Exercise 2

Theory of Demand

1. **(SE&IE)** Consider two goods, X and Y. Let U(X, Y) = XY. Py = \$1, Px = \$4, and I = \$72. **Recall the answer from Exercise 1.** The consumer's initial optimal bundle (Bundle A) is where X* = 9 and Y* = 36. When Px' = \$9, the consumer's new optimal bundle (Bundle C) is where X* = 4 and Y* = 36.
   a. In this case of price rise, how will the Slutsky's method compensate income of the consumer? Increasing or decreasing it? By how much?

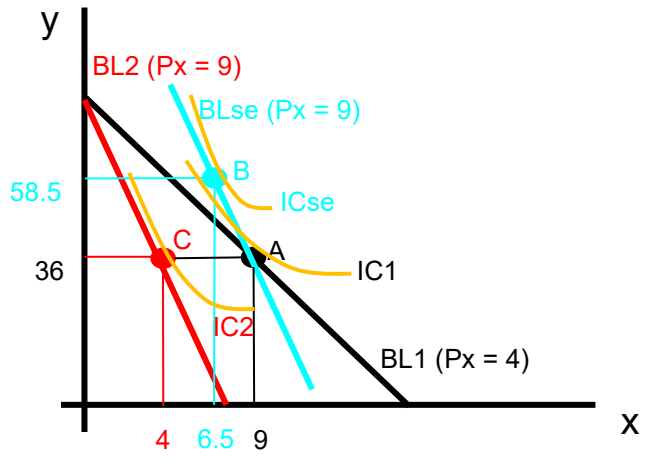
      The Slutsky's method compensates income so that the consumer will have just enough income to purchase the original bundle after the price change.

      BL1: $P_x x + P_y y = I$ → 4*9 + 1*36 = 72

      BL2: $P'_x x + P_y y = I'$ → 9*9 + 1*36 = 117

      Income compensation = I' - I = 117 - 72 = 45 → Increased by 45

   b. Find numerical values of the SE and IE using the Slutsky's method, and draw a consumer choice diagram to show the two effects.



Optimality Cond.: $\frac{MU_x}{MU_y} = \frac{P'_x}{P_y} \rightarrow \frac{y}{x} = \frac{9}{1} \rightarrow$ y = 9x.

Budget Line: $P'_x x + P_y y = I' \rightarrow$ 9x + y = 117.

Solving them: 18x = 117 → x = 6.5, y = 58.5.

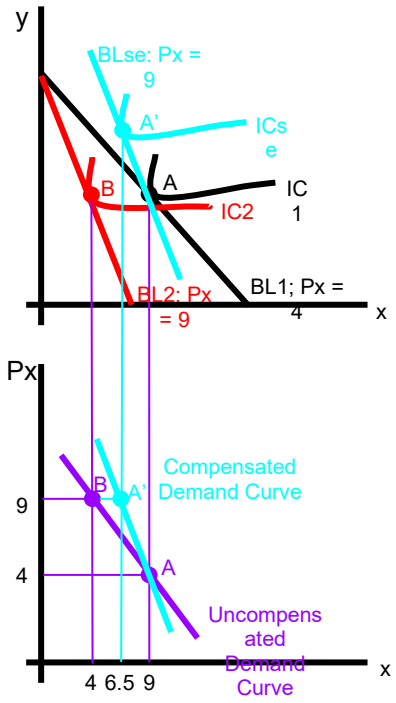
We will consider the SE and IE on good X only.

SE causes this consumer to decrease consumption on x from 9 units to 6.5 units. SE = - 2.5 units.

IE causes this consumer to decrease consumption on x from 6.5 units to 4 units. IE = - 2.5 units.

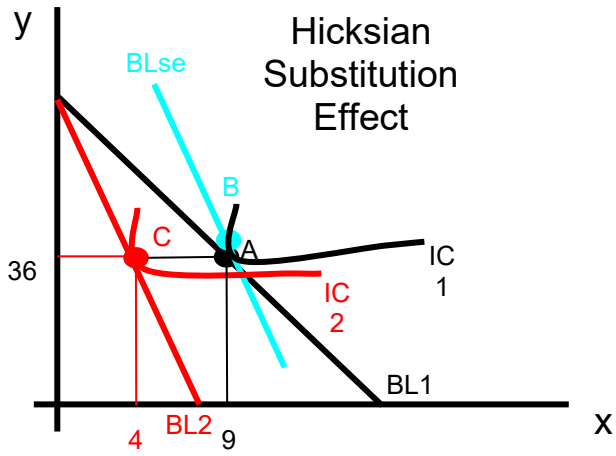
TE = SE + IE = - 5 units. That is, when Px rises, X falls by 5 units.

c. Draw the uncompensated and compensated demand curves. What is the difference between the two curves?



The uncompensated demand curve is derived from the total effect (SE+IE) whilst the compensated demand curve is derived from the substitution effect. Due to the price rise, the original bundle becomes unachievable. Therefore, compensated demand will compensate income to reach the original bundle. Hence, when Px = $9, compensated demand curve shows that consumer will buy 6.5 units (4 units for uncompensated demand). This is because the consumer has been income-compensated after the price rise.

d. Explain the difference between the Hicks' and Slutsky's methods in identifying the substitution effect. On the same diagram from Part b), illustrate the Hicks' method.



SE: A → B, IE: B → C

Hicksian SE: compensate income so that the consumer can afford the original utility.

→ Draw an imaginary budget line that represents the new price and is tangent to the original IC. The decomposition bundle can be found where the new budget line is tangent to the original IC.

Slutskian SE: compensate income so that the consumer can afford the original bundle.

→ draw an imaginary budget line that represents the new price and passes through the original bundle. The decomposition bundle can be found where the new budget line is tangent to some new IC.

Applications

1. **(Labor Supply)** With relevant diagrams, explain why some people may work less when they are richer. Make sure to refer to the substitution and income effects.

   Please check the lecture slide.

2. **(Voucher and Cash Subsidy)** With relevant diagrams, discuss the pros and cons between vouchers and cash subsidies? Is there any case where the consumer will be indifferent between the two?

   Please check the lecture slide.

3. **(Voucher and Cash Subsidy)** Paul consumes only two goods, pizza (P) and hamburgers (H) and considers them to be perfect substitutes, as shown by his utility function: U(P, H) = P + 4H. The price of pizza is $3 and the price of hamburgers is $6, and Paul's monthly income is $300. Knowing that he likes pizza, Paul's grandmother gives him a birthday gift certificate of $60 redeemable only at Pizza Hut. Though Paul is happy to get this gift, his grandmother did not realize that she could have made him exactly as happy by spending far less than she did. How much would she have needed to give him in cash to make him just as well off as with the gift certificate?

   Hints

   i. First, show that Paul will only consume hamburgers (a corner solution).
   ii. With the voucher from his grandma, he is going to buy 60/3 = 20 pizzas.
   iii. Find his utility from consuming only hamburger from the money he has and 20 pizzas from the voucher. Then, find the cash subsidy that would allow him to reach the same utility by consuming only hamburgers.

   $\frac{MU_P}{P_x}$ = ⅓ , $\frac{MU_H}{P_H}$ = 4/6 = ⅔ → MUh/Ph is always greater than MUp/Pp

   → This consumer consumes only hamburger.

   With no voucher nor subsidy → Pizza = 0 unit, Hamburger = 300/6 = 50 units.

   With voucher → Pizza = 20, Hamburger = 50, Utility = 20 + 4*50 = 220.

   To reach U = 220, He needs to eat Hamburgers = 220/4 = 55 units.

   To get 5 more hamburgers, he needs the cash subsidy = 5*6 = $30.

4. **(Revealed Preference)** A "rational" consumer with an income of $24 always buys two goods, X and Y. In the first period, Px = $4 and Py = $2, and the consumer spends $20 on Y and $4 on X. In the second period, Px' = $3 and Py' = $3, and the consumer spends $18 on Y and $6 on X.

   a. What is meant by "rational"?
   b. Using the revealed preference principle and a diagram, explain whether the consumer is better off or worse off in the second period. Make sure to include the inequality.
   c. Suppose there is another bundle (called Bundle C) in the second period with 6 units of X and 2 units of Y. On the previous diagram, draw three indifference curves passing through the three bundles to show the preference. Explain how the consumer would rank Bundle C.

Rational means “maximizing one’s objective (which is utility for consumers)”.

First period: he buys 1X and 10Y (Bundle A). Second period: he buys 2X and 6Y (Bundle B).
The consumer is worse off in the second period.
In the first period, he chooses A when B is affordable since $P_xX_a + P_yY_a \geq P_xX_b + P_yY_b$ or $24 \geq 20$.
Thus, he must prefer A to B and $U(A) > U(B)$.
In the second period, he chooses B but A is NOT affordable, so we cannot use the revealed preference principle to infer anything.

Since in the second period, the consumer buys B when C is affordable, the consumer must prefer B to C and $U(B) > U(C)$. The IC passing through A will be the highest. The IC passing through C will be the lowest. The IC passing through B will be in-between.