I}}$

6. (a) $|\underline{\mathbf{A}}| = 9(x^2 - 3)$, $|\underline{\mathbf{B}}| = 36$, $|\underline{\mathbf{C}}| = \frac{1}{2}$, $x = \pm 2$;

(b) $|\underline{\mathbf{A}}| = 54$, $\left|-2\underline{\mathbf{A}}^{T}\right| = 864$, $|\underline{\mathbf{D}}| = \frac{1}{3}$

(c) $x_4 = -\frac{11}{3}$

7. $I = 1/4$
8. $I = 1/3$
9. $Q = 6,647.08$ and $Q_{avg}=190$. (Quantity needs to be integer.)