

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

$$\bar{X} = \frac{\sum x_i}{n} \quad \bar{X} = \frac{621}{8} = 77.625 \quad \bar{Y} = \frac{\sum y_i}{n} \quad \bar{Y} = \frac{25.7}{8} = 3.2125$$

Table 1.a

Student	Y_i	X_i	$(x_i - \bar{X})(y_i - \bar{Y})$
1	2.8	63	$(-14.625)(-0.4125) = 6.0328$
2	3.4	72	$(-5.625)(0.1875) = -1.0547$
3	3	78	$(0.375)(-0.2125) = -0.0797$
4	3.5	81	$(3.375)(0.2875) = 0.9703$
5	3.6	87	$(4.375)(0.3875) = 1.6828$
6	3.0	75	$(-2.625)(-0.2125) = 0.5578$
7	2.7	75	$(-2.625)(-0.5125) = 1.3453$
8	3.7	90	$(12.375)(0.4875) = 6.0328$
			<u>17.4375</u>

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1 . (Note: $NIID$ = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{X})(y_i - \bar{Y})}{\sum (x_i - \bar{X})^2}$$

$$\hat{\beta}_1 = \frac{17.4375}{511.875}$$

$$\hat{\beta}_1 = 0.0340659$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_0 = 3.2125 - 0.0340659(77.625)$$

$$\hat{\beta}_0 = 0.5681319$$

1.2 For each observation i , find $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$. $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$$\hat{Y}_1 = 2.7143$$

$$\hat{u}_1 = 0.0857$$

$$\hat{Y}_2 = 3.0209$$

$$\hat{u}_2 = 0.3791$$

$$\hat{Y}_3 = 3.2252$$

$$\hat{u}_3 = -0.225$$

$$\hat{Y}_4 = 3.3275$$

$$\hat{u}_4 = 0.175$$

$$\hat{Y}_5 = 3.5319$$

$$\hat{u}_5 = 0.0681$$

$$\hat{Y}_6 = 3.1231$$

$$\hat{u}_6 = -0.1231$$

$$\hat{Y}_7 = 3.1231$$

$$\hat{u}_7 = -0.4231$$

$$\hat{Y}_8 = 3.6340$$

$$\hat{u}_8 = 0.066$$

$$\sum_{i=0}^N \hat{u}_i = 0$$

$$\sum \hat{u}_i = 0.0027$$

rounding error

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

$$var(\hat{u}_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2}$$

$$var(\hat{u}_i) = \sigma^2 = 0.0857^2 + 0.3791^2 + (-0.225^2) + 0.175^2 + 0.0681^2 + (-0.1231^2) + (-0.4231^2) + 0.066^2$$

$$var(\hat{u}_i) = \sigma^2 = \frac{0.4355}{6} = 0.0726$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{\sum x_i^2} = \frac{\sigma^2}{\sum (x_i - \bar{x})^2}$$

$$var(\hat{\beta}_1) = \frac{0.0726}{511.875} = 0.0001$$

$$var(\hat{\beta}_0) = \frac{\sum x_i^2}{n \sum x_i^2} \sigma^2$$

$$var(\hat{\beta}_0) = \sum x_i^2 \cdot \frac{\sigma^2}{n \sum x_i^2}$$

$$var(\hat{\beta}_0) = 48,717 \cdot 0.0001$$

$$var(\hat{\beta}_0) = 48,717$$

2. Data is listed in the table

X_i	Y_i	$(x_i - \bar{x})(y_i - \bar{y})$
10	0	$(-10)(-9.1) = 91$
12	2	$(-8)(-7.1) = 56.8$
14	5	$(-6)(-4.1) = 24.6$
16	6	$(-4)(-3.1) = 12.4$
18	7	$(-2)(-2.1) = 4.2$
22	10	$2(0.9) = 1.8$
24	10	$4(0.9) = 3.6$
26	15	$6(5.9) = 35.4$
28	16	$8(6.9) = 55.2$
30	20	$10(10.9) = 109$

$$\bar{x} = \frac{200}{10}$$

$$\bar{x} = 20$$

$$\bar{y} = \frac{91}{10}$$

$$\bar{y} = 9.1$$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 394$$

$$\sum (x_i - \bar{x})^2 = 440$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of

β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$\hat{\beta}_1 = \frac{394}{440}$$

$$\hat{\beta}_1 = 0.8955$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

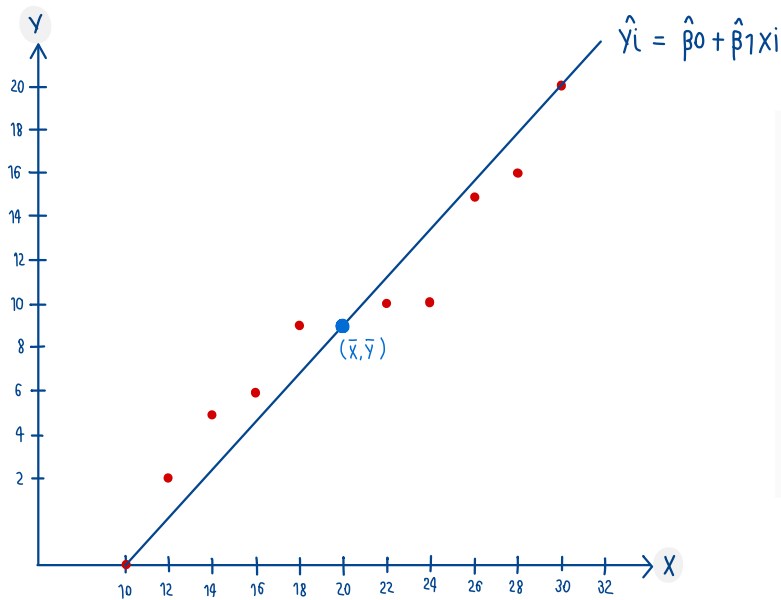
$$\hat{\beta}_0 = 9.1 - 0.8955(20)$$

$$\hat{\beta}_0 = -8.8091$$

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$. $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \mid \hat{u}_i = Y_i - \hat{Y}_i$

$\hat{Y}_1 = 0.1455$	$\hat{Y}_4 = 5.5182$	$\hat{Y}_7 = 12.6818$	$\hat{Y}_{10} = 18.0545$
$\hat{u}_1 = -0.1455$	$\hat{u}_4 = 0.4818$	$\hat{u}_7 = -2.6818$	$\hat{u}_{10} = 1.9455$
$\hat{Y}_2 = 1.9364$	$\hat{Y}_5 = 7.3091$	$\hat{Y}_8 = 14.4727$	From the calculated data, $\sum \hat{u}_i = 0$
$\hat{u}_2 = 0.0636$	$\hat{u}_5 = -0.3091$	$\hat{u}_8 = 0.5273$	
$\hat{Y}_3 = 3.7273$	$\hat{Y}_6 = 10.8909$	$\hat{Y}_9 = 16.2636$	
$\hat{u}_3 = 1.2727$	$\hat{u}_6 = -0.8909$	$\hat{u}_9 = -0.2636$	

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



From 2.1, $\bar{X} = 20, \bar{Y} = 9.1$

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

$$\hat{Y}_i = -8.8091 + 0.8955(20)$$

$$\hat{Y}_i = 9.1$$

$\therefore$ The line passes through $\bar{X}, \bar{Y}$

2.4 If $X_i = 16$, what is the predicted Y?

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = -8.8091 + 0.8955(16) = 5.5189$$

2.5 Find $\text{var}(\hat{u}_i), \text{var}(\hat{\beta}_0), \text{var}(\hat{\beta}_1)$

$$\text{var}(\hat{u}_i) = s^2 = \frac{\sum \hat{u}_i^2}{n-2}$$

$$\text{var}(\hat{u}_i) = s^2 = \frac{(-0.1455)^2 + 0.0636^2 + 1.2727^2 + 0.4818^2 + (-0.3091)^2 + (-0.8909)^2 + (-2.6818)^2 + 0.5273^2 + (-0.2636)^2}{10-2}$$

$$\text{var}(\hat{u}_i) = s^2 = \frac{14.0909}{10-2} = 1.7614$$

$$\text{var}(\hat{\beta}_1) = \frac{s^2}{\sum X_i^2} = \frac{1.7614}{440} = 0.004$$

$$\text{var}(\hat{\beta}_0) = \frac{\sum X_i^2}{n \sum X_i^2} s^2$$

$$\text{var}(\hat{\beta}_0) = \frac{4400 \cdot 0.004}{10}$$

$$\text{var}(\hat{\beta}_0) = 1.7614$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum_i (y_i - \bar{y})(X_i - \bar{X})}{\sum_i (X_i - \bar{X})^2}, \quad \text{then let's } A = X_i - \bar{X} \text{ and } K = \frac{X_i - \bar{X}}{\sum_i (X_i - \bar{X})^2}$$

$$\hat{\beta}_1 = \sum_i (y_i - \bar{y}) K_i$$

$$\begin{aligned} \hat{\beta}_1 &= \sum_i (\beta_1 X_i + u_i - \beta_1 \bar{X}) K_i \\ &= \sum_i (\beta_1 X_i - \beta_1 \bar{X}) K_i + \sum_i u_i K_i \\ &= \beta_1 \sum_i (X_i - \bar{X}) K_i + \sum_i u_i K_i \end{aligned}$$

$$= \beta_1 \sum_i A \cdot \frac{A}{\sum_i A^2} + \sum_i u_i K_i$$

$$= \beta_1 \cdot \frac{\sum_i A^2}{\sum_i A^2} + \sum_i u_i K_i$$

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_i u_i K_i]$$

$$= \beta_1 + E(\sum_i u_i K_i) \quad \leftarrow \text{SLR4: } E(u_i | X_i) = 0$$

SLR4 to make it to be zero

this assumption takes the value of X as given (or fixed) so, we can treat X as a constant. therefore, the error has expected value of zero given any value of X variables.

$$E(\hat{\beta}_1) = \sum_i K_i E(u_i) \quad \begin{array}{l} \text{function of } X_i \\ \text{zero} \end{array}$$

$$\text{so, } E(\hat{\beta}_1) = \beta_1$$

$\therefore$ From the proof, we know that $\hat{\beta}_1$ is an unbiased estimator under assumption SLR 1-4