Solution key HW#3

CHAPTER 6: RISK AVERSION AND CAPITAL ALLOCATION TO RISKY ASSETS

PROBLEM SETS

2. (b) A higher borrowing rate is a consequence of the risk of the borrowers' default. In perfect markets with no additional cost of default, this increment would equal the value of the borrower's option to default, and the Sharpe measure, with appropriate treatment of the default option, would be the same. However, in reality there are costs to default so that this part of the increment lowers the Sharpe ratio. Also, notice that answer (c) is not correct because doubling the expected return with a fixed risk-free rate will more than double the risk premium and the Sharpe ratio.

5. When we specify utility by $U = E(r) - 0.5A\sigma^2$, the utility level for T-bills is: 0.07

The utility level for the risky portfolio is:

$$U = 0.12 - 0.5 \times A \times (0.18)^2 = 0.12 - 0.0162 \times A$$

In order for the risky portfolio to be preferred to bills, the following must hold:

$$0.12 - 0.0162A > 0.07 \Rightarrow A < 0.05/0.0162 = 3.09$$

A must be less than 3.09 for the risky portfolio to be preferred to bills.

6. Points on the curve are derived by solving for E(r) in the following equation:

$$U = 0.05 = E(r) - 0.5A\sigma^2 = E(r) - 1.5\sigma^2$$

The values of E(r), given the values of σ^2, are therefore:

σ	σ^2	E(r)
0.00	0.0000	0.05000
0.05	0.0025	0.05375
0.10	0.0100	0.06500
0.15	0.0225	0.08375
0.20	0.0400	0.11000
0.25	0.0625	0.14375

The bold line in the graph on the next page (labeled Q6, for Question 6) depicts the indifference curve.

10. The portfolio expected return and variance are computed as follows:

(1) W_{Bills}	(2) r_{Bills}	(3) W_{Index}	(4) r_{Index}	$r_{Portfolio}$ (1)×(2)+(3)×(4)	$\sigma_{Portfolio}$ (3) × 20%	$\sigma^2_{Portfolio}$
0.0	5%	1.0	13.0%	13.0% = 0.130	20% = 0.20	0.0400
0.2	5%	0.8	13.0%	11.4% = 0.114	16% = 0.16	0.0256
0.4	5%	0.6	13.0%	9.8% = 0.098	12% = 0.12	0.0144
0.6	5%	0.4	13.0%	8.2% = 0.082	8% = 0.08	0.0064
0.8	5%	0.2	13.0%	6.6% = 0.066	4% = 0.04	0.0016
1.0	5%	0.0	13.0%	5.0% = 0.050	0% = 0.00	0.0000

13. Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

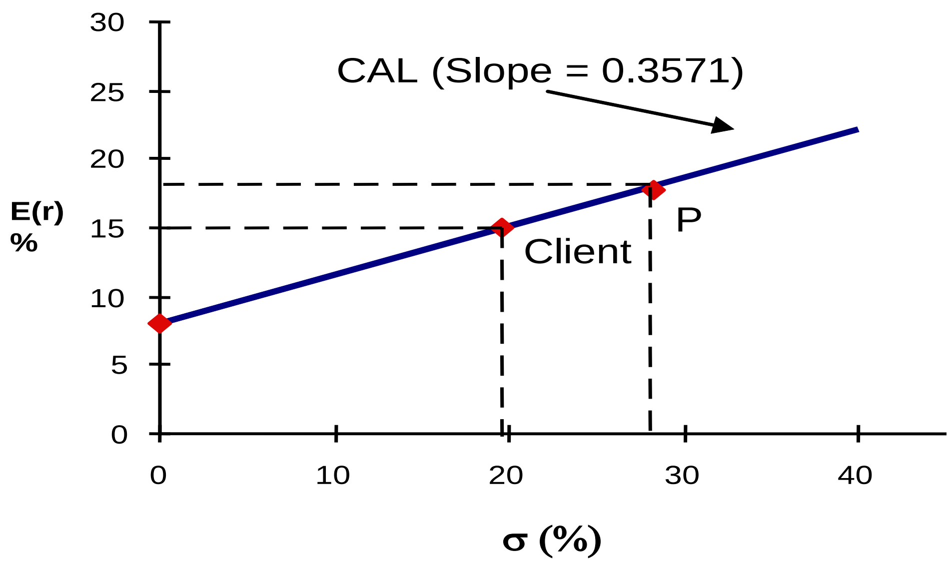
14. Investment proportions:

	30.0% in T-bills
0.7 × 25% =	17.5% in Stock A
0.7 × 32% =	22.4% in Stock B
0.7 × 43% =	30.1% in Stock C

15. Your reward-to-volatility ratio: $S = \frac{.18 - .08}{.28} = 0.3571$

Client's reward-to-volatility ratio: $S = \frac{.15 - .08}{.196} = 0.3571$

16.



19. a. $y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.18 - 0.08}{3.5 \times 0.28^2} = \frac{0.10}{0.2744} = 0.3644$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 8 + 10 \times y^* = 8 + (0.3644 \times 10) = 11.644\%$

$\sigma_C = 0.3644 \times 28 = 10.203\%$

21. a. $E(r_C) = 8\% = 5\% + y \times (11\% - 5\%) \Rightarrow y = \frac{.08 - .05}{.11 - .05} = 0.5$

b. $\sigma_C = y \times \sigma_P = 0.50 \times 15\% = 7.5\%$

c. The first client is more risk averse, allowing a smaller standard deviation.