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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^{5}(a+bx_i) = \sum_{i=1}^{5} a + b\sum_{i=1}^{5} x_i = 5 \cdot a + b\sum_{i=1}^{5} x_i$

$= 5a + bX_1 + bX_2 + bX_3 + bX_4 + bX_5$

b. $\sum_{y=0}^{5} f(x+y) = f[(x+0)+(x+1)+(x+2)+(x+3)+(x+4)]$

$= f(5x+10)$

c. $\sum_{i=1}^{10} i^2 = 1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2$

$= 385$

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y) = \sum_{x=1}^{2}\left[(2x+2)+(2x+3)+(2x+4)\right]$

$= \sum_{x=1}^{2}(6x+9) = (6(1)+9)+(6(2)+9) = 36$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b 0.0625	b 0.125	2.25b 0.28125	2b 0.25	1.5b 0.1875	0.5b 0.0625	0.25b 0.03125

** when b is constant number

a. Find the value of b

$0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$

$b = 0.125$

b. Find the answer for $P(X \le 2)$

$= (P=-2)+(P=-1)+(P=0)+(P=1)+(P=2)$

$= 0.0625 + 0.125 + 0.28125 + 0.25 + 0.1875$

$= 0.90625$

c. Find the answer for $P(-2 \le X \le 3)$

$= 0.96875$

d. Find the answer for $P(X \ge 1)$

$= 0.53125$

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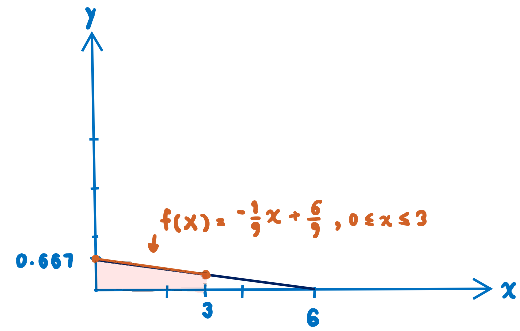
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$\int_1^3 -\frac{1}{9}x + \frac{6}{9} = -\frac{1}{9}x^2 + \frac{6}{9}x \Big|_1^3$$

$$= \left[-\frac{1}{9}(3)^2 + \frac{6}{9}(3)\right] - \left[-\frac{1}{9}(1)^2 + \frac{6}{9}(1)\right]$$

$$= \frac{4}{9}$$

c. Find the answer for $P(X \ge 2)$

$$\int_2^3 -\frac{1}{9}x + \frac{6}{9} = -\frac{1}{9}x^2 + \frac{6}{9}x \Big|_2^3$$

$$= \left[-\frac{1}{9}(3)^2 + \frac{6}{9}(3)\right] - \left[-\frac{1}{9}(2)^2 + \frac{6}{9}(2)\right]$$

$$= \frac{7}{9}$$

d. Find the expected value of X

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_1^3 x \cdot \left(-\frac{1}{9}x + \frac{6}{9}\right) dx$$

$$= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx = -\frac{1}{18}x^3 + \frac{6}{9}x^2 \Big|_0^3$$

$$= \left[-\frac{1}{18}(3)^3 + \frac{6}{9}(3)^2\right] = \frac{9}{2}$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

Throwing a dice

Coin Toss	y \ x	1	2	3	4	5	6	
	1	1/12	1/12	1/12	1/12	1/12	1/12	1/2
	0	1/12	1/12	1/12	1/12	1/12	1/12	1/2
		1/6	1/6	1/6	1/6	1/6	1/6	1

   b. Find the marginal probability distribution function (PDF) of X

The marginal prob of X $P(X=x)$ is represented in green

   c. Find the marginal probability distribution function (PDF) of Y

The marginal prob of Y $P(Y=y)$ is represented in yellow

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

x	P(X=x|Y=1)
1	$\frac{1}{12}/\frac{1}{2} = \frac{1}{6}$
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

   e. Find the expected value of X given Y is equal to 1

$$E(X \mid Y=1) = \sum x_i \, P(X=x_i \mid Y=1) = \sum \frac{x_i P(X=x_i, Y=1)}{P(Y=1)} = \frac{1}{P(Y=1)} \sum x_i \, P(X=x_i, Y=1)$$

$$= \frac{1}{0.5}\left[\left(1\cdot\frac{1}{12}\right)+\left(\cancel{2}\cdot\frac{1}{\cancel{12}_6}\right)+\left(\cancel{3}\cdot\frac{1}{\cancel{12}_4}\right)+\left(\cancel{4}\cdot\frac{1}{\cancel{12}_3}\right)+\left(5\cdot\frac{1}{12}\right)+\left(\cancel{6}!\,\frac{1}{\cancel{12}_2}\right)\right] = \frac{7}{2}$$

   f. Find the variance of X given Y is equal to 1

$$V(X \mid Y=1) = \sum \left(X - E(X \mid Y=1)\right)^2 \cdot P(X \mid Y=1)$$

$$= \left[\left(\left(1-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right)+\left(\left(2-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right)+\left(\left(3-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right)+\left(\left(4-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right)+\left(\left(5-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right)+\left(\left(6-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right)\right]$$

$$= \left(\frac{25}{4}\cdot\frac{1}{6}\right)+\left(\frac{9}{4}\cdot\frac{1}{6}\right)+\left(\frac{1}{4}\cdot\frac{1}{6}\right)+\left(\frac{1}{4}\cdot\frac{1}{6}\right)+\left(\frac{9}{4}\cdot\frac{1}{6}\right)+\left(\frac{25}{4}\cdot\frac{1}{6}\right)$$

$$= \frac{120}{24} = \frac{10}{3}$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$

$$= \frac{1}{N} E(x_1 + x_2 + x_3)$$

$$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3)\right]$$

$$= \frac{1}{3}\left[\mu_x + \mu_x + \mu_x\right]$$

$$= \frac{1}{3} \cdot 3\mu_x$$

$$= \mu_x$$

$$Var(\bar{x}) = var\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$

$$= \frac{1}{N^2} Var(x_1 + x_2 + x_3)$$

$$= \frac{1}{N^2}\left[V(X_1) + V(X_2) + V(X_3) + 2Cov(x_1, x_2) + 2Cov(x_1, x_3) + 2Cov(x_2, x_3)\right]$$

$$= \frac{1}{3^2}\left[\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2\right]$$

$$= \frac{\sigma_x^2}{4}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$

$$= \frac{1}{N} E(x_1 + x_2 + x_3 + x_4)$$

$$= \frac{1}{4} \cdot 4\mu_x$$

$$= \mu_x$$

$$var(\bar{x}) = var\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$

$$= \frac{1}{N^2} var(x_1 + x_2 + x_3 + x_4)$$

$$= \frac{1}{16} \cdot 4\sigma_x^2$$

$$= 0.25\sigma_x^2$$

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$\tilde{x} = \frac{1}{4}(0.5x_1 + x_2 + 0.5x + 2x_4)$

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$E(\tilde{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$

$= \frac{1}{4}E(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$= \frac{1}{4}\left[0.5E(x_1) + E(x_2) + 0.5E(x_3) + 2E(x_4)\right]$

$= \frac{1}{4}(0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$

$= \frac{1}{4}\cdot 4\mu_x = \mu_x$

thus $\tilde{x}$ is unbiased estimator of μ

$Var(\tilde{x}) = var\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$

$= \frac{1}{4^2}Var(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$= \frac{1}{4^2}\left[.25\,Var(x_1) + Var(x_2) + .25\,Var(x_3) + var(x_4)\right]$

$= \frac{1}{4^2}\left[.25\sigma_x^2 + \sigma_x^2 + .25\sigma_x^2 + 4\sigma_x^2\right]$

$= \frac{5.5\sigma_x^2}{16}$

$= 0.34\sigma_x^2$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$Var(\bar{x}) = 0.25\sigma_x^2$

$Var(\tilde{x}) = 0.34\sigma_x^2$

$var(\tilde{x}) > var(\bar{x})$ so, $\bar{x}$ is a better estimator because $\bar{x}$ has smaller variance indicates that data point are close to the mean