# Statistical Concepts and Risk Management

1. A firm holds two $50 million bonds with call dates this week.
   The probability that Bond A will be called is 0.80.
   The probability that Bond B will be called is 0.30.
   The probability that at least one of the bonds will be called is *closest to*?

*Answer:*
*We calculate the probability that at least one of the bonds will be called using the addition rule for probabilities:*

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where $P(A \text{ and } B) = P(A) \times P(B)$

$P(A \text{ or } B) = 0.80 + 0.30 - (0.8 \times 0.3) = 0.86$

2. Jessica Fassler, options trader, recently wrote two put options on two different underlying stocks (AlphaDog Software and OmegaWolf Publishing), both with a strike price of $11.50. The probabilities that the prices of AlphaDog and OmegaWolf stock will decline below the strike price are 65% and 47%, respectively. The probability that at least one of the put options will fall below the strike price is approximately?

*Answer:*
*We calculate the probability that at least one of the options will fall below the strike price using the addition rule for probabilities (A represents AlphaDog, O represents OmegaWolf):*

$P(A \text{ or } O) = P(A) + P(O) - P(A \text{ and } O)$, where $P(A \text{ and } O) = P(A) \times P(O)$

$P(A \text{ or } O) = 0.65 + 0.47 - (0.65 \times 0.47) =$ approximately 0.81

3. Thomas Baynes has applied to both Harvard and Yale. Baynes has determined that the probability of getting into Harvard is 25% and the probability of getting into Yale (his father's alma mater) is 42%. Baynes has also determined that the probability of being accepted at both schools is 2.8%. What is the probability of Baynes being accepted at either Harvard or Yale?

*Answer:*
*Using the addition rule, the probability of being accepted at Harvard or Yale is equal to:*

$P(\text{Harvard}) + P(\text{Yale}) - P(\text{Harvard and Yale}) = 0.25 + 0.42 - 0.028 = 0.642$ or 64.2%.

4. There is a 30% chance that the economy will be good and a 70% chance that it will be bad. If the economy is good, your returns will be 20% and if the economy is bad, your returns will be 10%. What is your expected return?

*Answer:*
*Expected value is the probability weighted average of the possible outcomes of the random variable. The expected return is:* $((0.3) \times (0.2)) + ((0.7) \times (0.1)) = (0.06) + (0.07) = 0.13$.

5. There is a 90% chance that the economy will be good next year and a 10% chance that it will be bad. If the economy is good, there is a 60% chance that XYZ Incorporated will have EPS of \$4.00 and a 40% chance that their earnings will be \$3.00. If the economy is bad, there is an 80% chance that XYZ Incorporated will have EPS of \$2.00 and a 20% chance that their earnings will be \$1.00. What is the firm's expected EPS?

*Answer:*
*The expected EPS is calculated by multiplying the probability of the economic environment by the probability of the particular EPS and the EPS in each case. The expected EPS in all four outcomes are then summed to arrive at the expected EPS:*
$(0.90 \times 0.60 \times \$4.00) + (0.90 \times 0.40 \times \$3.00) + (0.10 \times 0.80 \times \$2.00) + (0.10 \times 0.20 \times \$1.00) = \$2.16 + \$1.08 + \$0.16 + \$0.02 = \$3.42.$

6. There is a 40% probability that the economy will be good next year and a 60% probability that it will be bad. If the economy is good, there is a 50 percent probability of a bull market, a 30% probability of a normal market, and a 20% probability of a bear market. If the economy is bad, there is a 20% probability of a bull market, a 30% probability of a normal market, and a 50% probability of a bear market. What is the probability of a bull market next year?

*Answer:*
*Because a good economy and a bad economy are mutually exclusive, the probability of a bull market is the sum of the joint probabilities of (good economy and bull market) and (bad economy and bull market):* $(0.40 \times 0.50) + (0.60 \times 0.20) = 0.32$ *or 32%.*

7. An investor is considering purchasing ACQ. There is a 30% probability that ACQ will be acquired in the next two months. If ACQ is acquired, there is a 40% probability of earning a 30% return on the investment and a 60% probability of earning 25%. If ACQ is not acquired, the expected return is 12%. What is the expected return on this investment?

*Answer:*
$E(r) = (0.70 \times 0.12) + (0.30 \times 0.40 \times 0.30) + (0.30 \times 0.60 \times 0.25) = 0.165.$

8. Tully Advisers, Inc., has determined four possible economic scenarios and has projected the portfolio returns for two portfolios for their client under each scenario. Tully's economist has estimated the probability of each scenario, as shown in the table below. Given this information, what is the standard deviation of returns on portfolio A?

| Scenario | Probability | Return on Portfolio A | Return on Portfolio B |
|---|---|---|---|
| A | 15% | 18% | 19% |
| B | 20% | 17% | 18% |
| C | 25% | 11% | 10% |
| D | 40% | 7% | 9% |

*Answer:*
$E(R) = 11.65\%$
$\sigma^2 = 0.0020506$
$= 0.15(0.18 - 0.1165)^2 + 0.2(0.17 - 0.1165)^2 + 0.25(0.11 - 0.1165)^2 + 0.4(0.07 - 0.1165)^2$
$\sigma = 0.0452836$

9. Joe Mayer, CFA, projects that XYZ Company's return on equity varies with the state of the economy in the following way:

| State of Economy | Probability of Occurrence | Company Returns |
|---|---|---|
| Good | .20 | 20% |
| Normal | .50 | 15% |
| Poor | .30 | 10% |

The standard deviation of XYZ's expected return on equity is *closest to:*

*Answer:*

*In order to calculate the standard deviation of the company returns, first calculate the expected return, then the variance, and the standard deviation is the square root of the variance.*

*The expected value of the company return is the probability weighted average of the possible outcomes: (0.20)(0.20) + (0.50)(0.15) + (0.30)(0.10) = 0.145.*

*The variance is the sum of the probability of each outcome multiplied by the squared deviation of each outcome from the expected return:*

$(0.2)(0.20\ 0.145)^2 + (0.5)(0.15\ 0.145)^2 + (0.3)(0.10.145)^2$

$= 0.000605 + 0.0000125 + 0.0006075 = 0.001225.$

*The standard deviation is the square root of 0.001225 = 0.035 or 3.5%.*

10. The covariance of the returns on investments X and Y is 18.17. The standard deviation of returns on X is 7%, and the standard deviation of returns on Y is 4%. What is the value of the correlation coefficient for returns on investments X and Y?

*Answer:*

*The correlation coefficient = Cov (X,Y) / [(Std Dev. X)(Std. Dev. Y)] = 18.17 / 28 = 0.65*

11. The returns on assets C and D are strongly correlated with a correlation coefficient of 0.80. The variance of returns on C is 0.0009, and the variance of returns on D is 0.0036. What is the covariance of returns on C and D?

*Answer:*

$r = Cov(C,D) / (\sigma_c \times \sigma_d)$

$\sigma_c = (0.0009)^{0.5} = 0.03$

$\sigma_d = (0.0036)^{0.5} = 0.06$

$0.8(0.03)(0.06) = 0.00144$

12. The covariance of returns on two investments over a 10year period is 0.009. If the variance of returns for investment A is 0.020 and the variance of returns for investment B is 0.033, what is the correlation coefficient for the returns?

*Answer:*

*The correlation coefficient is:*

$Cov(A,B) / [(Std\ Dev\ A)(Std\ Dev\ B)] = 0.009 / [(\sqrt{0.02})(\sqrt{0.033})] = 0.350.$

13. The volatility of a stock price is 30% per annum. What is the standard deviation of the percentage price change in one week?

*Answer: 4.23% or 4.16%*

14. The volatility of an asset is 25% per annum. What is the standard deviation of the percentage price change in one trading day? Assuming a normal distribution, estimate 95% confidence limits for the percentage price change in one day.

*Answer:*

*The standard deviation of the percentage price change in one day is 1.57%. The 95% confidence limits are from -3.09% to +3.09%.*

15. Why do traders assume 252 rather than 365 days in a year when using volatilities?

*Answer:*

*Volatility is much higher when markets are open than when they are closed. Traders therefore measure time in trading days rather than calendar days when applying volatility.*

16. Suppose that observations on an exchange rate at the end of the last 11 days have been 0.7000, 0.7010, 0.7070, 0.6999, 0.6970, 0.7003, 0.6951, 0.6953, 0.6934, 0.6923, 0.6922. Estimate the daily volatility with the theoretical formula and simplified version.

*Answer:*

*The theoretical approach gives 0.547% per day. The simplified approach gives 0.530% per day.*

17. The number of visitors to a website follows the power law with $\alpha = 2$. Suppose that 1 % of sites get 500 or more visitors per day. What percentage of sites get (a) 1000 and (b) 2000 or more visitors per day?

*Answer:*

*(a) 0.25%*

*(b) 0.0625%.*

18. Assume that S&P 500 at close of trading yesterday was 1,040 and the daily volatility of the index was estimated as 1% per day at that time. The parameters in a GARCH(1,1) model are $\omega = 0.000002$, $\alpha = 0.06$, and $\beta = 0.92$. If the level of the index at close of trading today is 1,060, what is the new volatility estimate?

*Answer:*

*With the usual notation,* $u_{n-1} = 20/1040$ *= 0.01923, so that*

$$\sigma_n^2 = 0.000002 + 0.06 \times 0.019232 + 0.92 \times 0.012 = 0.0001162$$

*This gives* $\sigma_n^2$ *= 0.01078. The new volatility estimate is therefore 1.078% per day.*

19. The parameters of a GARCH(1,1) model are estimated as $\omega = 0.000004$, $\alpha = 0.05$, and $\beta = 0.92$. What is the long-run average volatility? If the current volatility is 20% per year, what is the expected volatility in 20 days?

*Answer:*

*The long-run average variance rate is* $\omega/(1-\alpha-\beta)$ *or 0.000004/0.03 = 0.0001333. The long-run average volatility is* $\sqrt{0.0001333}$ *or 1.155%. The equation describing the way the variance rate reverts to its long-run average is:* $E[\sigma_{n+k}^2] = V_L + (\alpha+\beta)^k(\sigma_n^2 - V_L)$.

*If the current volatility is 20% per year,* $\sigma_n = 0.2/\sqrt{252} = 0.0126$. *The expected variance rate in 20 days is* $0.0001333 + 0.97^{20}(0.0126^2 - 0.0001333) = 0.0001417$. *The expected volatility in 20 days is therefore* $\sqrt{0.0001417}$ *=0.0121, or 1.21% per day.*

20. Suppose that GARCH(1,1) parameters have been estimated as $\omega$ = 0.000003, $\alpha$ =0.04, and $\beta$ =0.94. The current daily volatility is estimated to be 1%. Estimate the daily volatility in 30 days.

*Answer:*

*In this case, $V_L$ = 0.00015 and the expected variance rate in 30 days is 0.000123. The volatility is 1.11% per day.*

21. Suppose that GARCH(1,1) parameters have been estimated as $\omega$ = 0.000002, $\alpha$ =0.04, and $\beta$ =0.94. The current daily volatility is estimated to be 1.3%. Estimate the volatility per annum that should be used to price a 20-day option.

*Answer:*

*In this case, $V_L$ =0.0001, a = 0.0202, T = 20, and V(0) = 0.000169, so that the volatility is 19.88%.*

22. Suppose that observations on a stock price (in US dollars) at the end of each of 15 consecutive weeks are as follows: 30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2 Estimate the stock price volatility.

*Answer:*

23. Suppose that the price of gold at close of trading yesterday was $300 and its volatility was estimated as 1.3% per day. The price at the close of trading today is $298. Update the volatility estimate using the GARCH(1,1) model with $\omega$=0.000002, $\alpha$ = 0.04, and $\beta$ = 0.94.

*Answer:*