

Solution: Quiz 2 (Part 2)

1. Let $B = \{1, \frac{1}{2}, \frac{1}{4}\}$. Let the set of all positive real numbers \mathbb{R}^+ be the domain for the predicate $P(y)$ which is defined as

$$P(y) : \sim \{\forall x \in B, 2xy > 1\}.$$

Find the truth set of the predicate $P(y)$.

Solution: First, we can simplify $P(y)$ by finding the negation of $\{\forall x \in B, 2xy > 1\}$:

$$P(y) : \sim \{\forall x \in B, 2xy > 1\} = [\exists x \in B, 2xy \leq 1].$$

To find the truth set T_p of $P(y)$, we have to consider all elements in \mathbb{R}^+ that make $P(y)$ true.

Consider each element of $B = \{1, \frac{1}{2}, \frac{1}{4}\}$.

Note that $y \in \mathbb{R}^+$.

(i) For $x = 1$, $2xy = 2(1)y \leq 1$ is true when $y \leq \frac{1}{2}$ or $y \in (-\infty, 1/2] \cap \mathbb{R}^+ = (0, 1/2]$.

(ii) For $x = \frac{1}{2}$, $2xy = 2(1/2)y \leq 1$ is true when $y \leq 1$ or $y \in (-\infty, 1] \cap \mathbb{R}^+ = (0, 1]$.

(iii) For $x = \frac{1}{4}$, $2xy = 2(1/4)y \leq 1$ is true when $y \leq 2$ or $y \in (-\infty, 2] \cap \mathbb{R}^+ = (0, 2]$.

Note that the truth set T_p of $P(y)$ is

$$T_p = \{y \in \mathbb{R}^+, P(y) \text{ is true.}\}$$

$P(y) = [\exists x \in B, 2xy \leq 1]$ is true when there is at least one $x \in B$ make $2xy \leq 1$ true which occurs when

$$y \in (0, 1/2] \cup (0, 1] \cup (0, 2] = (0, 2].$$

That is the truth set is

$$T_p(y) = (0, 2] = \{x \in \mathbb{R}, 0 < x \leq 2\}.$$