

# **Simple macroeconomic model**

EC320

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# Simple macroeconomic model

- Simple Keynesian Cross
  - Partial equilibrium: good market
  - Financial market is treated as given.
  - Implications: emphasizing at the role of government sector
- IS-LM model
  - General equilibrium: good and money market
  - Financial market is endogenized. (Determined within the model.)
  - Implications: emphasizing at the role of fiscal and monetary policy

# Simple macroeconomic model

- Solving for equilibrium solution of endogenous variables.
- Some comparative static analysis: analyze behavior of endogenous variables.
- Policy issues
  - Multipliers
  - Crowding-out effects
  - Policy effectiveness.

# A simple Keynesian Cross model

$Y = C + I \rightarrow$  equilibrium condition

$C = C(Y) = a + bY \rightarrow$  behavior of consumption

$I = I_0 \rightarrow$  exogenous investment (take it as given.)

Exogenous:  $I_0$ .

Endogenous:  $C$  and  $Y$ .

Parameters:  $a$  (autonomous consumption),  $b$  (mpc)

# A numerical example

$$Y = C + I$$

$$C = C(Y) = 10 + 0.5 * Y$$

$$I = 100$$

Exogenous:  $I_0$ .

Endogenous:  $C$  and  $Y$ .

Parameters:  $a, b$

# A numerical example: model solution

- Solving for the solution of endogenous variables, we yield that

$$Y = \underline{\hspace{10em}}.$$

$$C = \underline{\hspace{10em}}.$$

# Simple Keynesian Cross model: general solution

- In the general form,

$$Y = \underline{\hspace{10em}}$$

$$C = \underline{\hspace{10em}}$$

- What does the solution mean?
  - Y and C depend on “a,b” and “the given level of investment.”
  - If any of these change, “Y” and “C” would change correspondingly.

# Simple Keynesian Cross model:

## Comparative static

- Sensitivity analysis
  - What happen if  $I_0$  increases to  $I'_0$  ? Can you compare the change in  $Y$  and  $C$ , before and after?

$$\Delta Y = Y' - Y = \frac{1}{1-b} * (I'_0 - I_0)$$

$$\Delta C = C' - C = \frac{b}{1-b} * (I'_0 - I_0)$$

- $I'_0 = 110$ ,  $\Delta Y = ?$  and  $\Delta C = ?$

# Extensions

- Endogenous investment
  - Investment also depends on level of income
  - Income-accelerator framework.
- Government sector
  - Typically treated as an exogenous variable.
  - Alternatively,  $G$  might be tied to the level of GDP.
- Taxation
  - Consumption depends on disposable income.
  - Income tax
- Effect of interest rate on private spending
  - Consumption and investment could also depend on income
- Open economy
  - Equilibrium condition must include net export, i.e.  $X - M$ .
  - Need to model “ $X$ ” and “ $M$ ”.

# Extended Keynesian Cross model

$$Y = C + I + G \longrightarrow \text{Equilibrium condition}$$

$$C = C(Y_d, r)$$

Consumption function

- Positively vary with disposable income
- Negatively vary with interest rate

$$Y_d = Y - T$$

$$I = I(Y, r)$$

Investment function

- Positively vary with income
- Negatively vary with interest rate

$$T = T(Y)$$

$$G = G_0$$

Exogenous variable

# Example

- Equilibrium (1)

$$Y = C + I + G$$

- Consumption function (2)

$$C = a + bY_d$$

- Disposable income (3)

$$Y_d = Y - T$$

- Tax rule (4)

$$T = T_0 + t * Y$$

- Investment function (5)

$$I = I_0 - I_1 r$$

- Government spending (6)

$$G = G_0$$

# Example

- Substitute (2), (5), (6) into (1), this yields us

$$Y = a + bY_d + I_0 - I_1r + G_0 \quad (7)$$

- Substituting (4) into (3), this yields us

$$Y_d = Y - T_0 - t * Y \quad (8)$$

- Substituting (8) into (7), this yields us

$$\begin{aligned} Y &= a + b(Y - T_0 - t * Y) + I_0 - I_1r + G_0 \\ &= a - bT_0 + I_0 - I_1r + G_0 + b(1 - t)Y \end{aligned} \quad (9)$$

- Solving for “Y” in terms of other variables,

$$Y = \frac{1}{1 - b(1 - t)} [a - bT_0 + I_0 - I_1r + G_0]$$

- Explaining the general form of endogenous level of “Y” for given values of parameters and exogenous variables, aka, **the IS equation.**

# Example

- What can we play with the solution?
  - “Y” responds to changes of whatever on the right!
  - Sensitivity analysis

# Sensitivity analysis

- $a=1$  (old value)  $\rightarrow a'=2$  (new value)

- Old equilibrium

$$- Y^{old} = \frac{1}{1-b(1-t)} [1 - bT_0 + I_0 - I_1r + G_0]$$

- New equilibrium

$$- Y^{new} = \frac{1}{1-b(1-t)} [2 - bT_0 + I_0 - I_1r + G_0]$$

- Change in  $Y = \frac{1}{1-b(1-t)} (2-1)$

# Sensitivity analysis

- That is, one unit change in “a” causes an increase in “y” by  $\frac{1}{1-b(1-t)} > 1$  units.
- This means that “a” has a multiplicative effect of “Y”.
- We therefore call  $\frac{1}{1-b(1-t)}$  as the **multiplier** of “a”, the autonomous consumption.

# Multiplier for $I_0$ ?

- Do the same, we yield that multiplier of  $I_0$  (autonomous investment) is equal to,

$$\frac{1}{1 - b(1 - t)}$$

- Coefficient attached to the variable  $I_0$  and  $a$ .

# Multiplier of government variables

$$Y = \frac{1}{1 - b(1 - t)} [a - bT_0 + I_0 - I_1r + G_0]$$

- Multiplier of  $T_0$ :  $\frac{\Delta Y}{\Delta T_0} = \underline{\hspace{2cm}}$
- Multiplier of  $G_0$ :  $\frac{\Delta Y}{\Delta G_0} = \underline{\hspace{2cm}}$

# Multiplier of government variables

- Notice something?
  - In the absolute term, tax has smaller size of multiple effect than government purchase.
  - Why?
  - Intuitively, tax cut doesn't increase output in the first round. Tax cut indirectly increases output through an increase in consumption.
  - On the other hand, government spending increases output in the first round, suddenly right after the increase in  $G$ .

# What about “b”?

- No formula can be given for now. Need to find by using derivative. (Will come back to its closed-form formula later)
- Just live with the *Brute-force* method:
  - You must compare the two values “y”.
  - New Y (with new “b”) – Initial Y (with initial “b”).
- Intuitively, the change should be positive when “b = MPC” increases.

# A numeric example

- Equilibrium

$$Y = C + I + G$$

- Consumption

$$C = 48 + 0.8Y$$

- Investment

$$I = 98 - 75r$$

# A numeric example

- Questions:
  - a. Derive the IS equation
  - b. Solve for equilibrium output when  $r = 0.1$  (=10%)
  - c. Calculate multiplier of autonomous investment.

# IS-LM model

- So far, we have seen a simple Keynesian cross model that “ $r$ ” is exogenously given.
  - An increase in “ $r$ ” reduces aggregate expenditure, and hence the level of income in the equilibrium.
- Is the assumption that “ $r$ ” an exogenous variable sensible?
- IS-LM model is an extension of the Keynesian cross.
  - It endogenizes the value of interest rate by modeling how interest rate is determined in the equilibrium.

# IS-LM model

- Interest rate is determined from the equilibrium in money market, where money demand is equal to money supply.
- Since money demand also depends on income, equilibrium level of interest rate in the money market also depends on the level of income.
- As a result, income and interest rate will be simultaneously determining each other in the general equilibrium.

# Money market

- Equilibrium

$$M^d = M^s$$

- Money demand equation

$$M^d = L_0 + L_1 Y - L_2 r$$

- Money supply

$$M^s = M_0^s$$

# Equilibrium in money market

- Putting everything into the equilibrium condition

$$L_0 + L_1 Y - L_2 r = M_0^S$$

- Suppose that “Y” is given for now. Write “r” in terms of “Y” and “ $M_0^S$ ”, we yield the **LM equation**,

$$r = \frac{1}{L_2} (L_0 + L_1 Y - M_0^S)$$

# IS-LM equilibrium

- $(Y^*, r^*)$  that is the simultaneous solution of the system of equations that is characterized by

- IS equation

$$Y = \frac{1}{1 - b(1 - t)} [a - bT_0 + I_0 - I_1r + G_0]$$

- LM equation

$$r = \frac{1}{L_2} (L_0 + L_1Y - M_0^s)$$

To get a better sense, consider the following numerical example

- $C = 0.8(Y - T)$
- $T = 1,000$
- $I = 800 - 20r$
- $G = 1,000$
- $Y = C + I + G$
- $M^d = 0.4Y - 40r$
- $M^s = 1,200$

# Example

- Write numerical formula for IS equation
- Write numerical formula for LM equation
- Solve for endogenous equilibrium solution of “Y”, “C”, “I” and “saving”.
- What happen to equilibrium “Y” when G increases by 200.
- Back to when G is equal to 1000. What happen to equilibrium “Y” when money supply increases by 200.
- Measuring in term of the change in output, which policy is more effective? Explain the reason.

# Back to general form

- $(Y^*, r^*)$  that is the simultaneous solution of the system of equations that is characterized by

- IS equation

$$Y = \frac{1}{1 - b(1 - t)} [a - bT_0 + I_0 - I_1r + G_0]$$

- LM equation

$$r = \frac{1}{L_2} (L_0 + L_1Y - M_0^s)$$

# A bit tedious...

- $Y = \frac{1}{1-b(1-t)} [a - bT_0 + I_0 - I_1 r + G_0]$
- $Y = \frac{1}{1-b(1-t)} \left[ a - bT_0 + I_0 - I_1 \left( \frac{1}{L_2} (L_0 + L_1 Y - M_0^s) \right) + G_0 \right]$
- Suppose that  $x = \frac{1}{1-b(1-t)}$
- Remember that "x" is related to the multiplier effect.

# A bit tedious...

- $Y =$   
$$x \left[ a - bT_0 + I_0 - I_1 \left( \frac{1}{L_2} (L_0 + L_1 Y - M_0^S) \right) + G_0 \right]$$

- $Y =$   
$$\frac{x}{1+x\left(I_1\frac{L_1}{L_2}\right)} \left[ a - bT_0 + I_0 - \frac{I_1}{L_2} L_0 + \frac{I_1}{L_2} M_0^S + G_0 \right]$$

# Multiplier of G

- Multiplier of G is  $\frac{x}{1+x\left(I_1\frac{L_1}{L_2}\right)}$ 
  - $\left(I_1\frac{L_1}{L_2}\right) > 0$ .
  - This implies that  $\frac{x}{1+x\left(I_1\frac{L_1}{L_2}\right)} < x$ .
  - What is  $x$  ?
  - Compare with the previous case that “r” is given,  
multiplier effect of G has smaller size!

# Multiplier of G

- Obviously, an increase in “G” causes an increase in “y”.
- If interest rate were kept fixed, effect of “G” would be very strong.
- However, as “y” increases, interest rate tends to move along.
- This would lower the investment, and thus reducing aggregate expenditure.
- So, it partially offsets the effect of “G”.
- Income wouldn’t be changing that much, comparing to when you don’t have this feedback effect.
- Economically, this is called the “crowding-out effect”

# Multiplier of other variables

- Multiplier of tax is  $-b \left[ \frac{x}{1+x\left(I_1 \frac{L_1}{L_2}\right)} \right]$
- Multiplier of  $I_0$  is the same as multiplier of G
- Multiplier of M is  $\frac{x}{1+x\left(I_1 \frac{L_1}{L_2}\right)} \frac{I_1}{L_2}$

# Relative effectiveness of G and M

- Which policy is more effective, between an increase in government and an increase in money supply?
- To be more precise, which one can increase income more, for the same amount of change?
- Compare multipliers the two.

# Relative effectiveness of G and M

- Compare the multipliers of M and G

$$\frac{\text{multiplier of } M}{\text{multiplier of } G} = \frac{I_1}{L_2}$$

- $I_1 > L_2 \Rightarrow$  “M” is better than “G”. (vice versa)
- Intuition:
  - High value of  $I_1$  implies that investment is very sensitive to interest rate.
  - High value of  $L_2$  implies that demand money is less sensitive to income. Crowding out effect is not strong.

# Back to the previous example

- Does it make sense for the answer that you have solved for?