Quiz 2: Date: May 5, 2022 from 11.00-12.30

Question 1 (40 marks) Score....................

Consider the Muliperiod model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$\max_{C_s,\omega_s,\forall_t} \; E_t\left[\sum_{s=t}^{T-1}\delta^s\left(\frac{C_s^{(1-\gamma)}}{1-\gamma}\right)+\delta^T\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$

Assume that there is no wage income $(y_t = 0\,\forall t)$ and a constant risk-free rate return asset , $R_{ft} = R_f$. Also asssume that n=1 and the return of a single risky asset, R_{rt},is independently and identically distributed over time. Denote the proportion of wealth invested in the risky asset at date t as ω_t.

Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-1, C^*_{T-1} and ω^*_{T-1}., and give an explicit expression for C^*_{T-1}

$$\max_{C_s,\omega_s,\forall_t} \; E_t\left[\sum_{s=t}^{T-1}\delta^s\left(\frac{C_s^{(1-\gamma)}}{1-\gamma}\right)+\delta^T\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$

$; \; U(C_t,t) = \delta^s\left(\frac{C_s^{(1-\gamma)}}{1-\gamma}\right)$

$U_c(C_t,t) = \delta^s C_t^{-\gamma}$

FOC :

$U_c(C_{T-1},T-1) = E_{T-1}[B_W(W_T,T)R_{T-1}]$

$\delta^{T-1}(C_{T-1}^{-\gamma}) = E_{T-1}[\delta^T(R_{T-1}\cdot W_{T-1}^{-\gamma}]$

$= E_{T-1}[\delta^T(R_{T-1})\cdot(\delta_{T-1}\cdot R_{T-1})^{-\gamma}]$

$\delta^{T-1}\cdot C_{T-1}^{-\gamma} = \delta^T\cdot\delta_{T-1}^{-\gamma}\cdot R_{T-1}^{1-\gamma}$

$C^*_{T-1} = \delta\cdot W_{T-1}^{-\gamma}$ #

For the optimal condition of $\{w^*_{i,T-1}\}$

$E_{T-1}\left[\frac{R_{i,T-1}}{R^*_{T-1}}\right] = R_{f,T-1}E_{T-1}\left[\frac{1}{R^*_{T-1}}\right]$

$\delta^{T-1}(C_{T-1}^{-\gamma}) = R_{f,T-1}\delta^T E_{T-1}[R^*_{T-1}W_{T-1}^{-\gamma}]$

$1 = \frac{\delta W_{T-1}^{-\gamma}}{C_{T-1}^{-\gamma}} \underset{T-1}{E}[R^*_{T-1}]$ #

Score.....................

Question 1.2 (10 marks) Solve for the form of J(W_{T-1},T-1).

$$J(W_{T-1}, T-1) = \max_{C_{T-1}, \{W_{i,T-1}\}} E_{T-1}\left[\delta^{T-1}\left(\frac{C_{T-1}^{(1-\gamma)}}{1-\gamma}\right) + \delta^{T}\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$

$$= \max_{C_{T-1}, \{W_{i}, T-1\}} \delta^{T-1}\left(\frac{C_{T-1}^{(1-\gamma)}}{1-\gamma}\right) + E_{T-1}\left[\delta^{T}\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right] \quad //$$

Score.....................

Question 1.3 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-2, C^*_{T-2} and ω^*_{T-2}., and give an explicit expression for C^*_{T-2}

Score.....................

Question 1.4 (10 marks) Solve for the form of J(W_{T-2},T-2).Based on the pattern for T-1 and T-2, provide expressions for the optimal consumption and portfolio weight at any date T-t, t=1,2,3,....

$$J(W_{T-2}, T-2) = \max_{c_{T-2}, \{w_{i,T-2}\}} \delta^{T-2}\left(\frac{c_{T-2}^{(1-\gamma)}}{1-\gamma}\right) + E_{T-2}\left[\delta^{T-1}\left(\frac{c_{T-1}^{(1-\gamma)}}{1-\gamma}\right) + \delta^{T}\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$