8 ALTERNATIVE THEORIES OF ENDOGENOUS GROWTH

In the preceding seven chapters, we have laid out the basic questions of economic growth and some of the main answers provided by economic research. The next two chapters depart from this flow in two important directions. This chapter examines alternative theories of endogenous growth that have been proposed; in this sense, it could be read immediately following Chapter 5 or even Chapter 3. Chapter 9 turns to a question that has received much attention in the history of economic thought: the sustainability of long-run growth in the presence of finite natural resources. It, too, could be read any time after Chapter 3.

In this book, we have purposely limited ourselves to a few closely related models in an effort to formulate a general theory of growth and development. One result of this method of exposition is that we have not been able to discuss a large number of the growth models that have been developed in the last fifteen years. This chapter presents a brief discussion of some of these other models.

The models described so far all have the implication that changes in government policies, such as subsidies to research or taxes on investment, have *level* effects but no long-run *growth* effects. That is, these policies raise the growth rate temporarily as the economy grows to a higher level of the balanced growth path. But in the long run, the growth rate returns to its initial level.

Originally, the phrase "endogenous growth" was used to refer to models in which changes in such policies could influence the growth rate permanently.[1] Differences in growth rates across countries were thought to reflect permanent differences in underlying growth rates. This is not the point of view presented in this book. Nevertheless, it is important to understand how these alternative models work. Developing such an understanding is the primary goal of this chapter. After we have presented the mechanisms at work, we will discuss some of the evidence for and against these models.

8.1 A SIMPLE ENDOGENOUS GROWTH MODEL: THE "AK" MODEL

One of the simplest models that allows for endogenous growth (in the sense that policies can influence the long-run growth rate) is easily derived by considering the original Solow model of Chapter 2. Consider our first exposition of that model, in which there is no exogenous technological progress (i.e., $g \equiv \dot{A}/A = 0$). However, modify the production function so that $\alpha = 1$:

$$Y = AK, \tag{8.1}$$

where A is some positive constant.[2] It is this production function that gives the AK model its name.[3] Recall that capital is accumulated as individuals save and invest some of the output produced in the economy

[1]According to *Merriam Webster's Collegiate Dictionary*, "endogenous" means "caused by factors inside the organism or system." Technological change is clearly endogenous in this sense in the models we have discussed in the later chapters of this book. However, without (exogenous) population growth, per capita income growth eventually stops. For this reason, models such as that presented in Chapter 5 are sometimes referred to as "semi-endogenous" growth models.

[2]The careful reader will notice that strictly speaking, with $\alpha = 1$, the production function in Chapter 2 should be written as $Y = K$. It is traditional in the model we are presenting to assume that output is *proportional* to the capital stock rather than exactly equal to the capital stock.

[3]Romer (1987) and Sergio Rebelo (1991) were early expositors of this model.

rather than consuming it:

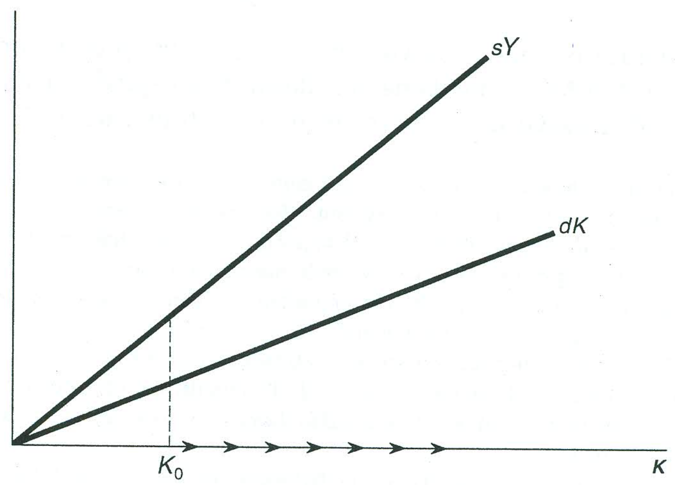
$$\dot{K} = sY - dK, \tag{8.2}$$

where s is the investment rate and d is the rate of depreciation, both assumed to be constant. We assume that there is no population growth, for simplicity, so that we can interpret upper-case letters as per capita variables (e.g., assume the economy is populated by only one person).

Now consider the familiar Solow diagram, drawn for this model in Figure 8.1. The dK line reflects the amount of investment that has to occur just to replace the depreciation of the capital stock. The sY curve is total investment as a function of the capital stock. Notice that because Y is linear in K, this curve is actually a straight line, a key property of the AK model. We assume that total investment is larger than total depreciation, as drawn.

Consider an economy that starts at point K_0. In this economy, because total investment is larger than depreciation, the capital stock grows. Over time, this growth continues: at every point to the right of K_0, total investment is larger than depreciation. Therefore, the capital stock is always growing, and growth in the model never stops.

FIGURE 8.1 **THE SOLOW DIAGRAM FOR THE AK MODEL**



The explanation for this perpetual growth is seen by comparing this figure to the original Solow diagram in Chapter 2. There, you will recall, capital accumulation was characterized by diminishing returns because $\alpha < 1$. Each new unit of capital that was added to the economy was slightly less productive than the previous unit. This meant that eventually total investment would fall to the level of depreciation, ending the accumulation of capital (per worker). Here, however, there are *constant returns* to the accumulation of capital. The marginal product of each unit of capital is always A. It does not decline as additional capital is put in place.

This point can be shown mathematically, as well. Rewrite the capital accumulation equation (8.2) by dividing both sides by K:

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - d.$$

Of course, from the production function in equation (8.1), $Y/K = A$, so that

$$\frac{\dot{K}}{K} = sA - d.$$

Finally, taking logs and derivatives of the production function, one sees that the growth rate of output is equal to the growth rate of capital, and therefore

$$g_Y \equiv \frac{\dot{Y}}{Y} = sA - d.$$

This simple algebra reveals a key result of the AK growth model: the growth rate of the economy is an increasing function of the investment rate. Therefore, government policies that increase the investment rate of this economy permanently will increase the growth rate of the economy permanently.

This result can be interpreted in the context of the Solow model with $\alpha < 1$. Recall that in this case, the sY line is a curve, and the steady state occurs when $sY = dK$ (since we have assumed $n = 0$). The parameter α measures the "curvature" of the sY curve: if α is small, then the curvature is rapid, and sY intersects dK at a "low" value of K^*. On the other hand, the larger α is, the further away the steady-state value, K^*, is from K_0. This implies that the transition to steady state is longer. The case of $\alpha = 1$ is the limiting case, in which the transition dynamics never end. In this way, the AK model generates growth endogenously. That is, we need not assume that anything in the model grows at some exogenous rate in order to generate per capita growth — certainly not technology, and not even population.

8.2 INTUITION AND OTHER GROWTH MODELS

The AK model generates endogenous growth because it involves a fundamental linearity in a differential equation. This can be seen by combining the production function and the capital accumulation equation of the standard Solow model (with the population normalized to one):

$$\dot{K} = sAK^{\alpha} - dK.$$

If $\alpha = 1$, then this equation is linear in K and the model generates growth that depends on s. If $\alpha < 1$, then the equation is "less than linear" in K, and there are diminishing returns to capital accumulation. If we divide both sides by K, we see that the growth rate of the capital stock declines as the economy accumulates more capital:

$$\frac{\dot{K}}{K} = sA\frac{1}{K^{1-\alpha}} - d.$$

Another example of how linearity is the key to growth can be seen by considering the exogenous growth rate of technology in the Solow model. Our standard assumption in that model can be written as

$$\dot{A} = gA.$$

This differential equation is linear in A, and permanent changes in g increase the growth rate permanently in the Solow model with exogenous technological progress. Of course, changes in government policies do not typically affect the exogenous parameter g, so we do not think of this model as generating endogenous growth. What these two examples show, however, is the close connection between linearity in a differential equation and growth.[4]

[4]In fact, this intuition can be a little misleading in more complicated models. For example, in a model with two differential equations, one can be "less than linear" but if the other is "more than linear" then the model can still generate endogenous growth. See Mulligan and Sala-i-Martin (1993).

Other endogenous growth models can be created by exploiting this intuition. For example, another very famous model in this class is a model based on human capital, created by Robert E. Lucas, Jr., the 1995 Nobel laureate in economics. The Lucas (1988) model assumes a production function similar to the one we used in Chapter 3:

$$Y = K^{\alpha}(hL)^{1-\alpha},$$

where h is human capital per person. Lucas assumes that human capital evolves according to

$$\dot{h} = (1 - u)h,$$

where u is time spent working and $1 - u$ is time spent accumulating skill. Rewriting this equation slightly, one sees that an increase in time spent accumulating human capital will increase the growth rate of human capital:

$$\frac{\dot{h}}{h} = 1 - u.$$

Notice that h enters the production function of this economy just like labor-augmenting technological change in the original Solow model of Chapter 2. So there is no need to solve this model further. It works just like the Solow model in which we call A human capital and let $g = 1 - u$. Therefore, in the Lucas model, a policy that leads to a permanent increase in the time individuals spend obtaining skills generates a permanent increase in the growth of output per worker.

8.3 EXTERNALITIES AND AK MODELS

We showed in Chapter 4 that the presence of ideas or technology in the production function means that production is characterized by increasing returns to scale. Then we argued that the presence of increasing returns to scale requires the introduction of imperfect competition: if capital and labor were paid their marginal products, as they would be in a world with perfect competition, no output would remain to compensate for the accumulation of knowledge.

There is an alternative way of dealing with the increasing returns that allows us to maintain perfect competition in the model. By the argument just given, individuals cannot be compensated for accumulating knowledge. However, if the accumulation of knowledge is itself an accidental by-product of other activity in the economy, it may still occur. That is, the accumulation of knowledge may occur because of an *externality*.

Consider a by-now-standard production function for an individual firm:

$$Y = BK^{\alpha}L^{1-\alpha}. \tag{8.3}$$

In this equation, there are constant returns to capital and labor. Hence, if B is accumulated endogenously, production is characterized by increasing returns.

Suppose that individual firms take the level of B as given. However, assume that in reality, the accumulation of capital generates new knowledge about production in the economy as a whole. In particular, suppose that

$$B = AK^{1-\alpha}, \tag{8.4}$$

where A is some constant. That is, an accidental by-product of the accumulation of capital by firms in the economy is the improvement of the technology that firms use to produce. An individual firm does not recognize this effect when it accumulates capital because it is small relative to the economy. This is the sense in which technological progress is *external* to the firm. Firms do not accumulate capital because they know it improves technology; they accumulate capital because it is a useful input into production. Capital is paid its private marginal product $\alpha Y/K$. However, it just so happens that the accumulation of capital provides an extraordinarily useful and unexpected benefit to the rest of the economy: it results in new knowledge.[5]

Combining equations (8.3) and (8.4), we obtain

$$Y = AKL^{1-\alpha}. \tag{8.5}$$

[5] This externality is sometimes called external "learning by doing." Firms learn better ways to produce as an accidental by-product of the production process. Kenneth Arrow (1962), the 1972 Nobel Prize winner in economics, and Marvin Frankel (1962) first formalized this process in a growth model.

Assuming that the population of this economy is normalized to one, this is exactly the production function considered at the beginning of this chapter.

To summarize, there are two basic ways to deal with the increasing returns to scale that are required if one wishes to endogenize the accumulation of knowledge: imperfect competition and externalities. One can drop the assumption of perfect competition and model the accumulation of knowledge as resulting from the intentional efforts of researchers who search for new ideas. Alternatively, one can maintain perfect competition and assume that the accumulation of knowledge is an accidental by-product — an externality — of some other activity in the economy, such as capital accumulation.

As is evident from the order of presentation and the time spent developing each alternative, the opinion of this author is that knowledge accumulation is more accurately modeled as the desired outcome of entrepreneurial effort rather than as an accidental by-product of other activity. One need not observe for long the research efforts in Silicon Valley or the biotechnology firms of Route 128 in Boston to see the importance of the intentional search for knowledge. Some other evidence comparing these two approaches will be presented in the next section.

First, however, it is worth noting that the externalities approach to handling increasing returns is sometimes appropriate, even in a model in which knowledge results from intentional R&D. Recall that in Chapter 5 we used imperfect competition to handle the increasing returns associated with the production of final output. However, we also used the externalities approach in handling a different production function, that for new knowledge. Consider a slight variation of the production function for knowledge in Chapter 5. In particular, let's rewrite equation (5.4) assuming $\lambda = 1$:

$$\dot{A} = \delta L_A A^{\phi}. \tag{8.6}$$

Externalities are likely to be very important in the research process. The knowledge created by researchers in the past may make research today much more effective; recall the famous quotation by Isaac Newton about standing on the shoulders of giants. This suggests that ϕ may be greater than 0.

Notice that with $\phi > 0$, the production function for new knowledge given in equation (8.6) exhibits increasing returns to scale. The return to labor is one, and the return to A is ϕ, for total returns to scale of $1+\phi$. In Chapter 5, we treated A^{ϕ} as an externality. Individual researchers take A^{ϕ} as given when deciding how much research to perform, and they are not compensated for the "knowledge spillover" to future researchers that their research creates. This is simply an application of using the externalities approach to handle increasing returns.

8.4 EVALUATING ENDOGENOUS GROWTH MODELS

What this brief presentation of some alternative endogenous growth models shows is that it is relatively easy to write down models in which permanent changes in government policies generate permanent changes in growth rates for an economy. Of course, it is also easy to write down models in which this is not true, as we have done throughout this book. Which is a better way to think about economic growth? Do changes in government policies have permanent effects on the rate of economic growth?

At some level, the answer to this question must surely be "Yes." For example, we know that economic growth rates have increased in the last two hundred years relative to what they were for most of history. In Chapter 4, we presented the argument of a number of economic historians, such as Douglass North: this increase was due in large part to the establishment of property rights that allowed individuals to earn returns on their long-term investments.

However, this general feature of economic growth is predicted by models such as that in Chapter 5, where government policies in general do not affect the long-term growth rate. For instance, if we do not allow inventors to earn returns on their inventions (e.g., through a 100 percent tax), no one will invest and the economy will not grow.

The question, then, is more narrow. For example, if the government were to provide an additional 10 percent subsidy to research, education, or investment, would this have a permanent effect on the growth rate of the economy, or would it "only" have a level effect in the long run? Another way of asking this same question is the following: If the government were to provide an additional subsidy to research or investment, growth rates would rise for a while, according to many models. However, for how long would growth rates remain high? The answer could be 5 or 10 years, 50 or 100 years, or an infinite amount of time. This way of asking the question illustrates that the distinction between whether policy has permanent or transitory effects on growth is somewhat misleading. We are really interested in how long the effects last.

One can use this reasoning as an argument in favor of models in which the effects are transitory. A very long transitory effect can come arbitrarily close to a permanent effect. However, the reverse is not true: a permanent effect cannot approximate an effect that lasts only for 5–10 years.

The recent literature on economic growth provides other reasons to prefer models in which changes in conventional government policies are modeled as having level effects instead of growth effects. The first reason is that there is virtually no evidence supporting the hypothesis that the relevant differential equations are "linear." For example, consider the simple AK model presented earlier in this chapter. This model requires us to believe the exponent on capital, α, is one. Recall that conventional estimates of the capital share using growth accounting suggest that the capital share is about 1/3. If one tries to broaden the concept of capital to include human capital and externalities, one can raise the exponent to 2/3 or perhaps 4/5. However, there is very little evidence to suggest the coefficient is one.[6]

Another example can be seen in the research-based models of economic growth like those presented in Chapter 5. Recall that if the differential equation governing the evolution of technology is linear, then the model predicts that an increase in the size of the economy (measured, for example, by the size of the labor force or the number of researchers) should increase per capita growth rates. For example, with $\lambda = 1$ and $\phi = 1$, the production function for ideas can be written as

$$\frac{\dot{A}}{A} = \delta L_A.$$

Again, there is a great deal of empirical evidence that contradicts this prediction. Recall from Chapter 4 that the number of scientists and engineers engaged in research, a rough measure of L_A, has grown enormously over the last forty years. In contrast, growth rates have averaged

[6] See, for example, Barro and Sala-i-Martin (1992) and Mankiw, Romer, and Weil (1992).

about 1.8 percent for the entire time.[7] The evidence favors a model that is "less than linear" in the sense that $\phi < 1$.

Yet another example is found by considering more carefully the U.S. experience in the last century. There have been large movements in many variables that the endogenous growth literature highlights as important. For example, investment rates in education (measured, say, by the average educational attainment of each generation) have increased enormously over the past century. In 1940, for example, fewer than one out of four adults had completed high school; by 1995, however, more than 80 percent of adults had a high school diploma. Investment rates in equipment such as computers have increased greatly. Since 1950, the fraction of the labor force that consists of scientists and engineers engaged in formal R&D has increased by a factor of almost three. Despite these changes, average growth rates in the United States are no higher today than they were from 1870 to 1929 (recall Fact 5 in Chapter 1).[8]

One final piece of evidence comes from observing differences across countries instead of differences over time within a country. A number of models in which policies can have growth effects predict that long-run growth rates should differ permanently across countries. The simple AK model and the Lucas model presented in this chapter, for example, share this prediction: differences in investment rates and differences in the rate at which individuals accumulate skills lead to permanent differences in growth rates. However, although economic policies vary substantially across countries, these differences are not always associated with differences in growth rates. Between 1960 and 1997, for example, the United States, Bolivia, and Malawi all grew at roughly the same rate. The large differences in economic policies across these countries are reflected in levels of income, not growth rates.

8.5 WHAT IS ENDOGENOUS GROWTH?

It is fairly easy to construct models in which permanent changes in conventional government policies have permanent effects on an economy's long-run growth rate. However, the view in this book is that these

[7] Jones (1995a) develops this argument in more detail.
[8] This evidence is emphasized by Jones (1995b).

models are not the best way to understand long-run growth. On the other hand, the development of these models and the empirical work by economists to test and understand them have been tremendously useful in shaping our understanding of the growth process.

Long-run growth may not be endogenous in the sense that it can be easily manipulated at the whim of a policymaker. However, this is not to say that exogenous growth models like the Solow model provide the last word. Rather, we understand economic growth as the endogenous outcome of an economy in which profit-seeking individuals who are allowed to earn rents on the fruits of their labors search for newer and better ideas. The process of economic growth, in this sense, is clearly endogenous.

EXERCISES

1. *Population growth in the AK model.* Consider the AK model in which we do not normalize the size of the labor force to one.

   (a) Using the production function in equation (8.5) and the standard capital accumulation equation, show that the growth rate of output depends on L.

   (b) What happens if L is growing at some constant rate n?

   (c) Specify the form of the externality in equation (8.4) differently to avoid this implication.

   (d) Does labor affect production?

2. *Physical investments in the Lucas model.* Does a permanent increase in s_K have a growth effect or a level effect in the Lucas model? Why?

3. *Market structure in the Lucas model.* Think about the market structure that underlies the Lucas model. Do we need perfect or imperfect competition? Do we need externalities? Discuss.

4. *Growth over the very long run.* Historical evidence suggests that growth rates have increased over the very long run. For example, growth was slow and intermittent prior to the Industrial Revolution. Sustained growth became possible after the Industrial Revolution,

with average growth rates of per capita income in the nineteenth century of approximately 1 percent per year. Finally, in the twentieth century, more rapid growth has emerged. Discuss this evidence and how it can be understood in endogenous growth models (in which standard policies can affect long-run growth) and semi-endogenous growth models (in which standard policies have level effects in the long run).

5. *The idea production function.* What is the economic justification for thinking that the production function for new ideas takes the form given in equation (8.6)? In particular, why might this production function exhibit increasing returns to scale?