Total Costs (TC),
Total fixed cost (TFC or FC)
Total variable costs (TVC or VC)

$TC = TFC + TVC$

or $TC = FC + VC$

FC = costs that do not vary w/ outputs you produce.

VC = costs that do vary w/ outputs you produce.

Average total cost (Atc or AC)
Average fixed cost (AFC)
Average variable cost (AVC)

From $TC = FC + VC$ — (1)

Dividing (1) throughout by Q gives

$$\frac{TC}{Q} = \frac{FC}{Q} + \frac{VC}{Q}$$

$$\boxed{AC = AFC + AVC} \quad \text{— (2)}$$

variable cost per unit of output

Fixed cost per unit of output

average cost
or cost per unit of output
or unit cost

in business term

AC, AFC, AVC → unit is Baht/piece

Baht/unit

Baht/

Marginal cost (MC)

MC = extra cost we incur when we produce an extra unit of output.

$$\boxed{MC = \frac{\Delta TC}{\Delta Q} = \frac{TC_{NEW} - TC_{OLD}}{Q_{NEW} - Q_{OLD}}} \quad (1)$$

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta(FC + VC)}{\Delta Q} = \frac{\Delta FC + \Delta VC}{\Delta Q} = \cancel{\frac{\Delta FC}{\Delta Q}}^{0} + \frac{\Delta VC}{\Delta Q}$$

$$\boxed{MC = \frac{\Delta VC}{\Delta Q}} \quad (2)$$

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Variation of Short-Run Cost with Output

$FC/Q + VC/Q = TC/Q$

Output, q	Fixed Cost, FC	Variable Cost, VC	Total Cost, TC	Marginal Cost, MC	Average Fixed Cost, $AFC = F/q$	Average Variable Cost, $AVC = VC/q$	Average Cost, $AC = TC/q$
0	48	0	48				
1	48	25	73	25	48 +	25 =	73
2	48	46	94	21	24 +	23 =	47
3	48	66	114	20	16	22	38
4	48	82	130	16	12	20.5	32.5
5	48	100	148	18	9.6	20	29.6
6	48	120	168	20	8	20	28
7	48	141	189	21	6.9	20.1	27
8	48	168	216	27	6	21	27
9	48	198	246	30	5.3	22	27.3
10	48	230	278	32	4.8	23	27.8
11	48	272	320	42	4.4	24.7	29.1
12	48	321	369	49	4.0	26.8	30.8

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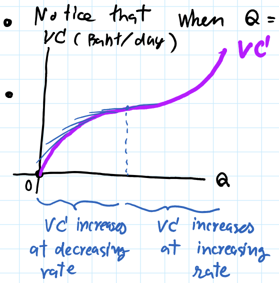
Fact#1 (on FC')

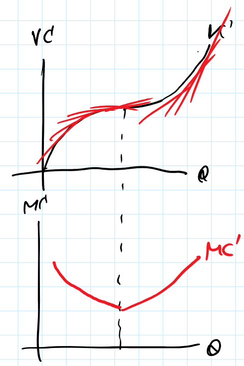
- FC' = 48. It does not change when Q changes.
- Notice that even when Q = 0, FC' = 48.
- 

Fact#2 (on VC')

- When the Firm produces more Q, the firm pays more VC'!

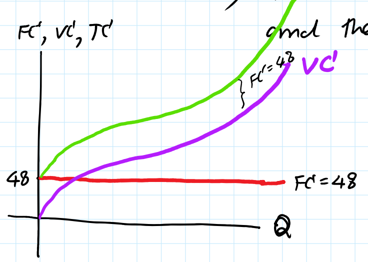
  i.e., Q↑ → VC'↑
- Notice that when Q = 0, VC' = 0.
- 

- When Q↑, VC'↑
- slope of VC' = $\frac{\Delta VC'}{\Delta Q}$ = MC'!



Fact#3 : (on TC')

- When Q↑, TC'↑
- As TC' = FC' + VC', TC' increases at decreasing rate first and then increases at increasing rate.



Notice that slope of TC' = $\frac{\Delta TC'}{\Delta Q}$

which is equal to MC' !!!

So slope of TC' is MC' as well.

Grand conclusion : $\boxed{\text{slope of } VC' = \text{slope of } TC' = MC'}$

Fact#4 : (on MC')

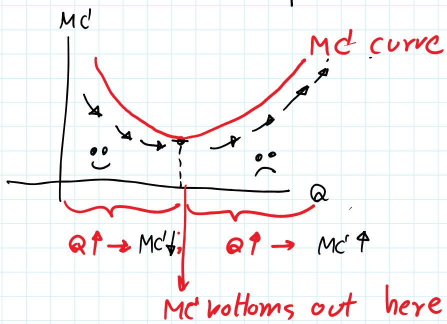
- $MC' = \frac{\Delta TC'}{\Delta Q}$ or $MC' = \frac{\Delta VC'}{\Delta Q}$

  ↑ slope of TC' curve ↑ slope of MC curve.

- MC' is U-shaped : as Q↑, MC' Falls first,

MC'

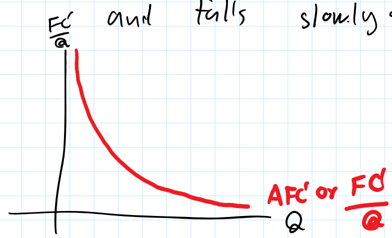
- MC is U-shaped: as Q↑, MC Falls first, bottoms out, and then rises.



Fact#5 (ON AFC)

- $AFC = \frac{FC}{Q}$ (Fixed cost per unit of cookie)

- As Q↑, $\frac{FC}{Q}$ Falls sharply at the begining and falls slowly at large amount of Q.



(EX) $FC = 1{,}000{,}000$ USD

IF $Q=1$, $\frac{FC}{Q} = \frac{1{,}000{,}000}{1} = 1{,}000{,}000$ USD / CD

IF $Q=2$, $\frac{FC}{Q} = \frac{1{,}000{,}000}{2} = 500{,}000$ USD / CD

⋮

IF $Q=10$, $\frac{FC}{Q} = \frac{1{,}000{,}000}{10} = 100{,}000$ USD / CD

⋮

IF $Q=1{,}000{,}000$ $\frac{FC}{Q} = \frac{1{,}000{,}000}{1{,}000{,}000} = 1$ USD/CD.

IF $Q = 100{,}000{,}000$, $\frac{FC}{Q} = \frac{1{,}000{,}000}{100{,}000{,}000} = 1$ CENT / CD.

Notice that As the Firm produces more CDs, Fixed cost is <u>shared</u> by each unit of CDs.

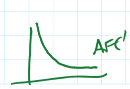
Each unit of CDs helps sharing the fixed cost!

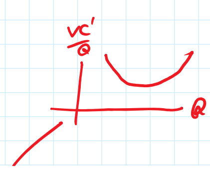
We call this "<u>Spreading Effect</u>"

Recall this $\pi = TR - TC$

profit per unit $\frac{\pi}{Q} = \frac{TR}{Q} - \frac{TC}{Q}$

$\pi = P \cdot Q - TC$





our concern

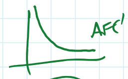
profit per unit

or

profit margin

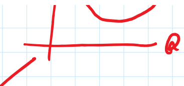
$$\frac{\pi}{Q} = \frac{P\cdot \cancel{Q}}{\cancel{Q}} - \frac{TC}{Q}$$

$$\frac{\pi}{Q} = P - AC$$



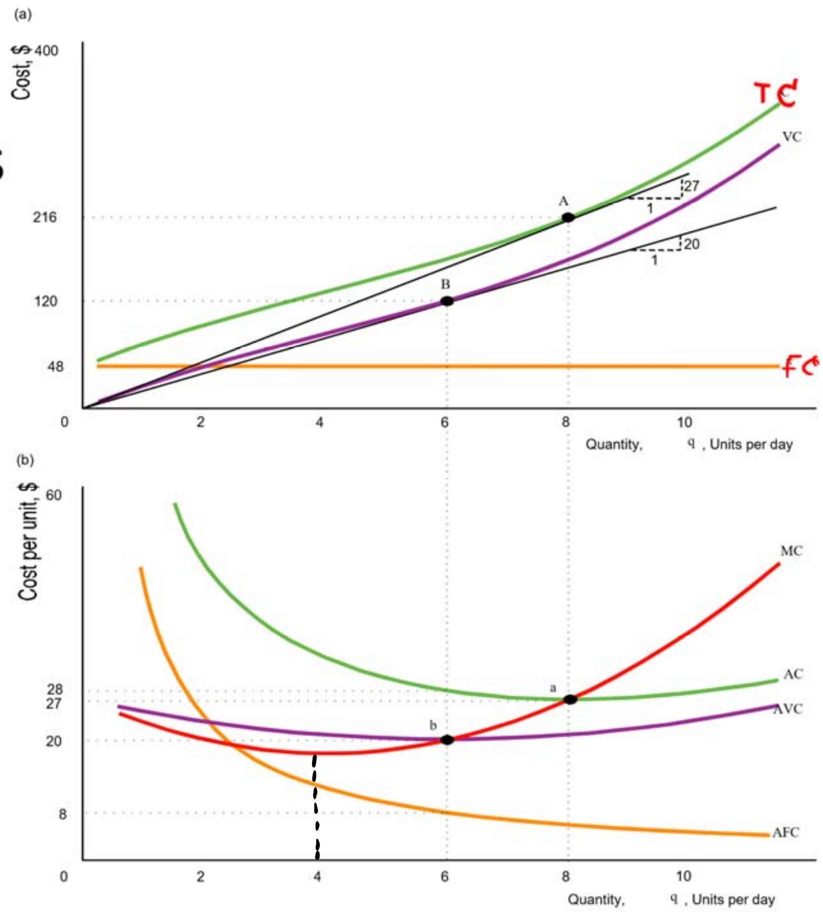
AFC + AVC

(no need to worry about $\frac{FC}{Q}$ as it is always falling when you produce more



our concern is about $\frac{VC}{Q}$ (why?)

Short-Run Cost Curves



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Facts about TC, FC, VC

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Facts about AFC
Why AFC has rectangular hyperbola shape?
(Spreading effect)

Fact#6

Facts about AVC: Why AVC is U-shaped? (Diminishing return effect)

$AVC = \frac{VC}{Q}$ [variable cost per unit of output (cookies)]

unit: baht paid to owners of variable inputs

$AVC = \frac{VC}{Q} = \frac{w \cdot L}{Q} = w \cdot \frac{L}{Q} = w \cdot \frac{1}{\frac{Q}{L}} = w \cdot \frac{1}{AP_L}$

So $\boxed{AVC = \frac{w}{AP_L}}$

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Given w is fixed (supposed), if $AP_L \uparrow$, AVC is falling and if $AP_L \downarrow$, AVC is rising

IF labor becomes more productive (i.e., $AP_L \uparrow$), $\frac{VC}{Q} \downarrow$. :)

IF labor becomes less productive (i.e., $AP_L \downarrow$), $\frac{VC}{Q} \uparrow$. :(

Ex: suppose $w = 300$ baht/person/day

IF $Q_1 = 1{,}000$ cookies

$L_1 = 10$ workers

$AP_{L_1} = \frac{Q_1}{L_1} = \frac{1000}{10}$ cookies/day / person/day $= 100$ cookies/person

$VC_1 = w \cdot L_1 = 300 \cdot 10 = 3000$ baht/day

$AVC_1 = \frac{VC_1}{Q_1} = \frac{3{,}000 \text{ baht}}{1{,}000 \text{ cookies}} = 3$ baht/cookie

$AP_L \uparrow$

IF $Q_2 = 1{,}200$ cookies

$L_2 = 11$ workers

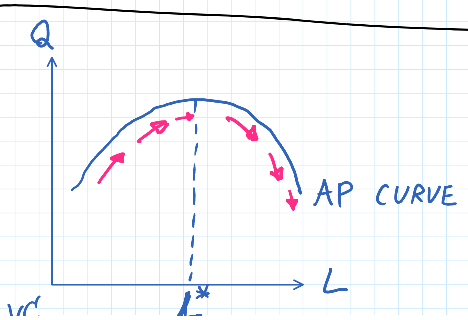
$AP_{L_2} = \frac{Q_2}{L_2} = \frac{1{,}200}{11}$ cookies/person $= 109.09$ cookies/person

$VC_2 = w \cdot L_2 = 300 \cdot 11 = 3{,}300$ baht/day

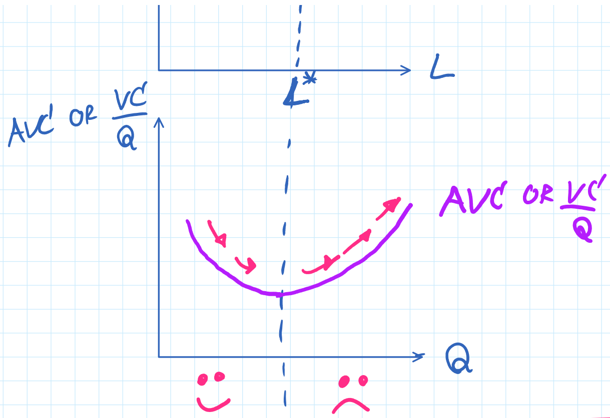
$AVC_2 = \frac{VC_2}{Q_2} = \frac{3{,}300}{1{,}200} = 2.75$ baht/cookie !!!

$\frac{VC}{Q} \downarrow$:)

PRODUCTION



$AVC = \frac{w}{\ }$



$$\boxed{AVC = \frac{w}{AP_L}}$$

$AP_L \uparrow \rightarrow AVC \downarrow$

$AP_L \downarrow \rightarrow AVC \uparrow$

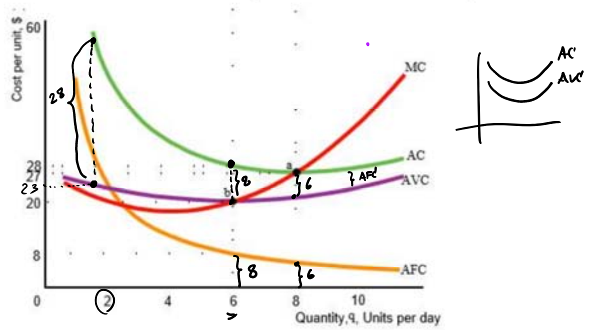
$\boxed{\text{AVC curve is "a mirror image" of AP curve.}}$

- If $MP > AP \rightarrow AP \uparrow \rightarrow AVC \downarrow$
  (Specializations and division of labor)

- If $MP < AP \rightarrow AP_L \downarrow \rightarrow AVC \uparrow$ (Diminishing Return effect causes this to happen!)
  due to the fact that Law of Diminishing MP (Mismatch between the use of workers and the fixed amount of machines (k) arises

Facts about AC: Why AC is U-shaped?

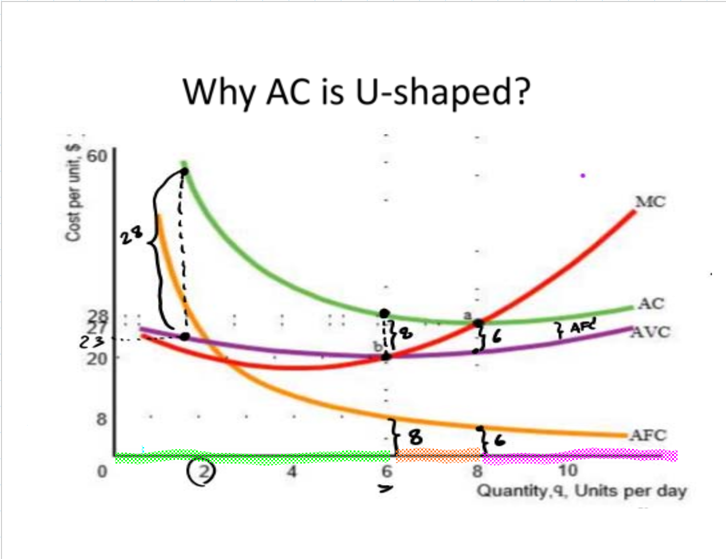
#7



① AC curve can be constructed by "vertical sum" of AVC and AFC.

② Vertical gap or Vertical distance of AC and AVC is equal to... AFC. That's way the gap we are talking about gets narrower as we produce more output.

③ Notice that AVC "bottoms out" first (at Q = 6).
AC "bottoms out" second (at Q = 8).
MC "cuts" at the bottom of AVC valley and then at the bottom of AC valley.



From $0 < Q < 6$: $AFC\downarrow + AVC\downarrow = AC\downarrow$

- Thanks to benefit of "spreading effect" ($AFC\downarrow$)
- Thanks to benefit of "specialization" ($AVC\downarrow$)

* $MP_L > AP_L \rightarrow AP\uparrow \rightarrow AVC\downarrow$ $\left[AVC = \frac{w}{AP_L} \right]$

From $6 < Q < 8$: $AFC\downarrow + AVC\uparrow = AC\downarrow$

It must be that $\Downarrow AFC$ is stronger than $\Uparrow AVC$

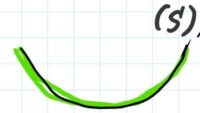
- Thanks to "spreading effect" ($AFC\Downarrow$) ++
- Diminishing Return effect starts to operate. : $MP_L < AP_L \rightarrow AP\downarrow \rightarrow AVC\Uparrow$ —
- However, spreading effect still "dominates" Diminishing Return effect.

As a result, we still observe that AC is still falling :)

From $Q > 8$, $AFC\downarrow + AVC\uparrow = AC\uparrow$

Since DMR effect ($\uparrow AVC$) > SPREADING EFFECT ($\downarrow AFC$), then AC $\uparrow$:(

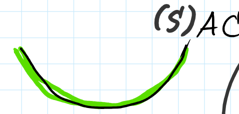
That's why AC has U-shape!



in the short run production

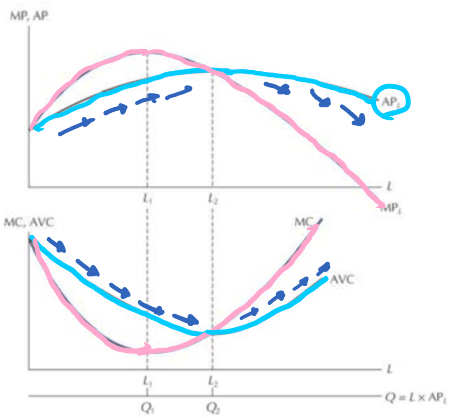
($\uparrow$AVC) ($\downarrow$AFC)

That's why AC has U-shape!



in the short run production

The Relationship Between MP, AP, MC, and AVC



Recall that...

On Production	On cost of production		
TP	TC	FC	VC
AP_L	AC	AFC	AVC
MP_L	MC		

#1 (AP_L — AVC) #2 (MP_L — MC)

$$\boxed{AVC = \frac{w}{AP_L}}$$

$$\boxed{MC = \frac{w}{MP_L}}$$

#1 → when AP ↑, AVC ↓ :)
when AP ↓, AVC ↑ :(

#2 → when MP ↑, MC ↓ :)
when MP ↓, MC ↑ :(

- AVC curve is a mirror image of AP curve.
- MC curve is a mirror image of MP curve.
- Looking at how AP & MP are related : when MP > AP, AP ↑
  when MP < AP, AP ↓
- Looking at how MC & AVC are related : when MC < AVC, AVC ↓
  when MC > AVC, AVC ↑

Note Proof on $\boxed{MC = \frac{w}{MP_L}}$

$$MC = \frac{\Delta VC}{\Delta Q} = \frac{\Delta\, w \cdot L}{\Delta Q} = w \cdot \frac{\Delta L}{\Delta Q} = w \cdot \frac{1}{\frac{\Delta Q}{\Delta L}} = w \cdot \frac{1}{MP_L}$$

$$\boxed{MC = \frac{w}{MP_L}}$$

end of the proof.

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Swans Reflecting Elephants (1937) is a painting by the Spanish surrealist Salvador Dalí.

AP, MP

AVC' MC'

Average-Marginal Relationship holds when we consider MC and AC.

Term	Definition	Mathematical Description
Explicit costs	Costs that require an outlay of money by the firm	
Implicit costs	Costs that do not require an outlay of money by the firm	
Fixed costs	Costs that do not vary with the quantity of output produced	*FC*
Variable costs	Costs that vary with the quantity of output produced	*VC*
Total cost	The market value of all the inputs that a firm uses in production	$TC = FC + VC$
Average fixed cost	Fixed cost divided by the quantity of output	$AFC = FC / Q$
Average variable cost	Variable cost divided by the quantity of output	$AVC = VC / Q$
Average total cost	Total cost divided by the quantity of output	ATC or $AC = TC / Q$
Marginal cost	The increase in total cost that arises from an extra unit of production	$MC = \Delta TC / \Delta Q$

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Production and Costs in the Long Run

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output

inputs

- $Q=f(L,K)$ → Production Function in the long run
- Where Q = output, L= amount of labor, and K= amount of capital
- Let w = wage (price of labor) and r = rental rate (price of capital)
- Isocost: wL+rK= TC

labor cost capital cost

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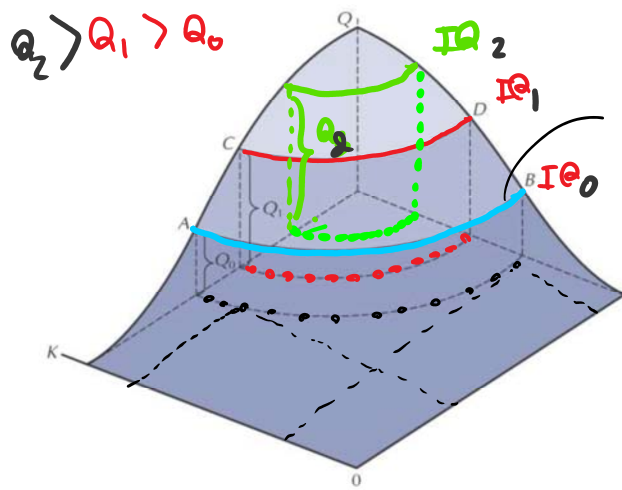
. All inputs can be varied (= No Fixed input)

[w/ consumer choice chapter
$P_x \cdot X + P_y \cdot Y = M$]

The Production Mountain



ISOQUANT : a collection of (L, K) that gives a manager the same quantity of output (Q)

equal quantity

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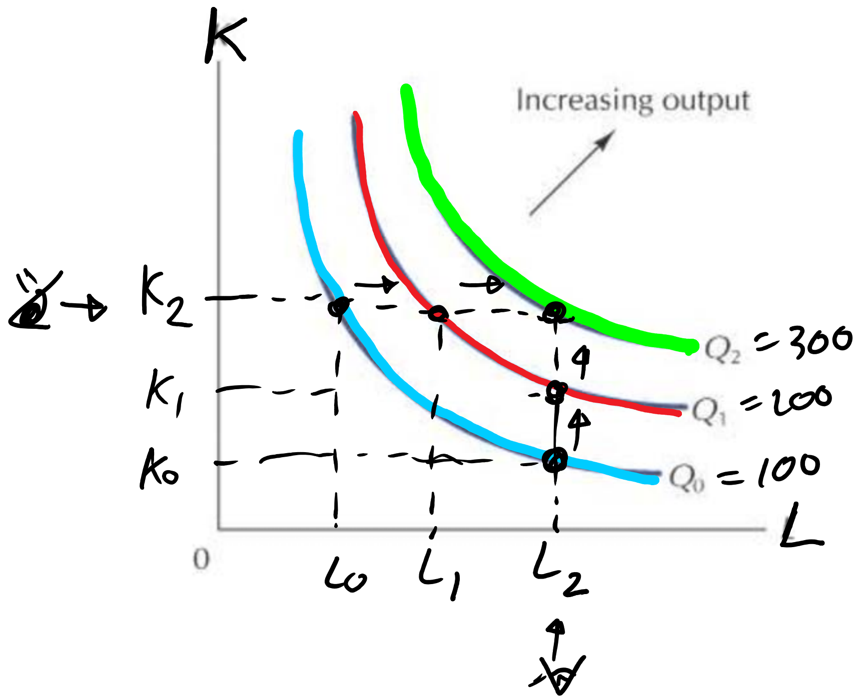
The Isoquant Map Derived from the Production Mountain



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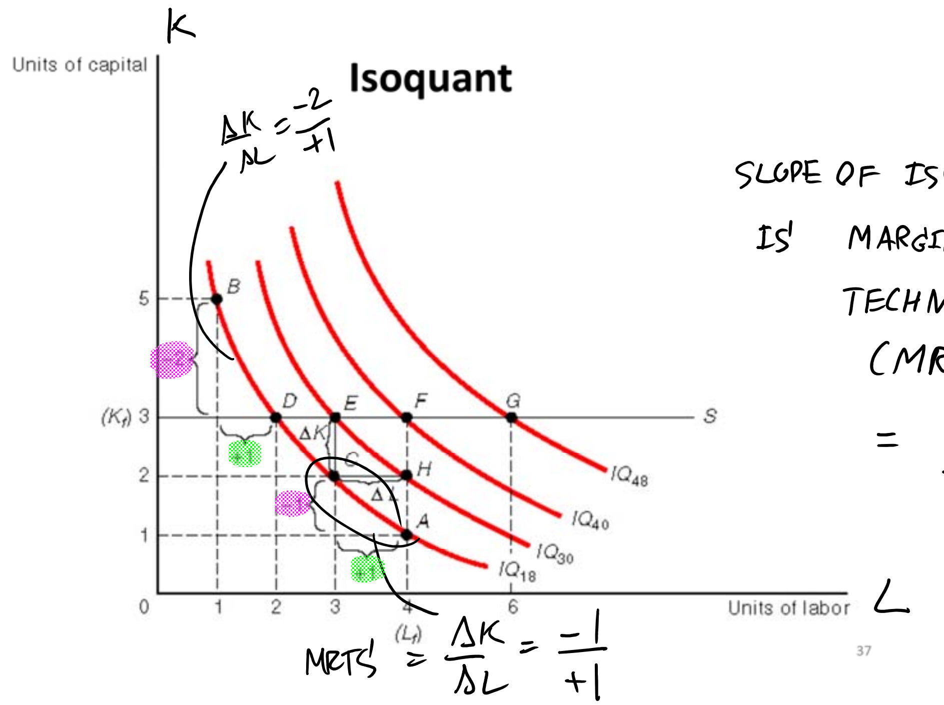


SLOPE OF ISOQUANT IS MARGINAL RATE OF TECHNICAL SUBSTITUTION ($MRTS_{LK}$)

$$= \frac{\Delta K}{\Delta L} \Big| _{Q = \bar{Q}}$$

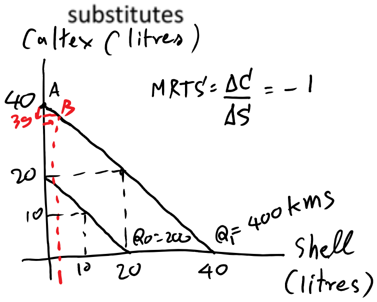
such that

OUTPUT IS MAINTAINED (= CONSTANT)

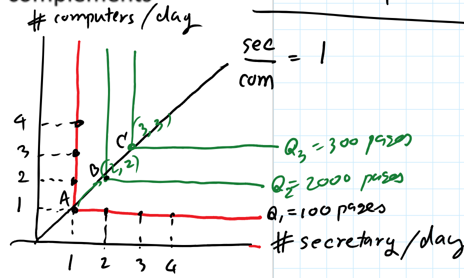
Notice that on an Isoquant, MRTS is diminishing.

MRTS tells us about ability of manager to substitute L and K so that he still gets the same output

Isoquant for perfect substitutes



Isoquant for perfect complements



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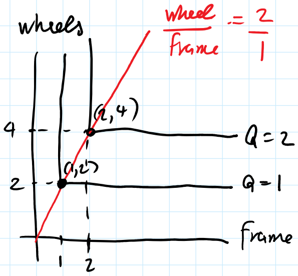
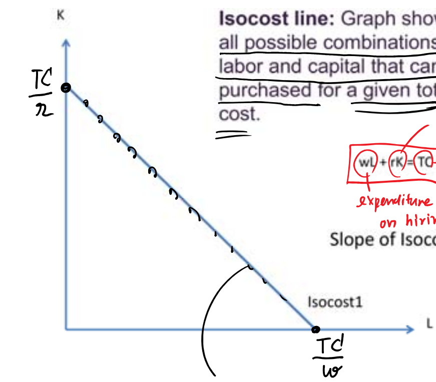
$\boxed{\text{Fixed proportion } 1:1}$



$\frac{wheel}{frame} = \frac{2}{1}$ or $\boxed{Wheel = 2 \cdot Frame}$



Isocost line: Graph showing all possible combinations of labor and capital that can be purchased for a given total cost.

$wL + rK = TC$

expenditure on renting K

Total cost

expenditure on hiring L

Slope of Isocost $= -\dfrac{TC/r}{TC/w} = -\dfrac{w}{r}$

= ratio of input prices

= relative price of inputs.

Ex: TC = 10,000 baht/day

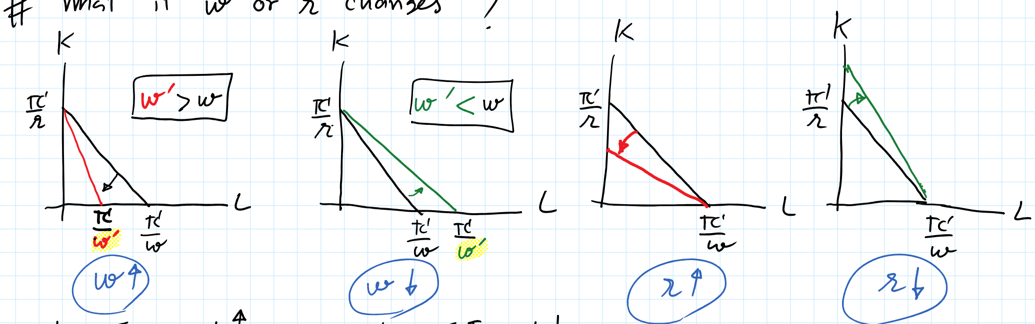
w = 200 baht/คน/day

r = 500 baht/unit/day

$\frac{TC}{w} = \frac{10,000}{200} = 50$ คน/day

$\frac{TC}{r} = \frac{10,000}{500} = 20$ machines/day

What if w or r changes?



Slope of Isocost ↑

$\boxed{\frac{w'}{r} > \frac{w}{r}}$

∴ labor becomes more expensive relative to capital

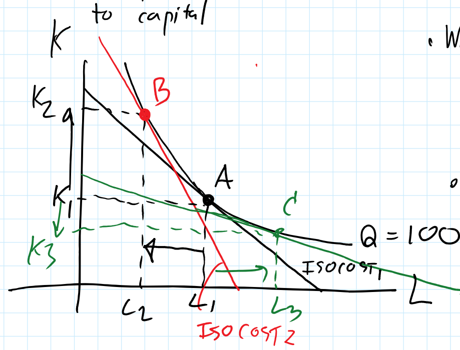
slope of Isocost ↓

$\boxed{\frac{w'}{r} < \frac{w}{r}}$

labor becomes cheaper relative to capital

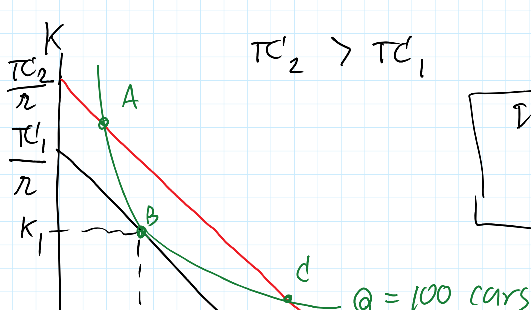
D-I-Y

Application



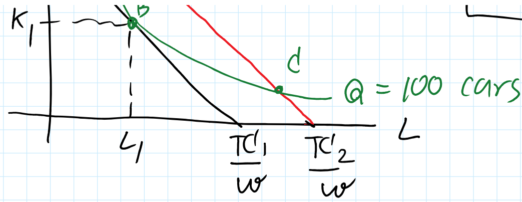
- When $\frac{w}{r}$ ↑, he moves from A → B: use less L and use more K
- When $\frac{w}{r}$ ↓, he moves from A → C: use more L and use less K.

By doing so, he could maintain Q = 100!



$TC'_2 > TC'_1$

Input mix at B (L_1, K_1) minimizes Total cost of producing 100 cars.



cost minimization problem

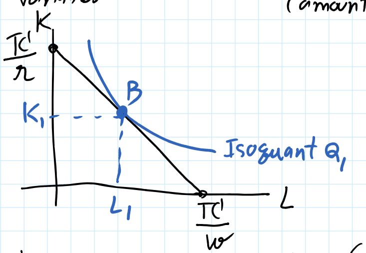
A producer

2 inputs : L & K

$P_L = w$ $P_K = r$

objective → $\boxed{\begin{array}{l}\text{Minimize } TC = w \cdot L + r \cdot K \\ \text{s.t.} \quad Q = Q_0\end{array}}$

decision variables : choose L & K (amounts)



slope of Isocost = $-\frac{w}{r}$ $\left(= -\frac{P_L}{P_K} \right)$

Slope of Isoquant = $MRTS_{LK} = \frac{\Delta K}{\Delta L}\Big|_{Q=\bar{Q}}$

At B: B is cost-minimizing input choice

At B : slope of isoquant = slope of Isocost

$$\boxed{MRTS_{LK} = -\frac{w}{r}}$$

↓

Cost minimizing Rule
or
Golden Rule of Cost Minimization

At B: $\boxed{\frac{MP_L}{MP_K} = \frac{w}{r}}$

At B: $\frac{MP}{w} = \frac{MP_K}{r}$

Recall this

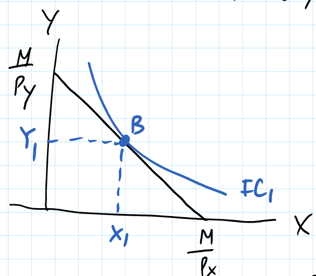
A Consumer

2 goods : X & Y

P_X P_Y

objective → $\boxed{\begin{array}{l}\text{Maximize } U(X, Y) \\ \text{s.t.} \quad P_X \cdot X + P_Y \cdot Y = M\end{array}}$

decision variables : choose X & Y (amounts)



slope of BL = $-\frac{P_X}{P_Y}$

slope of IC = $MRS_{XY} = \frac{\Delta Y}{\Delta X}\Big|_{U=\bar{U}}$

At B : B is utility maximizing choice

At B ; slope of IC = slope of BL

$$\boxed{MRS_{XY} = -\frac{P_X}{P_Y}}$$

↓

Rational spending rule
of
Golden Rule of Utility Maximization

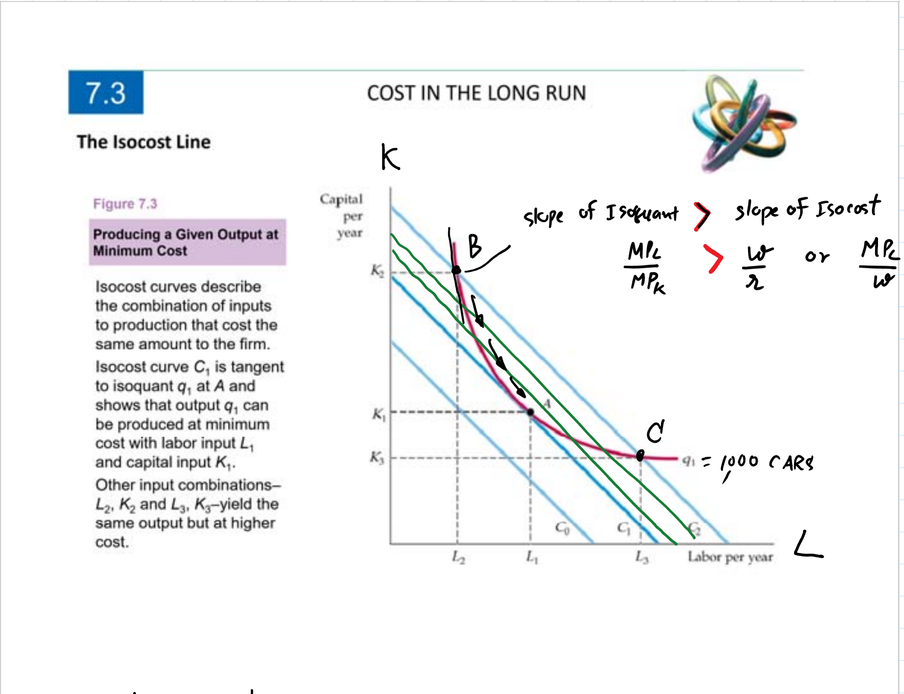
At B: $\boxed{\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}}$

OR

At B ; $\boxed{\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}}$ ***

At B: $\boxed{\frac{MP}{w} = \frac{MP_K}{r}}$

At B: $\boxed{\frac{MU_x}{P_x} = \frac{MU_y}{P_y}}$ ***



$Q = F(L, K)$

Fact#1 Given w, r, input combination $A(L_1, K_1)$ minimizes total cost of producing $q = q_1 = 1{,}000$

Your cost will be equal to C_1 baht.

Fact#2 At input combination $A(L_1, K_1)$, slope of Isoquant = slope of Isocost

$$MRTS_{LK} = -\frac{w}{r}$$

$$\frac{\Delta K}{\Delta L} = -\frac{w}{r}$$

or
$$-\frac{MP_L}{MP_K} = -\frac{w}{r}$$

$$\boxed{\frac{MP_L}{MP_K} = \frac{w}{r}}$$

or
$$\boxed{\frac{MP_L}{w} = \frac{MP_K}{r}}$$

this condition holds ONLY AT A!

$\frac{MP_L}{w}$ = additional output per baht when you spend on hiring a worker

Ex: $w = 300$ baht/ppl/day

$MP_L = 600$ cookies

$$\frac{MP_L}{w} = \frac{600}{300} = 2 \text{ cookies/baht}$$

$\frac{MP_K}{r}$ = additional output per baht when you spend on renting a machine.

Ex: $r = 600$ baht/machine/day

$MP_K = 200$ cookies

$\frac{MP_K}{r} = \frac{200}{600} = \frac{1}{3} = 0.33$ cookies / baht spent.

So If $\frac{MP_L}{w} > \frac{MP_K}{r}$, a smart manager should use more L and less K.

If $\frac{MP_L}{w} < \frac{MP_K}{r}$, a smart manager should use more K and less L.

Cost Minimization Rule

$$\boxed{\frac{MP_L}{w} = \frac{MP_K}{r}}$$

occurs at input choice $A(L_1, K_1)$

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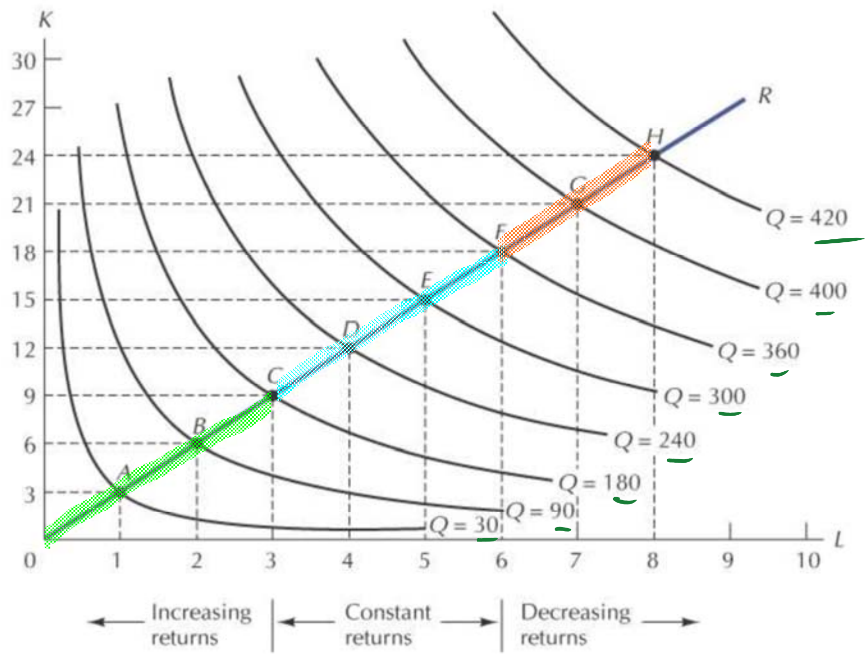
Figure 9-11: Returns to Scale Shown on the Isoquant Map



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doubling all inputs gives output more than double

$\%\Delta Q = \%\Delta L = \%\Delta K$

example: increase L and K by 10%. leads to, let's say, 10%. increase in output.

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Point	L	% change	K	% change	Q	% change	RTS
A	1	-	3	-	30	-	
B	2	**100.0%**	6	**100.0%**	90	**200%**	IRS
C	3	**50.0%**	9	**50.0%**	180	**100%**	IRS
D	4	**33.3%**	12	**33.3%**	240	**33%**	CRS
E	5	**25.0%**	15	**25.0%**	300	**25%**	CRS
F	6	**20.0%**	18	**20.0%**	360	**20%**	CRS
G	7	**16.7%**	21	**16.7%**	400	**11%**	DRS
H	8	**14.3%**	24	**14.3%**	420	**5%**	DRS

NOTICE THAT $\%\Delta L = \%\Delta K < \%\Delta Q$

$\%\Delta L = \%\Delta K = \%\Delta Q$

$\%\Delta L = \%\Delta K > \%\Delta Q$

Note: Calculated from Figure 9-11: Returns to Scale Shown on the Isoquant Map, Frank (2006).

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Math Note on Return to Scale

$F(cK, cL) > cF(L,K) \rightarrow$ IRS

$F(cK, cL) = cF(L,K) \rightarrow$ CRS

$F(cK, cL) < cF(L,K) \rightarrow$ DRS

Ex: consider $Q = F(K,L) = \underline{\underline{2KL}}$.

$$F(cK, cL) = 2(cK)(cL)$$
$$= c^2\, 2KL$$
$$= c^2\, F(K,L)$$

if $c=2$, then $F(2K, 2L) = 2^2 F(K,L)$

$$= 4\, F(K,L)$$

Then $Q = 2KL$ follows IRS.

Ex: $Q = mL^{\alpha}K^{\beta}$ where $\alpha+\beta=1$

$$F(cL, cK) = m\,(cL)^{\alpha}(cK)^{\beta}$$
$$= m\, c^{\alpha}L^{\alpha} \cdot c^{\beta}K^{\beta}$$
$$c^{\alpha+\beta}\, mL^{\alpha}K^{\beta}$$
$$= c^{\alpha+\beta} \cdot F(L,K)$$
$$= c \cdot F(L,K)$$

Since $F(cL, cK) = c\,F(L,K)$, then $Q = mL^{\alpha}K^{\beta}$ exhibits constant Return to scale. (CRS)

D-I-Y: check
$$Q = 4K^{\frac{1}{2}}L^{\frac{1}{2}} \quad \text{—— (1)}$$
$$Q = 4\cdot K + 2\cdot L \quad \text{—— (2)}$$
$$Q = K^{0.5}L^{0.5} \quad \text{—— (3)}$$

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Returns to Scale: The bigger, The better?

- There are **increasing returns to scale (economies of scale)** when long-run average total cost declines as output increases.
- There are decreasing returns to scale (**diseconomies of scale)** when long-run average total cost increases as output increases.
- There are **constant returns to scale** when long-run average total cost is constant as output increases.

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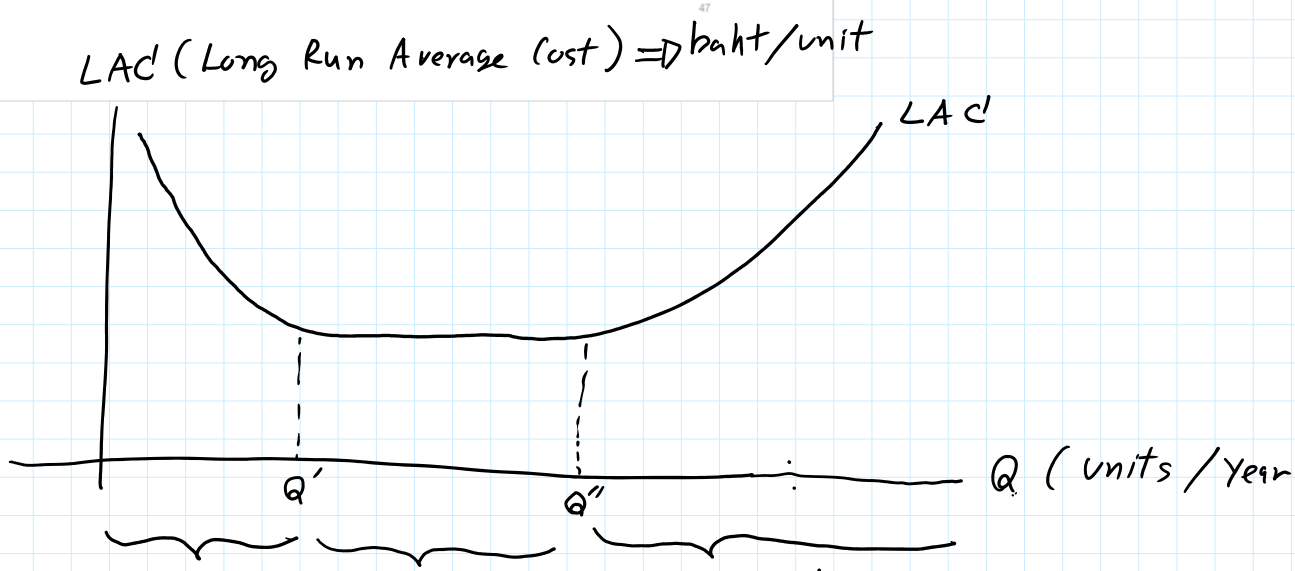


As Q↑, LAC↓
" Economies of scale

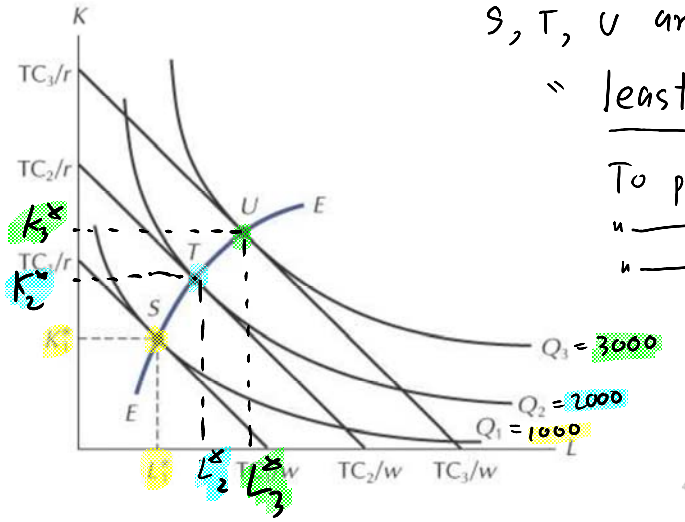
As Q↑, LAC is constant
↓
Constant Return to scale

As Q↑, LAC↑
" Diseconomies of scale "

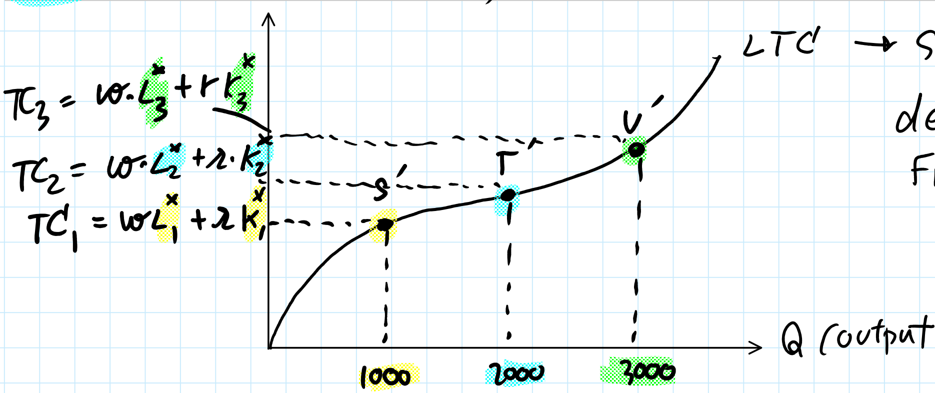
Figure 10.15: The Long-Run Expansion Path : A collection of least combinations

S, T, U are so called "least cost combinations"

To produce $Q_1 = 1000$, you use S
" ——— $Q_2 = 2000$, " ——— " T
" ——— $Q_3 = 3000$, " ——— " U



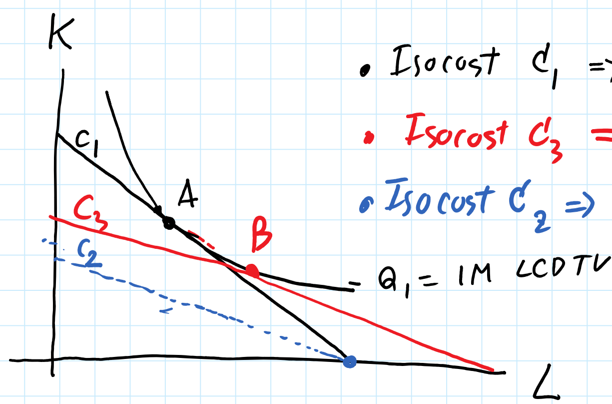
Minimized Total cost (baht/year)



LTC → shows relationship between desired amount of output Firm want to produce AND its minimized total cost.

Note $TC_3' > TC_2' > TC_1'$

Application



- Isocost C_1 ⇒ 50 million baht
- Isocost C_3 ⇒ 60 million baht
- Isocost C_2 ⇒ 50 million baht (why?)

Now, suppose price of capital (r) increases . . .