1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5,564$	$\sum_{i=1}^{n} X_i Y_i = 7,524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$ TSS	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$ RSS		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

Find $\hat{\beta}_2 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{-174.20}{1098.8} = -0.1585$

$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 21.03 - (-0.1538)(12.20) = 22.9637$

$\hat{y} = 22.9627 - 0.1585 x_i$

$\therefore \hat{\beta}_1 = 22.9637,\ \hat{\beta}_2 = -0.1582$

explain $\hat{\beta}_1 = 22.9637 \rightarrow$ if $x = 0$, y will be equals to 22.9627 units (β_1)

$\hat{\beta}_2 = -0.1585 \rightarrow$ if x increase 1 unit, y will be decrease 0.1585 units.

b) Find r^2 and explain its meaning.

$r^2 = \dfrac{ESS}{TSS}$

$= 1 - \dfrac{RSS}{TSS} \Rightarrow 1 - \dfrac{873.14}{882.97} = 0.01113$

$= 1 - \dfrac{\Sigma \hat{u}_i^2}{\Sigma y_i^2}$

The coefficient of determination (r^2) is fit very well on the regression line comparing to the estimator ($\bar{y}$) by 1.11 %

c) If $X_i = 5$, estimate the value of $\ddot{Y}_i$ and explain its meaning.

$E(\hat{y} \mid x = 5)$

$\hat{y} = 22.9637 - 0.1585(5) = 22.1712$

if $x_i = 5$, we expected $\hat{y} = 22.1712$

d) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

$$\text{var}(u_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{873.14}{30-2} = 31.1836$$

$$\text{var}(\hat{\beta}_1) = \frac{\sum x_i^2}{n \times \sum (x_i - \bar{x})^2} \cdot \sigma^2 = \frac{5564}{30 \times 1098.8} \cdot 31.1836 = 5.2635$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{31.1836}{1098.8} = 0.0284$$

$\therefore \text{var}(u_i) = 31.836$, $\text{var}(\hat{\beta}_1) = 5\ 2635$, $\text{var}(\hat{\beta}_2) = 0.0284$

e) Test the hypothesis whether β_2 coefficients are different from zero at 0.05 level of significance.

step 1 hypothesis

$H_0 : \beta_2 = 0$: null hypothesis

$H_1 : \beta_2 \neq 0$: alternative hypothesis

step2 calculate test statistic

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{Se\,\hat{\beta}_2} = \frac{\hat{\beta}_2 - \beta_2}{\sqrt{\text{var}(\hat{\beta}_2)}} = \frac{-0.1585 - 0}{\sqrt{0.0284}} = -0.9405$$

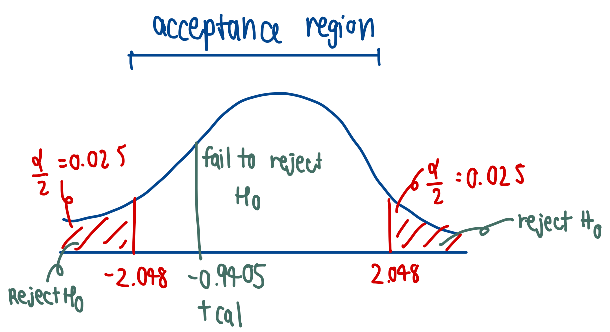
step3

$-\alpha = 0.05$

-Lower bound : $t_{\frac{\alpha}{2}} = -2.048$

-Upper bound : $t_{\frac{\alpha}{2}} = 2.048$

$t_{28,0.025} = 2.048$



step4 Conclusion

tcal lies within the acceptance region as $|t_{cal}| < |t_{cri}| = (0.9405 < 0.248)$ which mean we fail to reject the null hypothesis at the significant level of 95% or we cannot say for sure that β_2 is not zero 95 out of 100 times when we sample.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

step 1 hypothesis

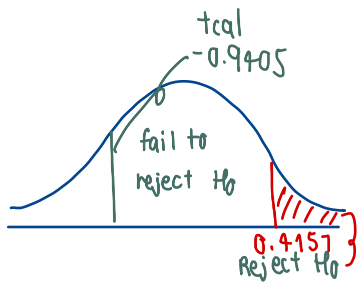
$H_0 : \hat{\beta}_2 \geq 0$

$H_1 : \hat{\beta}_2 < 0$

step2 calculate test statistic

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{se\,\hat{\beta}_2} = \frac{\hat{\beta}_2 - \beta_2}{\sqrt{var\,\hat{\beta}_2}} = \frac{0.1585 - 0}{\sqrt{0.0284}} = -0.9405$$

step3



- $\alpha = 0.01$

- The upper bound : $\beta_2 + t_{\frac{\alpha}{2}} \cdot \sigma\hat{\beta}_2$

$= 0 + (2.467 \cdot \sqrt{.0284}$

$= 0.4157$

step4 Conclusion

tcal lies within the acceptance region as $|t_{cal}| > |t_{cri}| = (0.9405 > 0.4157)$ which mean we fail to reject the null hypothesis at the significant level of 99%. or we cannot say for sure that β_2 is more than zero 99 out of 100 times when we sample.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\underset{\substack{(52)\\ se\,\hat{\beta}_1}}{\overset{\hat{\beta}_1}{\hat{Y}_i = 7{,}836}} - \underset{\substack{(411.8)\\ se\,\hat{\beta}_2}}{\overset{\hat{\beta}_2}{502.4}}X_i$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

Yes, as the regression function has the negative slope between Y (market price of car) and X (car aged). So that, when X increase, Y will decreased.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

Step1

$var(\hat{Y}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]$ given $\rightarrow X_i = 5 \rightarrow \hat{Y}_0 = 7{,}836 - 502.4(5) = 5{,}324$

$\rightarrow var(\hat{Y}_0) = 212{,}877 \left[\frac{1}{11} + \frac{(5-7.45)^2}{78.73}\right] = 35582.5345$

Step2

$\sigma_{\hat{Y}_0} = \sqrt{35582.5345} \approx 188.6333$

Step3 find the 95% CI

$\rightarrow t_{\frac{0.05}{2}} = t_{0.025,\,9}^{(n-2)} = 2.262$

$\circ\ Pr\left[\hat{Y}_0 - (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{Y}_0}) \leq Y_0 \leq \hat{Y}_0 + (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{Y}_0})\right]$

$\rightarrow Pr\left[5{,}324 - (2.262 \cdot 188.6333) \leq Y_0 < 5{,}324 + (2.262 \cdot 188.633)\right]$

$= 4{,}897.3115 \leq Y_0 \leq 5750.6885$

$\therefore$ When his car is 5 years old, the market estimated price would be in the range between USD 4,897.3115 to USD 5,750.6885 at the significant level of 95%.

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

The SRF will change, only if we multiply or divide them. As in this question, the changed of the amout will be multiply by 10.

SRF : $Y = 7836 - 502.4(10X)$

d) Calculate the elasticity of market price when a car is 10 years old.

elasticity of price $= \frac{dY}{dX} \times \frac{X}{Y}$

note: $\hat{Y}$ at 10 year $= 7836 - 502.4(10)$

$= 2{,}812$

So, the elasticity of price $= -502.4 \times \frac{10}{2812} = |-1.7866| = 1.7866$

$\therefore$ so, the elasticity of market price equals to 1.7866

which mean the times' used car is highly elastic to the price