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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and \; \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and \; var(\hat{\beta}_2)$

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

$\bar{x} = 77.625$

$\bar{Y} = 3.2125$

$\hat{Y}_i = 0.5655 + 0.03406 x_i$

Table 1

x_i^2	$\hat{u}_i^2$	Student	Y_i (GPA)	X_i (point)	$x_i - \bar{x}$	$Y_i - \bar{Y}$	$(x_i-\bar{x})(Y_i-\bar{Y})$	$(x_i-\bar{x})^2$	$\hat{Y}_i$	$\hat{u}_i$ ($Y - \hat{Y}_i$)
3969	0.25122	1	2.8	63	-14.625	-0.4125	6.0328	213.8906	2.71128	0.50122
5184	0.0379	2	3.4	72	-5.625	0.1875	-1.0547	31.6406	3.01782	0.19468
6084	0.0001	3	3.0	78	0.375	-0.2125	-0.0797	0.1406	3.22218	-0.00968
6561	0.0125	4	3.5	81	3.375	0.2875	0.9703	11.3906	3.32436	-0.11186
7569	0.0999	5	3.6	87	9.375	0.3875	3.6328	87.8906	3.52872	-0.31622
5625	0.0086	6	3.0	75	-2.625	-0.2125	0.5578	6.8906	3.12	0.0925
5625	0.0086	7	2.7	75	-2.625	-0.5125	1.3453	6.8906	3.12	0.0925
8100	0.1751	8	3.7	90	12.375	0.4875	6.0328	153.1406	3.6309	-0.4184
48,717	0.59392						17.4374 ← $\Sigma(x_i-\bar{x})(Y_i-\bar{Y})$	511.8748 ← $\Sigma(x_i-\bar{x})^2$		0.02474

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

$\hat{\beta}_1, \hat{\beta}_2$ อธิบาย $\hat{\beta}_1, \hat{\beta}_2$

$$\hat{\beta}_1 = \bar{Y}_i - \hat{\beta}_2 \bar{x}$$

$$\hat{\beta}_2 = \frac{\Sigma(x_i-\bar{x})(Y_i-\bar{Y})}{\Sigma(x_i-\bar{x})^2} = \frac{17.4374}{511.8748} \approx 0.03406$$

$$\therefore \hat{\beta}_1 = (3.2125) - (0.0341)(77.625)$$
$$= 3.2125 - 2.647 \approx 0.5655$$

Interpret the regression ; Therefore $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$ so $\hat{Y}_i = 0.5655 + 0.0347 x_i$, if the economics exam point equal to zero means that the GPA of the students will be $\hat{\beta}_1$, 0.5655 on average and if the economics exam point change by 1 unit, the GPA of the students will increase by β_2, 0.03406 on average.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

$Y - \hat{Y}_i = \hat{u}_i$; $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$

$\hat{Y}_i = \bar{Y} - \hat{u}_i$; $\hat{Y}_i = 0.5655 + 0.03406 x_i$

$\sum_{i=1}^{n} \hat{u}_i \approx 0.02474 \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \hat{\sigma}_u^2 = \frac{\Sigma u_i^2}{n-k} = \frac{0.5939}{8-2} \approx 0.099$$

$$var(\hat{\beta}_1) = \frac{\sigma_u^2\, \Sigma x_i^2}{n\Sigma(x_i-\bar{x})^2} = \frac{(0.099)(48,717)}{8 \cdot (511.8748)} = 1.178$$

$$var(\hat{\beta}_2) = \frac{\sigma_u^2}{\Sigma(x_i-\bar{x})^2} = \frac{(0.099)}{511.8748} = 0.00019341$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

$\bar{x} = 20$
$\bar{y} = 9.1$

2. Data is listed in the table

X_i	Y_i	$x_i-\bar{x}$	$y_i-\bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$	$(x-\bar{x})^2$	$\hat{Y}_i$	$\hat{u}_i = Y_i-\hat{Y}_i$	u_i^2	x_i^2
10	0	-10	-9.1	91	100	0.15	-0.15	0.0225	100
12	2	-8	-7.1	56.8	64	1.94	0.06	0.0036	144
14	5	-6	-4.1	24.6	36	3.73	1.27	1.6129	196
16	6	-4	-3.1	12.4	16	5.52	0.48	0.2304	256
18	7	-2	-2.1	4.2	4	7.31	-0.31	0.0961	324
22	10	2	0.9	1.8	4	10.89	-0.89	0.7921	484
24	10	4	0.9	3.6	16	12.68	-2.68	7.1824	576
26	15	6	5.9	35.4	36	14.47	0.53	0.2809	676
28	16	8	6.9	55.2	64	16.26	-0.26	0.0676	784
30	20	10	10.9	109	100	18.05	1.95	3.8025	900
				394	440		0	14.091	4440

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

$\hat{\beta}_1, \hat{\beta}_2$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{x}$

$\hat{\beta}_2 = \dfrac{\Sigma(x_i-\bar{x})(Y_i-\bar{Y})}{\Sigma(x_i-\bar{x})^2} = \dfrac{394}{440} = 0.895$

$\hat{\beta}_1 = 9.1 - (0.895)(20)$
$= -8.8$

$\hat{\beta}_2 = 0.895$ means that if x change by 1 unit, on average, Y will change by 0.895 unit in the same direction.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

$Y_i - \hat{Y}_i = \hat{u}_i$

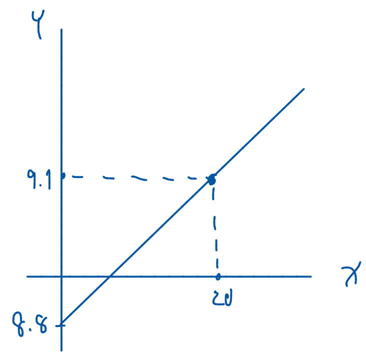
$\hat{Y}_i = -8.8 + 0.895 x_i$

and $\Sigma \hat{u}_i \approx 0$

$\Sigma(\cancel{\beta_1} + \cancel{\beta_2 x_i} - \cancel{\beta_1} - \cancel{\beta_2 x_i}) = \Sigma \hat{u}_i = 0$ and check in the table, it's true

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{x} \rightarrow \bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 20$
$= -8.8 + 0.895(20)$
$\bar{Y} = 9.1$



2.4 If $X_i = 18$, what is the predicted Y? $\rightarrow \hat{Y}_i$

$\hat{Y}_i = -8.8 + (0.895)(18)$
$= 7.31$ #

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

$var(\hat{u}_i) = var(u_i) = \dfrac{\Sigma u_i^2}{n-2} = \dfrac{14.091}{8} = 1.76137$
$\hat{\sigma}_u^2$

$var(\hat{\beta}_1) = \dfrac{\hat{\sigma}_u^2 \Sigma x_i^2}{n\Sigma(x_i-\bar{x})^2} = \dfrac{1.761375(4440)}{10 \cdot 440}$ #

$var(\hat{\beta}_2) = \dfrac{\hat{\sigma}_u^2}{\Sigma(x_i-\bar{x})^2} = \dfrac{1.761375}{440}$ #

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

and $E(\hat{\beta}_2)\beta_2$

$Y_i = \beta_1 + \beta_2 x_i + u_i$; $\bar{Y} = \beta_1$

unbiased estimator ; $E(\hat{\theta}) = \theta$

$E(\hat{\beta}_1) = \beta_1$

$E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{x})$

$\bar{Y} - \left(\frac{\Sigma(x_i - \bar{x})(Y_i - \bar{Y})}{\Sigma(x_i - \bar{x})^2}\right)\bar{x}$; $k_i = \frac{(x_i - \bar{x})}{\Sigma(x_i - \bar{x})^2}$

$\bar{Y} - \left(\frac{\Sigma(x_i - \bar{x})Y_i}{\Sigma(x_i - \bar{x})^2} - \cancel{\frac{\Sigma(x_i - \bar{x})\bar{Y}}{\Sigma(x_i - \bar{x})^2}}^{0}\right)\bar{x}$

$\frac{\bar{Y}\Sigma(x_i - \bar{x})}{\Sigma(x_i - \bar{x})^2}$

$= \bar{Y}\Sigma x_i - \bar{Y} \cdot \Sigma \bar{x}$

$= \frac{\Sigma Y}{n} \cdot \Sigma x_i - \frac{\Sigma Y}{n} \cdot \frac{\Sigma x}{\cancel{n}} \cdot \cancel{n}$

$= 0$

$\bar{Y} - \left(\frac{\Sigma(x_i - \bar{x})Y_i}{\Sigma(x_i - \bar{x})^2}\right)\bar{x}$

$\bar{Y} - (\Sigma k_i Y_i)\bar{x}$

$\bar{Y} - (\Sigma k_i(\beta_1 + \beta_2 x_i + u_i))\bar{x}$

$\bar{Y} - (\Sigma k_i \beta_1 + \Sigma k_i \beta_2 x_i + \Sigma k_i u_i)\bar{x}$

$\Sigma k_i = 0$ proof ; $\frac{\cancel{\Sigma(x_i - \bar{x})}^{0}}{\Sigma(x_i - \bar{x})^2} = 0$

$\frac{\Sigma x}{n} \cdot \Sigma x = \frac{\Sigma x_i^2}{n}$

$\Sigma k_i x_i = 1$ proof ; $\frac{\Sigma(x_i - \bar{x})x_i}{\Sigma(x_i - \bar{x})^2} = \frac{\cancel{\Sigma x_i^2 - \bar{x}\Sigma x_i}}{\Sigma x_i^2 - 2\bar{x}\Sigma x_i + \Sigma(\bar{x})^2} = 1$

$\to \Sigma x_i^2 - 2\frac{\Sigma x^2}{n} + \frac{(\Sigma x)^2}{n}$

$= \cancel{\Sigma x_i^2 - \frac{\Sigma x_i^2}{n}}$

$\therefore (0)(\beta_1) + (\beta_2)(1) + \frac{\Sigma x_i u_i}{\Sigma x_i^2}$

$\frac{1}{\Sigma x_i^2}(x_1 u_1 + x_2 u_2 + \ldots + x_n u_n)$

$= \frac{1}{\Sigma x_i^2}(\cancel{E(x_1 u_1)}^{0} + \cancel{E(x_2 u_2)}^{0} + \ldots + \cancel{E(x_n u_n)})$

$= 0$

$\therefore \beta_2$; $E(\beta_2$

$= \bar{Y} - \beta_2 \bar{x}$

$E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{x})$

$= E\left(\frac{\Sigma Y_i}{n} - \beta_2 \bar{x}\right)$

$= \frac{\Sigma(\beta_1 + \beta_2 x_i + u_i)}{n} - \beta_2 \bar{x}$

$= \frac{\Sigma \beta_1}{n} + \frac{\Sigma \beta_2 x_i}{n} + \cancel{\frac{\Sigma u_i}{n}}^{0 \text{ from Assumption}} - \beta_2 \bar{x}$

$= \frac{n\beta_1}{n} + \cancel{\frac{\beta_2 \Sigma x_i}{n}} - \cancel{\frac{\beta_2 \Sigma x_i}{n}} = \beta_1$

$\therefore E(\beta_1) = \beta_1 \to E(\hat{\beta}_1) = \beta_1$

$\therefore \hat{\beta}_1$ is unbias estimator of β_1 #