

FN312 Investment

The Capital Asset Pricing Model

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Outline

- Market portfolio
- Return and risk for individual securities

Reading:
Chapter 9

Capital Asset Pricing Model (CAPM)

- It is the equilibrium model that underlies all modern financial theory
- Derived using principles of diversification with simplified assumptions
- Markowitz, Sharpe, Lintner and Mossin are researchers credited with its development

Assumptions

1. Individual behavior

- a. Investors are rational, mean-variance optimizers.
- b. Their common planning horizon is a single period.
- c. Investors all use identical input lists, an assumption often termed **homogeneous expectations**. Homogeneous expectations are consistent with the assumption that all relevant information is publicly available.

2. Market structure

- a. All assets are publicly held and trade on public exchanges.
- b. Investors can borrow or lend at a common risk-free rate, and they can take short positions on traded securities.
- c. No taxes.
- d. No transaction costs.

Market Portfolio

Resulting Equilibrium Conditions

- All investors will hold the same portfolio for risky assets
 - Market portfolio
- Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value

Market Risk Premium

- The risk premium on the market portfolio is proportional to its risk and the degree of risk aversion:

$$E(R_M) = \bar{A}\sigma_M^2$$

where

σ_M^2 : the variance of the market portfolio

\bar{A} : is the average degree of risk aversion across investors

Why?

Return and Risk for Individual Securities

Return and Risk For Individual Securities

- An individual security's risk premium is a function of:
 - Its contribution to the risk of the market portfolio
 - The covariance of returns with the assets that make up the market portfolio

Individual Securities: Example

Portfolio Weights	w_1	w_2	...	w_{GE}	...	w_n
w_1	$\text{Cov}(R_1, R_1)$	$\text{Cov}(R_1, R_2)$...	$\text{Cov}(R_1, R_{GE})$...	$\text{Cov}(R_1, R_n)$
w_2	$\text{Cov}(R_2, R_1)$	$\text{Cov}(R_2, R_2)$...	$\text{Cov}(R_2, R_{GE})$...	$\text{Cov}(R_2, R_n)$
\vdots	\vdots	\vdots		\vdots		\vdots
w_{GE}	$\text{Cov}(R_{GE}, R_1)$	$\text{Cov}(R_{GE}, R_2)$...	$\text{Cov}(R_{GE}, R_{GE})$...	$\text{Cov}(R_{GE}, R_n)$
\vdots	\vdots	\vdots		\vdots		\vdots
w_n	$\text{Cov}(R_n, R_1)$	$\text{Cov}(R_n, R_2)$...	$\text{Cov}(R_n, R_{GE})$...	$\text{Cov}(R_n, R_n)$

- The contribution of GE's stock to the variance of the market portfolio is

$$w_{GE} [w_1 \text{Cov}(R_1, R_{GE}) + w_2 \text{Cov}(R_2, R_{GE}) + \dots + w_{GE} \text{Cov}(R_{GE}, R_{GE}) + \dots + w_n \text{Cov}(R_n, R_{GE})]$$

Individual Securities: Example

- Covariance of GE return with the market portfolio:

$$w_{GE} \sum_{i=1}^n w_i \text{Cov}(R_i, R_{GE}) = w_{GE} \text{Cov} \left(\sum_{i=1}^n w_i R_i, R_{GE} \right)$$

- The reward-to-risk ratio for GE would be:

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE} E(R_{GE})}{w_{GE} \text{Cov}(R_{GE}, R_M)}$$

Individual Securities: Example

- Reward-to-risk ratio for investment in market portfolio:

$$\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(R_M)}{\sigma^2(R_M)}$$

- The reward-to-risk ratio for GE would be:

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE} E(R_{GE})}{w_{GE} \text{Cov}(R_{GE}, R_M)}$$

- These ratios should equal:

$$\frac{E(R_{GE})}{\text{Cov}(R_{GE}, R_M)} = \frac{E(R_M)}{\sigma^2(R_M)}$$

Individual Securities: Example

- The risk premium for GE:

$$E(R_{GE}) = \frac{\text{Cov}(R_{Ge}, R_M)}{\sigma^2(R_M)} E(R_M)$$

- Restating, we obtain:

$$E(r_{GE}) = r_f + \beta_{GE} [E(r_M) - r_f]$$

Expected Return-Beta Relationship

- CAPM holds for the overall portfolio because:

$$E(r_P) = \sum_k w_k E(r_k) \text{ and}$$

$$\beta_P = \sum_k w_k \beta_k$$

- This also holds for the market portfolio:

$$E(r_M) = r_f + \beta_M \left[E(r_M) - r_f \right]$$

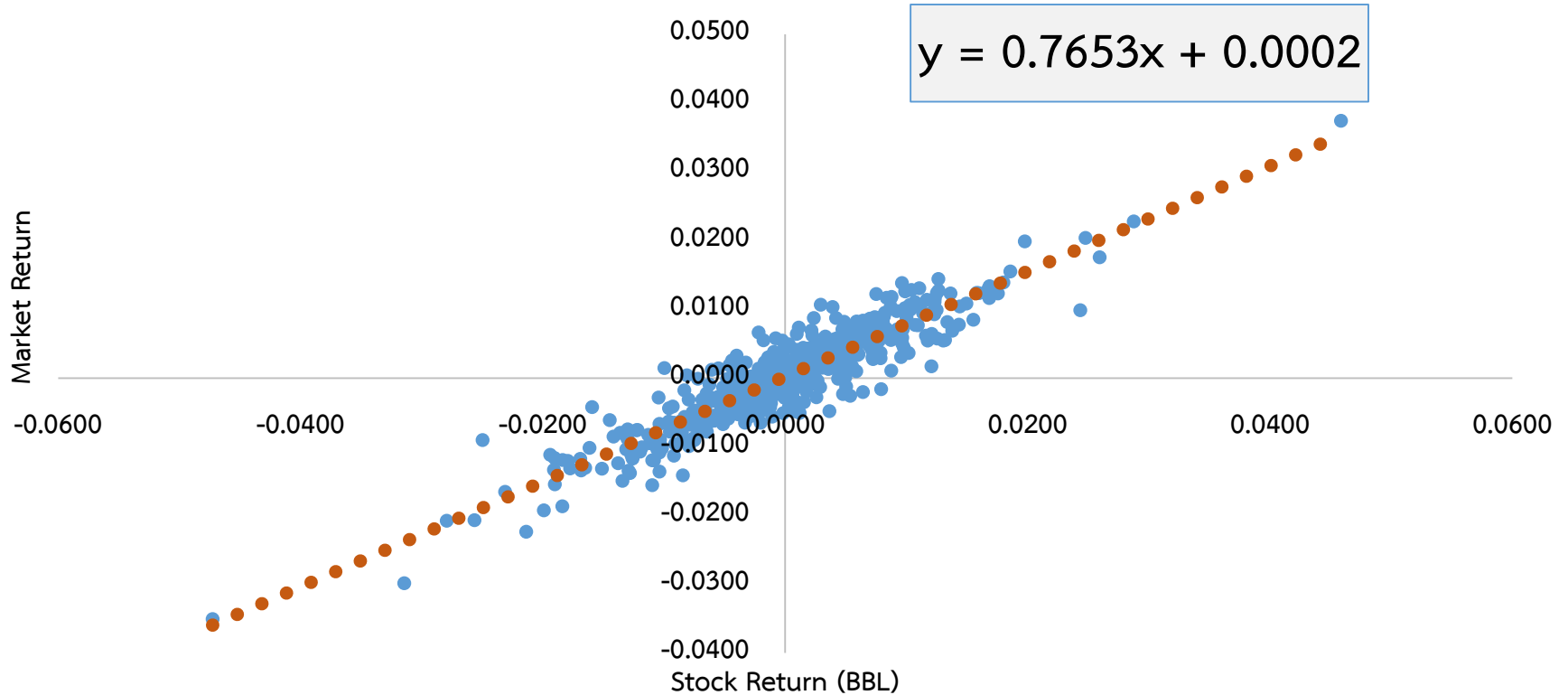
Single-Index Model and Realized Returns

- To move from expected to realized returns, use the index model in excess return form:

$$R_i = \alpha_i + \beta_i R_M + e_i$$

- The index model beta coefficient is the same as the beta of the CAPM expected return-beta relationship

Single-Index Model and Realized Returns



Example:

4. Here are data on two companies. The T-bill rate is 4% and the market risk premium is 6%.

Company	\$1 Discount Store	Everything \$5
Forecasted return	12%	11%
Standard deviation of returns	8%	10%
Beta	1.5	1.0

What would be the fair return for each company according to the capital asset pricing model (CAPM)?

The Security Market Line

$$E(r_M) = r_f + \beta_M [E(r_M) - r_f]$$

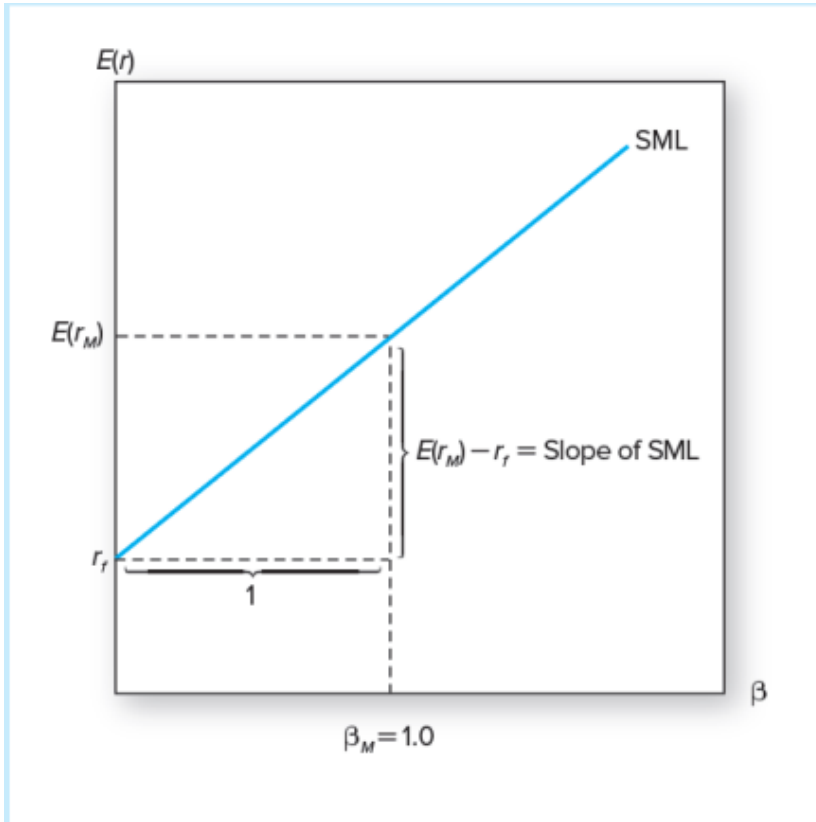


Figure 9.2 The security market line

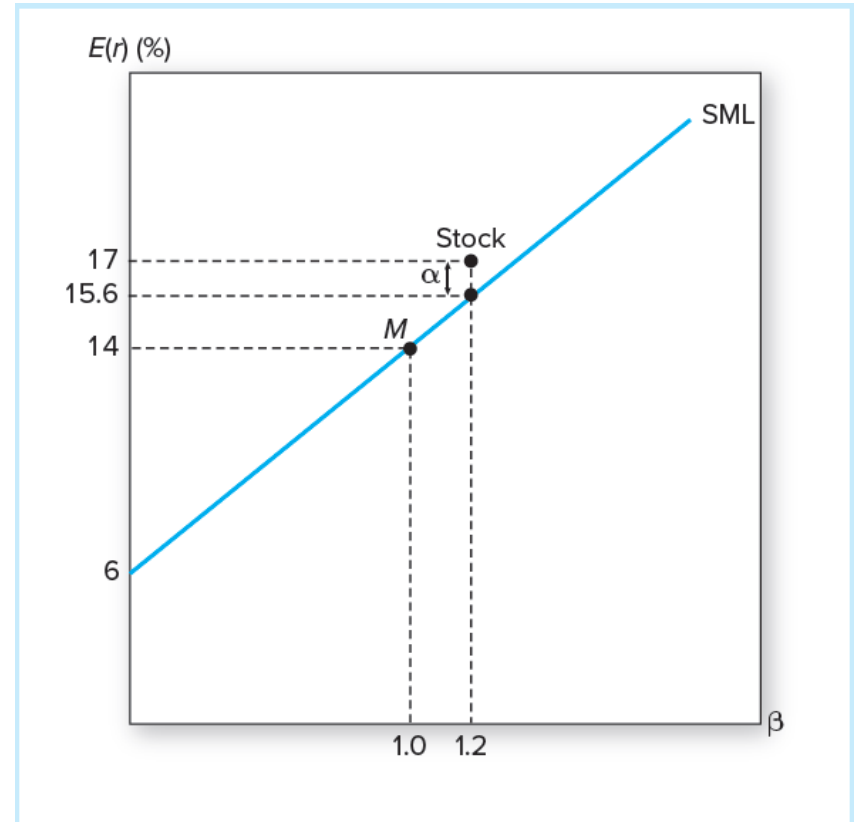


Figure 9.3 The SML and a positive-alpha stock

Example:

For Problems 10 through 16: If the simple CAPM is valid, which of the following situations are possible? Explain. Consider each situation independently.

10.

Portfolio	Expected Return	Beta
<i>A</i>	20%	1.4
<i>B</i>	25%	1.2

11.

Portfolio	Expected Return	Standard Deviation
<i>A</i>	30%	35%
<i>B</i>	40%	25%

12.

Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18%	24%
<i>A</i>	16%	12%

Example:

For Problems 17 through 19: Assume that the risk-free rate of interest is 6% and the expected rate of return on the market is 16%.

17. A share of stock sells for \$50 today. It will pay a dividend of \$6 per share at the end of the year. Its beta is 1.2. What do investors expect the stock to sell for at the end of the year?
18. I am buying a firm with an expected perpetual cash flow of \$1,000 but am unsure of its risk. If I think the beta of the firm is .5, when in fact the beta is really 1, how much *more* will I offer for the firm than it is truly worth?
19. A stock has an expected rate of return of 4%. What is its beta?

Extensions of the CAPM

1. Zero-Beta Model
 - Helps to explain positive alphas on low beta stocks and negative alphas on high beta stocks
2. Consideration of labor income and nontraded assets

Extensions of the CAPM

3. Merton's Multi-period Model and hedge portfolios
 - Incorporation of the effects of changes in the real rate of interest and inflation

4. Consumption-based CAPM
 - Rubinstein, Lucas, and Breeden
 - Investors allocate wealth between consumption today and investment for the future

Question?