(iii) Show that $\hat{\beta}_0$ can be written as $\hat{\beta}_0 = \beta_0 + \bar{u} - (\hat{\beta}_1 - \beta_1)\bar{x}$.
(iv) Use parts (ii) and (iii) to show that $\mathrm{Var}(\hat{\beta}_0) = \sigma^2/n + \sigma^2(\bar{x})^2/\mathrm{SST}_x$.
(v) Do the algebra to simplify the expression in part (iv) to equation (2.58). [*Hint*: $\mathrm{SST}_x/n = n^{-1}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2$.]

2.7 Using data from 1988 for houses sold in Andover, Massachusetts, from Kiel and McClain (1995), the following equation relates housing price (*price*) to the distance from a recently built garbage incinerator (*dist*):

$$\widehat{\log(price)} = 9.40 + 0.312 \log(dist)$$
$$n = 135, R^2 = 0.162.$$

(i) Interpret the coefficient on log(*dist*). Is the sign of this estimate what you expect it to be?
(ii) Do you think simple regression provides an unbiased estimator of the ceteris paribus elasticity of *price* with respect to *dist*? (Think about the city's decision on where to put the incinerator.)
(iii) What other factors about a house affect its price? Might these be correlated with distance from the incinerator?

2.8 (i) Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the intercept and slope from the regression of y_i on x_i, using n observations. Let c_1 and c_2, with $c_2 \neq 0$, be constants. Let $\tilde{\beta}_0$ and $\tilde{\beta}_1$ be the intercept and slope from the regression of c_1y_i on c_2x_i. Show that $\tilde{\beta}_1 = (c_1/c_2)\hat{\beta}_0$ and $\tilde{\beta}_0 = c_1\hat{\beta}_0$, thereby verifying the claims on units of measurement in Section 2.4. [*Hint:* To obtain $\tilde{\beta}_1$, plug the scaled versions of x and y into (2.19). Then, use (2.17) for $\tilde{\beta}_0$, being sure to plug in the scaled x and y *and* the correct slope.]
(ii) Now, let $\tilde{\beta}_0$ and $\tilde{\beta}_1$ be from the regression of $(c_1 + y_i)$ on $(c_2 + x_i)$ (with no restriction on c_1 or c_2). Show that $\tilde{\beta}_1 = \hat{\beta}_1$ and $\tilde{\beta}_0 = \hat{\beta}_0 + c_1 - c_2\hat{\beta}_1$.
(iii) Now, let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the OLS estimates from the regression $\log(y_i)$ on x_i, where we must assume $y_i > 0$ for all i. For $c_1 > 0$, let $\tilde{\beta}_0$ and $\tilde{\beta}_1$ be the intercept and slope from the regression of $\log(c_1y_i)$ on x_i. Show that $\tilde{\beta}_1 = \hat{\beta}_1$ and $\tilde{\beta}_0 = \log(c_1) + \hat{\beta}_0$.
(iv) Now, assuming that $x_i > 0$ for all i, let $\tilde{\beta}_0$ and $\tilde{\beta}_1$ be the intercept and slope from the regression of y_i on $\log(c_2x_i)$. How do $\tilde{\beta}_0$ and $\tilde{\beta}_1$ compare with the intercept and slope from the regression of y_i on $\log(x_i)$?

2.9 In the linear consumption function

$$\widehat{cons} = \hat{\beta}_0 + \hat{\beta}_1 inc,$$

the (estimated) *marginal propensity to consume* (MPC) out of income is simply the slope, $\hat{\beta}_1$, while the *average propensity to consume* (APC) is $\widehat{cons}/inc = \hat{\beta}_0/inc + \hat{\beta}_1$. Using observations for 100 families on annual income and consumption (both measured in dollars), the following equation is obtained:

$$\widehat{cons} = -124.84 + 0.853\ inc$$
$$n = 100, R^2 = 0.692.$$

(i) Interpret the intercept in this equation, and comment on its sign and magnitude.
(ii) What is the predicted consumption when family income is $30,000?
(iii) With *inc* on the *x*-axis, draw a graph of the estimated MPC and APC.