Introduction

	Consumption duality To explain how to obtain the utility with the budget we have	
	Primal problem	Dual problem
	Keeping our budget fixed and maximize utility	Keeping our utility fixed and minimize cost
Real world Example	What combination of goods should I purchase given amount of my budget and prices of goods?	What is the minimum budget that I need to purchase these goods to gain a certain level of utility?
In math term	$\max U(x_1, x_2)$ Subject to $P_1x_1 + P_2x_2 = m$	$\min P_1x_1 + P_2x_2$ Subject to $U(x_1, x_2) = \bar{U}$
For creating	Marshallian demand (SE+IE) >> can be upward sloping demand	Hicksian demand (SE only)

Derive demand

1. Find initial point
2. Find effect of price change for primal problem approach and dual problem approach

Step one: Find initial point

[using primal approach]

$$\text{Max } U(x, y)$$
$$st: g(x, y,) \quad = m$$

Let $U(x, y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$
Let $g(x, y,) = P_x x + P_y y$
Let x be quantity of goods x
Let y be quantity of goods y
Let P_x be price of goods x equals to 3
Let P_y be price of goods y equals to 2
Let m be total budget equals to 100

1. Set up Lagrangian functions.

Lagrangian function: $L(x,y,\lambda)=f(x,y,)+\lambda(m-g(x,y,))$

$$L = x^{\frac{1}{2}}y^{\frac{1}{2}} + \lambda\,(m - P_x x - P_y y)$$

$$L = x^{\frac{1}{2}}y^{\frac{1}{2}} + \lambda\,(100 - 3x - 2y)$$

$$\frac{\partial L}{\partial x} = \frac{1}{2}x^{-\frac{1}{2}}\,y^{\frac{1}{2}} - \lambda * 3 = 0 \text{ ----(1)}$$

$$\frac{\partial L}{\partial y} = \frac{1}{2}x^{\frac{1}{2}}\,y^{-\frac{1}{2}} - \lambda * 2 = 0 \text{ ----(2)}$$

$$\frac{\partial L}{\partial \lambda} = 100 - 3x - 2y = 0 \text{ ----(3)}$$

Solve for x and y

(1)/ (2): $x^{-1}y = \frac{3}{2} \longrightarrow y = \frac{3}{2}x$

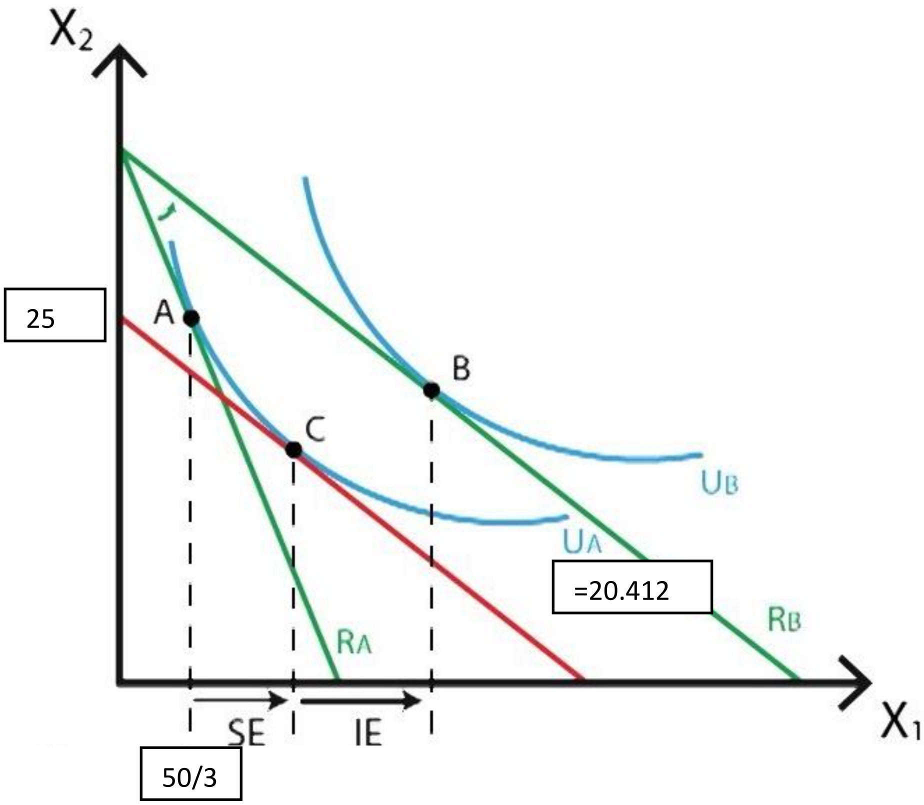
Plug $y = \frac{3}{2}x$ in (3) and solve for x and y

$$100 - 3x - 2(\frac{3}{2}x) = 0$$

$$x = \frac{50}{3}, y = 25$$

$Optimized\ utility\ level = U\left(x = \frac{50}{3}, y = 25\right) = \frac{50^{\frac{1}{2}}}{3} * 25^{\frac{1}{2}} = 20.41241$

Graphically

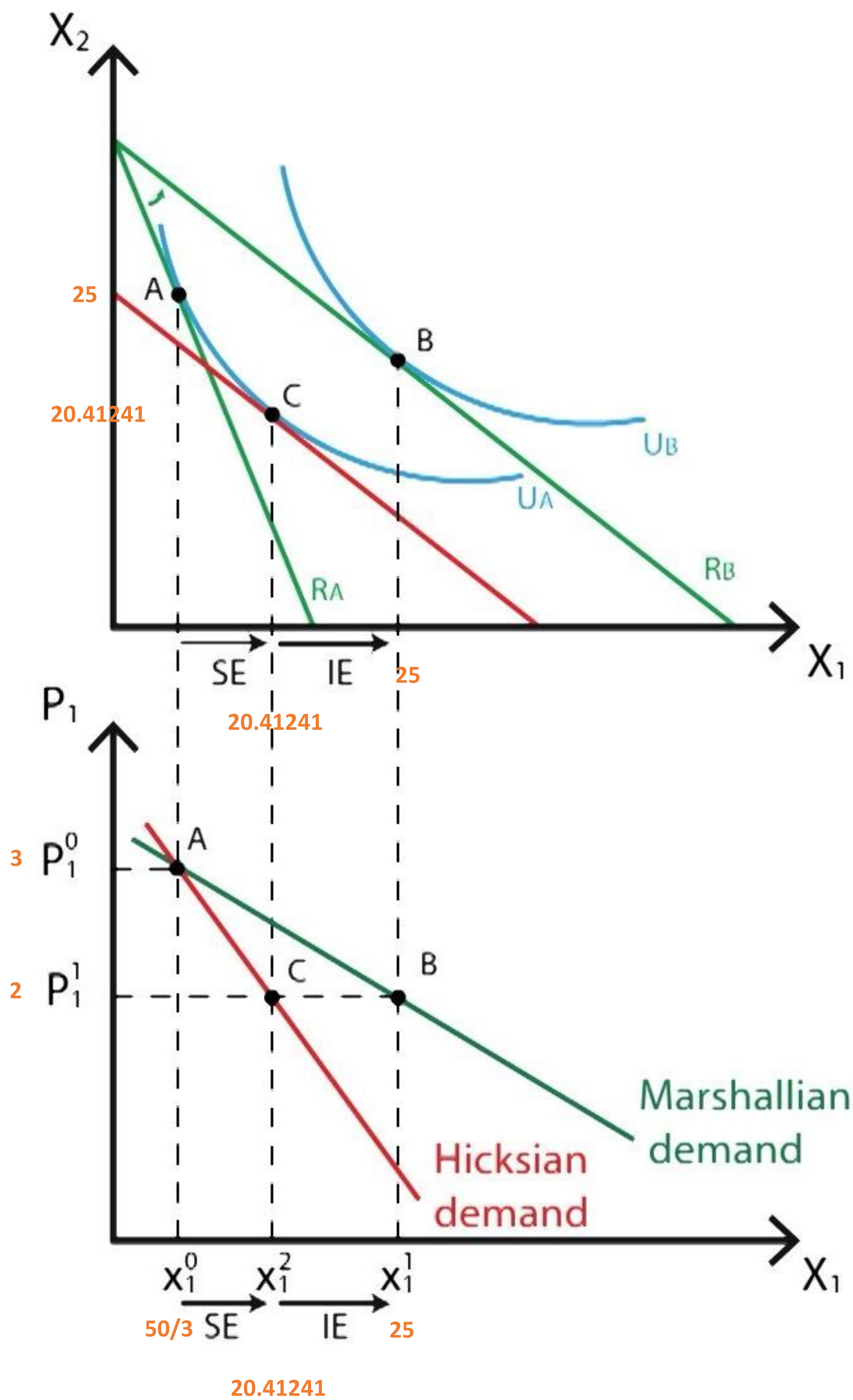


2. Suppose prices change to $P_x = 2, P_y = 2$
   a. We find point B by using primal approach
   b. We find point C by using dual approach

Using primal approach - Keeping our budget fixed (constraint) and maximize utility	Dual approach - Keeping our utility fixed and minimize utility
Maximize utility $U(x,y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$ Subject to fixed budget constraint: $P_x x + P_y y = m$	Minimize cost $P_x x + P_y y$ Subject to fixed utility constraint $x^{\frac{1}{2}}y^{\frac{1}{2}} = \bar{U} = 20.41241$ (utility level at initial point)
Max $L(x,y,\lambda) = x^{\frac{1}{2}}y^{\frac{1}{2}} - \lambda\,(P_x x + P_y y - m)$ Max $L(x,y,\lambda) = x^{\frac{1}{2}}y^{\frac{1}{2}} - \lambda\,(2x + 2y - 100)$ $\frac{\partial L}{\partial x} = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}} - \lambda * 2 = 0$ ----(1) $\frac{\partial L}{\partial y} = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}} - \lambda * 2 = 0$ ----(2) $\frac{\partial L}{\partial \lambda} = 2x + 2y - 100 = 0$ ----(3) Solve for x and y (1)/(2) $y = x$---(4) Plug x = y in (3) $2x + 2x = 100$ x =25 $y = 25$	$Min\ L(x,y,\lambda) = P_x x + P_y y - \lambda\,(x^{\frac{1}{2}}y^{\frac{1}{2}} - \bar{U})$ $Min\ L(x,y,\lambda) = 2x + 2y - \lambda\,(x^{\frac{1}{2}}y^{\frac{1}{2}} - 20.41241)$ $\frac{\partial L}{\partial x} = 2 - \lambda\,\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}} = 0$ ----(1) $\frac{\partial L}{\partial y} = 2 - \lambda\,\frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}} = 0$ ----(2) $\frac{\partial L}{\partial \lambda} = x^{\frac{1}{2}}y^{\frac{1}{2}} - 20.41241 = 0$ ----(3) Solve for x and y (1)/(2) $x^{-1}y = 1$ $x = y$ ----(4) Plug x =y in (3) $x^{\frac{1}{2}}x^{\frac{1}{2}} - 20.41241 = 0$ $x = 20.41241 = y$
$utility\ level = U(x,y) = 25^{\frac{1}{2}} * 25^{\frac{1}{2}} = 25$ << higher utility level	$utility\ level = U(x,y) = 20.41241^{\frac{1}{2}} *$ $20.41241^{\frac{1}{2}} = 20.41241$ << fixed utility level
Total cost =100 <<use up all budget	$Total\ cost;\ P_x x + P_y y$ $= 2 * 20.41241 + 2 * 20.41241$ $= 81.64964$

Summary: use for plotting demand curve

	x	y	Utility level	cost	
Initial point	50/3	25	20.41241	100	
Primal	25	25	25	100	<< same cost
Dual	20.41241	20.41241	20.41241	81.64964	<< same utility



X2
25
A
B
20.41241
C
UB
UA
RB
RA
X1
SE
IE
25
P1
20.41241
3 P1^0
A
2 P1^1
C
B
Marshallian
demand
Hicksian
demand
X1
x1^0
x1^2
x1^1
50/3 SE
IE
25
20.41241