Session 4-5 Theory of Risk and Return Exercise:

1. The following information is provided for a stock market:

	μ_j	β_j
Asset 1	6.6%	0.4
Asset 2	9.8%	1.2
Asset 3	12.2%	1.8

Notation: μ_j =expected rate of return on asset *j*; β_j =beta-coefficient for asset *j*, *j* = 1, 2, 3. In the context of the Capital Asset Pricing Model (CAPM), define the 'beta-coefficient', β_j, corresponding to asset *j*. Discuss how assets' beta-coefficients should be interpreted and explain how their values can be obtained in practice.
Answer:

2. The following information is provided for a stock market:

	σ_j	ρ_{jM}
Security *A*	50%	0.6
Security *B*	60%	−0.2
Market Portfolio	20%	1.0

Notation: σ_j = standard deviation of the rate of return on asset *j* = A and *j* = B; ρ_{jM} = correlation coefficient between the return on asset *j* and the return on the market portfolio. The mean rate of return on the market portfolio is 8% and the risk-free rate of return is 5%.

- In the Capital Asset Pricing Model, explain what is meant by the beta coefficient, β_j, for a security. Calculate the beta coefficients for the two securities from the given information.
- You are told that the mean rates of return for securities A and B are 7.5% and 4.6% respectively. What would you infer from this information in the context of the Capital Asset Pricing Model?

Answer:

3. The volatility of a stock price is 30% per annum. What is the standard deviation of the percentage price change in one week?

Answer: 4.23% or 4.16%

4. The volatility of an asset is 25% per annum. What is the standard deviation of the percentage price change in one trading day? Assuming a normal distribution, estimate 95% confidence limits for the percentage price change in one day.

Answer:
The standard deviation of the percentage price change in one day is 1.57%. The 95% confidence limits are from -3.09% to +3.09%.

5. Why do traders assume 252 rather than 365 days in a year when using volatilities?

Answer:

Volatility is much higher when markets are open than when they are closed. Traders therefore measure time in trading days rather than calendar days when applying volatility.

6. Suppose that observations on an exchange rate at the end of the last 11 days have been 0.7000, 0.7010, 0.7070, 0.6999, 0.6970, 0.7003, 0.6951, 0.6953, 0.6934, 0.6923, 0.6922. Estimate the daily volatility with the theoretical formula and simplified version.

Answer:

The theoretical approach gives 0.547% per day. The simplified approach gives 0.530% per day.

7. The number of visitors to a website follows the power law with $\alpha = 2$. Suppose that 1 % of sites get 500 or more visitors per day. What percentage of sites get (a) 1000 and (b) 2000 or more visitors per day?

Answer:

(a) 0.25%

(b) 0.0625%.

8. Assume that S&P 500 at close of trading yesterday was 1,040 and the daily volatility of the index was estimated as 1% per day at that time. The parameters in a GARCH(1,1) model are $\omega = 0.000002$, α =0.06, and $\beta = 0.92$. If the level of the index at close of trading today is 1,060, what is the new volatility estimate?

Answer:

With the usual notation, $u_{n-1} = 20/1040$ *= 0.01923, so that*

$$\sigma_n^2 = 0.000002 \; + \; 0.06 \times 0.01923^2 \; + \; 0.92 \times 0.01^2 \; = \; 0.0001162$$

This gives σ_n *= 0.01078. The new volatility estimate is therefore 1.078% per day.*

9. The parameters of a GARCH(1,1) model are estimated as $\omega = 0.000004$, $\alpha = 0.05$, and $\beta = 0.92$. What is the long-run average volatility? If the current volatility is 20% per year, what is the expected volatility in 20 days?

Answer:

The long-run average variance rate is $\omega/(1-\alpha-\beta)$ *or 0.000004/0.03 = 0.0001333. The long-run average volatility is* $\sqrt{0.0001333}$ *or 1.155%. The equation describing the way the variance rate reverts to its long-run average is:* $E[\sigma_{n+k}^2] = V_L + (\alpha+\beta)^k(\sigma_n^2 - V_L)$ *.*

If the current volatility is 20% per year, $\sigma_n = 0.2/\sqrt{252} = 0.0126$*. The expected variance rate in 20 days is* $0.0001333 + 0.97^{20}(0.0126^2 - 0.0001333) = 0.0001417$ *. The expected volatility in 20 days is therefore* $\sqrt{0.0001417}$ *=0.0121, or 1.21% per day.*

10. Suppose that GARCH(1,1) parameters have been estimated as $\omega = 0.000003$, α =0.04, and β =0.94. The current daily volatility is estimated to be 1%. Estimate the daily volatility in 30 days.

Answer:

In this case, V_L *= 0.00015 and the expected variance rate in 30 days is 0.000123. The volatility is 1.11% per day.*

11. Suppose that GARCH(1,1) parameters have been estimated as $\omega = 0.000002$, $\alpha = 0.04$, and $\beta = 0.94$. The current daily volatility is estimated to be 1.3%. Estimate the volatility per annum that should be used to price a 20-day option.

Answer:

In this case, V_L =0.0001, a = 0.0202, T = 20, and V(0) = 0.000169, so that the volatility is 19.88%.

12. Suppose that observations on a stock price (in US dollars) at the end of each of 15 consecutive weeks are as follows: 30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2 Estimate the stock price volatility.

Answer:

13. Suppose that the price of gold at close of trading yesterday was $300 and its volatility was estimated as 1.3% per day. The price at the close of trading today is $298. Update the volatility estimate using the GARCH(1,1) model with $\omega = 0.000002$, $\alpha = 0.04$, and $\beta =$ 0.94.

Answer: