CHAPTER 5

SECURITY-MARKET INDEXES

Answers to Questions

1. The purpose of security-market indexes is to provide a general indication of the aggregate market changes or market movements. More specifically, the indexes are used to derive market returns for a period of interest and then used as a benchmark for evaluating the performance of alternative portfolios. A second use is in examining the factors that influence aggregate stock price movements by forming relationships between market (series) movements and changes in the relevant variables in order to illustrate how these variables influence market movements. A further use is by technicians who use past aggregate market movements to predict future price patterns. Finally, a very important use is in portfolio theory, where the systematic risk of an individual security is determined by the relationship of the rates of return for the individual security to rates of return for a market portfolio of risky assets. Here, a representative index is used as a proxy for the market portfolio of risky assets.

2. A characteristic that differentiates alternative market indexes is the sample - the size of the sample (how representative of the total market it is) and the source (whether securities are of a particular type or a given segment of the population (NYSE, TSE). The weight given to each member plays a discriminatory role - with diverse members in a sample, it would make a difference whether the index is price-weighted, value-weighted, or unweighted. Finally, the computational procedure used for calculating return - i.e., whether arithmetic mean, geometric mean, etc.

3. A price-weighted series is an arithmetic average of current prices of the securities included in the sample - i.e., closing prices of all securities are summed and divided by the number of securities in the sample.

   A $100 security will have a greater influence on the series than a $25 security because a 10 percent increase in the former increases the numerator by $10 while it takes a 40 percent increase in the price of the latter to have the same effect.

4. A value-weighted index begins by deriving the initial total market value of all stocks used in the series (market value equals number of shares outstanding multiplied by current market price). The initial value is typically established as the base value and assigned an index value of 100. Subsequently, a new market value is computed for all securities in the sample and this new value is compared to the initial value to derive the percent change which is then applied to the beginning index value of 100.

5. Given a four security series and a 2-for-1 split for security A and a 3-for-1 split for security B, the divisor would change from 4 to 2.8 for a price-weighted series.

Stock	Before Split Price	After Split Prices
A	$20	$10
B	30	10
C	20	20
D	30	30
Total	100/4 = 25	70/x = 25
		x = 2.8

The price-weighted series adjusts for a stock split by deriving a new divisor that will ensure that the new value for the series is the same as it would have been without the split. The adjustment for a value-weighted series due to a stock split is automatic. The decrease in stock price is offset by an increase in the number of shares outstanding.

Before Split

Stock	Price/Share	# of Shares	Market Value
A	$20	1,000,000	$20,000,000
B	30	500,000	15,000,000
C	20	2,000,000	40,000,000
D	30	3,500,000	105,000,000
Total			$180,000,000

The $180,000,000 base value is set equal to an index value of 100.

After Split

Stock	Price/Share	# of Shares	Market Value
A	$10	2,000,000	$20,000,000
B	10	1,500,000	15,000,000
C	20	2,000,000	40,000,000
D	30	3,500,000	105,000,000
Total			$180,000,000

$$\text{New Index Value} = \frac{\text{Current Market Value}}{\text{Base Value}} \text{ x Beginning Index Value}$$

$$= \frac{180{,}000{,}000}{180{,}000{,}000} \text{ x } 100$$

$$= 100$$

which is precisely what one would expect since there has been no change in prices other than the split.

6. In an unweighted price index series (perhaps more appropriately called an "unweighted" or "equally-weighted" index), all stocks carry equal weight irrespective of their price and/or their value. One way to visualize an unweighted series is to assume that equal dollar amounts are invested in each stock in the portfolio, for example, an equal amount of $1,000 is assumed to be invested in each stock. Therefore, the investor would own 25

shares of GM ($40/share) and 40 shares of Coors Brewing ($25/share). An unweighted price index that consists of these two stocks would be constructed as follows:

Stock	Price/Share	# of Shares	Market Value
GM	$ 40	25	$1,000
Coors	25	40	1,000
Total			$2,000

A 20% price increase in GM:

Stock	Price/Share	# of Shares	Market Value
GM	$ 48	25	$1,200
Coors	25	40	1,000
Total			$2,200

A 20% price increase in Coors:

Stock	Price/Share	# of Shares	Market Value
GM	$ 40	25	$1,000
Coors	30	40	1,200
Total			$2,200

Therefore, a 20% increase in either stock would have the same impact on the total value of the index (i.e., in all cases the index increases by 10%. An alternative treatment is to compute percentage changes for each stock and derive the average of these percentage changes. In this case, the average would be 10% [(20% + 0%) / 2 = 10%]. So in the case of an unweighted price-index series, a 20% price increase in GM would have the same impact on the index as a 20% price increase of Coors Brewing.

7. Based upon the sample from which it is derived and the fact that is a value-weighted index, the Wilshire 5000 Equity Index is a weighted composite of the NYSE composite index, the AMEX market value series, and the NASDAQ composite index. We would expect it to have the highest correlation with the NYSE Composite Index because the NYSE has the highest market value.

8. The high correlations between returns for alternative NYSE price index series can be attributed to the **source** of the sample (i.e. stock traded on the NYSE). The four series differ in sample size, that is, the DJIA has 30 securities, the S&P 400 has 400 securities, the S&P 500 has 500 securities, and the NYSE Composite over 2,800 stocks. The DJIA differs in computation from the other series, that is, the DJIA is a price-weighted series where the other three series are value-weighted. Even so, there is strong correlation between the series because of similarity of types of companies.

9. The two price indexes (Tokyo SE and Nikkei) for the Tokyo Stock Exchange show a high positive correlation (0.82). However, the two indexes represent substantially

different sample sizes and weighting schemes. The Nikkei-Dow Jones Average consists of 225 companies and is a price-weighted series. Alternatively, the Tokyo SE encompasses a much larger set of 1,800 companies and is a value-weighted series.

The correlation between the Tokyo SE and the S&P 500 is substantially lower at 0.328.These results support the argument for diversification among countries.

10. Since the equal-weighted series implies that all stocks carry the same weight, irrespective of price or value, the results indicate that on average all stocks in the index increased by 23 percent. On the other hand, the percentage change in the value of a large company has a greater impact than the same percentage change for a small company in the value weighted index. Therefore, the difference in results indicates that for this given period, the smaller companies in the index outperformed the larger companies.

11. The bond-market series are more difficult to construct due to the wide diversity of bonds available. Also bonds are hard to standardize because their maturities and market yields are constantly changing. In order to better segment the market, you could construct five possible subindexes based on coupon, quality, industry, maturity, and special features (such as call features, warrants, convertibility, etc.).

12. Since the Merrill Lynch-Wilshire Capital Markets index is composed of a distribution of bonds as well as stocks, the fact that this index increased by 15 percent, compared to a 5 percent gain in the Wilshire 5000 Index indicates that bonds outperformed stocks over this period of time.

13. The Russell 1000 and Russell 2000 represent two different samples of stocks, segmented by size. The fact that the Russell 2000 (which is composed of the smallest 2,000 stocks in the Russell 3000) increased more than the Russell 1000 (composed of the 1000 largest capitalization U.S. stocks) indicates that small stocks performed better during this time period.

14. One would expect that the level of correlation between the various world indexes should be relatively high. These indexes tend to include the same countries and the largest capitalization stocks within each country.

15. High yield bonds (ML High Yield Bond Index) have definite equity characteristics. Consequently, they are more highly correlated with the NYSE composite stock index than with the ML Aggregate Bond Index.

16. Indexes with the broadest representation of U.S. stocks include the Wilshire 5000, the NYSE Composite, and possibly the Nasdaq composite. These indexes would be appropriate benchmarks for portfolio managers wishing to construct a broadly diversified portfolio.

CHAPTER 5

Answers to Problems

1(a).

$$DJIA = \sum_{i=1}^{30} P_{it} / D_{adj}$$

Day 1

Company	Price/Share
A	12
B	23
C	52

$$DJIA = \frac{12+23+52}{3} = \frac{87}{3} = 29$$

Day 2

Company	(Before Split) Price/Share	(After Split) Price/Share
A	10	10
B	22	44
C	55	55

$$DJIA = \frac{10+22+55}{3}$$

$$= \frac{87}{3} = 29$$

$$DJIA = \frac{10+44+55}{X}$$

$$29 = \frac{109}{X}$$

$$X = 3.7586 \text{ (new divisor)}$$

Day 3

Company	(Before Split) Price/Share	(After Split) Price/Share
A	14	14
B	46	46
C	52	26

$$DJIA = \frac{14+46+52}{3.7586} = 29.798$$

$$= \frac{112}{3.7586}$$

$$DJIA = \frac{14+46+26}{Y}$$

$$29.798 = \frac{86}{Y}$$

$$Y = 2.8861 \text{ (new divisor)}$$

Day 4

Company	Price/Share
A	13
B	47
C	25

$$DJIA = \frac{13 + 47 + 25}{2.8861}$$

$$= \frac{85}{2.8861} = 29.452$$

Day 5

Company	Price/Share
A	12
B	45
C	26

$$DJIA = \frac{12 + 45 + 26}{2.8861}$$

$$= \frac{83}{2.8861} = 28.759$$

1(b). Since the index is a price-weighted average, the higher priced stocks carry more weight. But when a split occurs, the new divisor ensures that the new value for the series is the same as it would have been without the split. Hence, the main effect of a split is just a repositioning of the relative weight that a particular stock carries in determining the index. For example, a 10% price change for company B would carry more weight in determining the percent change in the index in Day 3 after the reverse split that increased its price, than its weight on Day 2.

1(c). Student Exercise

2(a). Base = (\$12 x 500) + (\$23 x 350) + (\$52 x 250)
= \$6,000 + \$8,050 + \$13,000 = \$27,050

Day 1 = (\$12 x 500) + (\$23 x 350) + (\$52 x 250)
= \$6,000 + \$8,050 + \$13,000 = \$27,050

$Index_1$ = (\$27,050/\$27,050) x 10 = 10

Day 2 = (\$10 x 500) + (\$22 x 350) + (\$55 x 250)
= \$5,000 + \$7,700 + \$13,750 = \$26,450

$Index_2$ = (\$26,450/\$27,050) x 10 = 9.778

Day 3 = (\$14 x 500) + (\$46 x 175) + (\$52 x 250)
= \$7,000 + \$8,050 + \$13,000 = \$28,050

$Index_3$ = (\$28,050/\$27,050) x 10 = 10.370

Day 4 = (\$13 x 500) + (\$47 x 175) + (\$25 x 500)
= \$6,500 + \$8,225 + \$12,500 = \$27,225

$Index_4$ = (\$27,225/\$27,050) x 10 = 10.065

Day 5 = (\$12 x 500) + (\$45 x 175) + (\$26 x 500)
= \$6,000 + \$7,875 + \$13,000 = \$26,875

$Index_5$ = (\$26,875/\$27,050) x 10 = 9.935

2(b). The market values are unchanged due to splits and thus stock splits have no effect. The index, however, is weighted by the relative market values.

3. Price-weighted index $(PWI)_{2008}$ = (20 + 80+ 40)/3 = 46.67

To accounted for stock split, a new divisor must be calculated:
(20 + 40 + 40)/X = 46.67
X = 2.143 (new divisor after stock split)

Price-weighted $index_{2003}$ = (32 + 45 + 42)/2.143 = 55.53

VWI_{2002} = 20(100,000,000) + 80(2,000,000) + 40(25,000,000)
= 2,000,000,000 + 160,000,000 + 1,000,000,000
= 3,160,000,000

assuming a base value of 100 and 1998 as base period, then
(3,160,000,000/3,160,000,000) x 100 = 100

VWI_{2003} = 32(100,000,000) + 45(4,000,000) + 42(25,000,000)
= 3,200,000,000 + 180,000,000 + 1,050,000,000
= 4,430,000,000

assuming a base value of 100 and 2002 as period, then
(4,430,000,000/3,160,000,000) x 100 = 1.4019 x 100 = 140.19

3(a). Percentage change in PWI = (55.53 - 46.67)/46.67 = 18.99%

Percentage change in VWI = (140.19 - 100)/100 = 40.19%

3(b). The percentage change in VWI was much greater than the change in the PWI because the stock with the largest market value (K) had the greater percentage gain in price (60% increase).

3(c).

December 31, 2002

Stock	Price/Share	# of Shares	Market Value
K	$20	50.0	$1,000.00
M	80	12.5	1,000.00
R	40	25.0	1,000.00
Total			$3,000.00

December 31, 2003

Stock	Price/Share	# of Shares	Market Value
K	$32	50.0	$1,600.00
M	45	25.0*	1,125.00
R	42	25.0	1,050.00
Total			$3,775.00

(*Stock-split two-for-one during the year.)

$$\text{Percentage change} = \frac{3{,}775.00 - 3{,}000}{3{,}000} = \frac{775.00}{3{,}000} = 25.83\%$$

$$\begin{aligned}\text{Geometric average} &= [(1.60)\,(1.125)\,(1.05)]^{1/3} - 1 \\ &= [1.89]^{1/3} - 1 \\ &= 1.2364 - 1 \\ &= .2364 \text{ or } 23.64\%\end{aligned}$$

Unweighted averages are not impacted by large changes in stocks prices (i.e. price-weighted series) or in market values (i.e. value-weighted series).

4. Student Exercise

5(a). Given a three security series and a price change from period t to t+1, the percentage change in the series would be 42.85 percent.

	Period t	**Period t+1**
A	$ 60	$ 80
B	20	35
C	18	25
Sum	$ 98	$140
Divisor	3	3
Average	32.67	46.67

$$\text{Percentage change} = \frac{46.67 - 32.67}{32.67} = \frac{14.00}{32.67} = 42.85\%$$

5(b).

Period t

Stock	Price/Share	# of Shares	Market Value
A	$60	1,000,000	$ 60,000,000
B	20	10,000,000	200,000,000
C	18	30,000,000	540,000,000
Total			$800,000,000

Period t+1

Stock	Price/Share	# of Shares	Market Value
A	$80	1,000,000	$ 80,000,000
B	35	10,000,000	350,000,000
C	25	30,000,000	750,000,000
Total			$1,180,000,000

$$\text{Percentage change} = \frac{1{,}180 - 800}{800} = \frac{380}{800} = 47.50\%$$

5(c). The percentage change for the price-weighted series is a simple average of the differences in price from one period to the next. Equal weights are applied to each price change.

The percentage change for the value-weighted series is a weighted average of the differences in price from one period t to t+1. These weights are the relative market values for each stock. Thus, Stock C carries the greatest weight followed by B and then A. Because Stock B had the greatest percentage increase (75%) and the second largest weight, the percentage change would be larger for this series than the price-weighted series.

6(a).

Period t

Stock	Price/Share	# of Shares	Market Value
A	$60	16.67	$ 1,000,000
B	20	50.00	1,000,000
C	18	55.56	1,000,000
Total			$3,000,000

Period t+1

Stock	Price/Share	# of Shares	Market Value
A	$80	16.67	$ 1,333.60
B	35	50.00	1,750.00
C	25	55.56	1,389.00
Total			$4,472.60

$$\text{Percentage change} = \frac{4{,}472.60\ -\ 3{,}000}{3{,}000} = \frac{1{,}472.60}{3{,}000} = 49.09\%$$

6(b).

$$A = \frac{80-60}{60} = \frac{20}{60} = 33.33\%$$

$$B = \frac{35-20}{20} = \frac{15}{20} = 75.00\%$$

$$C = \frac{25\text{ - }18}{18} = \frac{7}{18} = 38.89\%$$

$$\text{Arithmetic average} = \frac{33.33\% + 75.00\% + 38.89\%}{3}$$

$$= \frac{147.22\%}{3} = 49.07\%$$

The answers are the same (slight difference due to rounding). This is what you would expect since Part A represents the percentage change of an equal-weighted series and Part B applies an equal weight to the separate stocks in calculating the arithmetic average.

6(c). Geometric average is the nth root of the product of n items.

$$\begin{aligned}\text{Geometric average} &= [(1.3333)\,(1.75)\,(1.3889)]^{1/3} - 1\\ &= [3.2407]^{1/3} - 1\\ &= 1.4798 - 1\\ &= .4798 \text{ or } 47.98\%\end{aligned}$$

The geometric average is less than the arithmetic average. This is because variability of return has a greater affect on the arithmetic average than the geometric average.