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EE320 Quiz

Question 1.

Question 1: Consider a firm with a production function $Q = \sqrt{K} + \sqrt{L}$ Assume that input price of L is \$1 and input price of K is \$2. Consider the following problem.

1.1 Does the production function exhibit constant return to scale? Explain.

1.2 In the short-run, K is fixed at 5 units, derive the short-run cost function.

1.3 Derive the long-run cost function. (Hint: you don't need to check the second-order derivative test.)

1.1 GUIDELINE ANSWER. (SKETCH)

- A function is homogenous degree k, if $F(tx_1, tx_2, .., tx_n) = t^k F(x_1, x_2, ..., x_n)$, where $t > 1$.
- A production function exhibits a constant returns to scale if k is equal to one, an increasing returns to scale if k is greater than one and a decreasing returns to scale if k is less than one.
- Show that $Q(tK, tL) = t^{0.5} Q(K, L)$. [A candidate need to show the work in the details.]
- $k = 0.5$, which is less than one. Therefore, the production function does not exhibit constant returns to scale.
- This production function exhibits decreasing returns to scale.

1.2 GUIDELINE ANSWER. (SKETCH)

- Define short run cost function.
- In the short run, capital (K) is fixed at 5.
- Short run cost function is $TC = wL + rK = (1)L + \bar{2}(5) = L + 10$.
- $Q = \sqrt{K} + \sqrt{L}$. Rearrage to find $L(Q)$.$\sqrt{L} = Q - \sqrt{5}$. $L = (Q - \sqrt{5})^2$.
- Substitute $L = (Q - \sqrt{5})^2$ into the short run cost function.
- $TC(Q) = (Q - \sqrt{5})^2 + 10 = Q^2 - 2\sqrt{5}Q + 15$. **ANS.**

1.3 GUIDELINE ANSWER. (SKETCH)

- Define long run cost function.
- In the long run, all inputs are variable.
- Long run cost function is $TC = wL + rK = L + 2K$.
- $Q = \sqrt{K} + \sqrt{L}$.
- $L = (Q - \sqrt{K})^2$.
- $TC = wL + rK = L + 2K$.
- Profit maximizing condition. $MRTS = \frac{MPL}{MPK} = \frac{\partial Q/\partial L}{\partial Q/\partial K} = \frac{w}{r} = \frac{1}{2}$.
- $MPL = \frac{\partial Q}{\partial L} = 0.5L^{-0.5}$. $MPK = \frac{\partial Q}{\partial L} = 0.5K^{-0.5}$

- $MRTS = \frac{MPL}{MPK} = \frac{\partial Q/\partial L}{\partial Q/\partial K} = \frac{0.5L^{-0.5}}{0.5K^{-0.5}} = \frac{K^{0.5}}{L^{0.5}} = \frac{w}{r} = \frac{1}{2}$. $\Rightarrow K = 0.25L$.
- $Q = \sqrt{0.25L} + \sqrt{L} = 0.5\sqrt{L} + \sqrt{L} = 1.5\sqrt{L}$. $\Rightarrow L = \frac{Q^2}{(1.5)^2}$
- $TC = wL + rK = L + 2K = 1L + 2(0.25)L = 1.5L$.
- $TC = 1.5 \times \frac{Q^2}{(1.5)^2} = \frac{Q^2}{1.5}$. **ANS**

Question 2:

Joe's utility function depends on consumption and leisure, and it is given by: $8\sqrt{C} + \ell$, where c is consumption ℓ is leisure. Each day Joe works n hours and spends the rest of the time on leisure; that is, ℓ+n=24. Suppose further that Joe's total income is the sum of a fixed non-wage income (Y_0) and a wage income (Y_n). The wage income (Y_n) is equal to wn, where w is the hourly wage rate. If Joe spends all of his income on the consumption, his budget constraint becomes: $p_c c = Y_0 + wn = Y_0 + w(24 - \ell)$, which can also be written as: $p_c c + w\ell = Y_0 + 24w$.

2.1. Use Lagrange method to derive the optimal values of $c*$ and $\ell*$ in terms of the parameters p_c,w and Y_0. State the condition under which the consumer demands some leisure (i.e., $\ell^* > 0$).

2.2. Based on your answer in (2.1), derive the impact of an increase in the hourly wage (w) on the demand for leisure ($\ell*$). Interpret its economic meaning.

2.3. Suppose now that the hourly wage rate (w) is \$24, the price of consumption (p_c) is \$12, and the fixed non-wage income (Y_0) is 384. Use Lagrange method to determine the levels of $c*$ and $\ell*$ that maximizes Joe's utility subject to the budget constraint, and determine the level of constrained maximum utility.

2.4. Verify your answers in (2.3) by using the second-order sufficient condition.

2.1 GUIDELINE ANSWER. (SKETCH)

- Define utility maximization by using Lagrange method.[show details]
- Solve the optimization problem. [show details]
- $c^* = \left(\frac{4w}{p_c}\right)^2$, $\ell^* = \frac{Y_0}{w} + 24 - \frac{16w}{p_c}$ and $\lambda^* = \frac{1}{w}$.
- Note that λ is shadow price of Y_0, which is the non-wage income.
- $\ell^* = \frac{Y_0}{w} + 24 - \frac{16w}{p_c} > 0. \Rightarrow Y_0 > \frac{16w^2}{p_c} - 24.$
- Economic intrepretation : leisure will be greater than zero if real wage ($\frac{w}{p}$) is high enough for a given value of non-wage income.
- [no need to provide the second order condition]

2.2 GUIDELINE ANSWER. (SKETCH)

- Impact of an increase in the hourly wage (w) on the demand for leisure ($\ell*$): $\frac{\partial \ell}{\partial w} = -\frac{Y_0}{w^2} - \frac{16}{p_c}$, which is negative. Economic intrepretation, as wage increases, opportunity cost of leisure increases and hence demand for leisure decreases. Wage is can be thought of as opportunity cost (price) of leisure.

2.3 GUIDELINE ANSWER. (SKETCH)

- Substitute hourly wage rate (w) is \$24, the price of consumption (p_c) is \$12, and the fixed non-wage income (Y_0) is 384 into the optimization function in 2.1. [Show details]
- Solve the optimization problem. [Show details]
- Find the solution. $c* = 64, \ell^* = 8, \lambda* = \frac{1}{24}$ and $u^* = 72$.

2.4 GUIDELINE ANSWER. (SKETCH)

- Find the second order condtion.
- Write down the border hessian matrix.
- Write down the condition for maximization.
- Calculate the determinant of border hessian matrix.
- Provide conclusion.