NOTE

Penguin.

1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

to find $Y^* \rightarrow Y_d^* \rightarrow C^* = a + bY_d^*$
$Y^* - T$

TAKE NOTE
HW

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^*.

$Y_d^* = Y - T$

$C^* = a + bY_d$

$Y = C + I + G$

$Y = a + b(Y - T_0) + I_0 + G_0$

$= a + bY - bT_0 + I_0 + G_0$

$Y - bY = a - bT_0 + I_0 + G_0$

$Y^* = \frac{1}{1-b}(a - bT_0 + I_0 + G_0)$

- Numerically, if a = 1, T_0 = \$0 I_0 = \$1, G_0 = \$1 and b = 0.5, this yields us,

$Y^* = \frac{1}{1-b}(a - bT_0 + I_0 + G_0)$

$= \frac{1}{1-0.5}(1 - 0.5(0) + 1 + 1)$

$= 2(3) = 6$ #

Question: What if G is now changed to \$2, how big is the change in Y* and C*?

Answer:

New Y* = 8 ➔

A \$1 increase in G causes an increase in Y* by 2

$$Y^* = \frac{1}{1-b}(a - bT_0 + I_0 + G_0)$$

$$= \frac{1}{1-0.5}(1 - 0.5(0) + 1 + 2)$$

$$= 2(4) = 8 \#$$

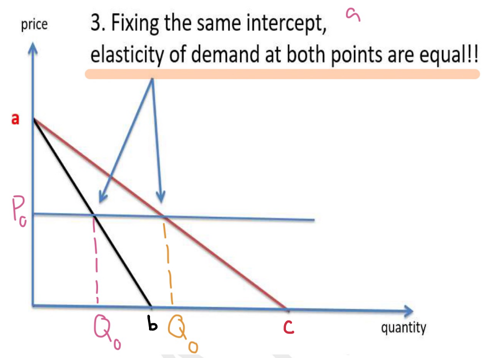
In the later part, we will try to figure out the value of *multiplier* when we don't assign any numerical values of exogenous variables.

- The idea is simple. *We just apply the derivative method to the equilibrium solution function.*
- That is, we calculate the value of $\frac{\partial Y^*}{\partial I_0}, \frac{\partial Y^*}{\partial G_0}, \frac{\partial Y^*}{\partial a}, \frac{\partial Y^*}{\partial b}$.

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that
(i) elasticity of supply is always greater than 1 if $c > 0$,
(ii) elasticity of supply is always eqaul to 1 if $c = 0$,
(iii) elasticity of supply is always less to 1 if $c < 0$.



2.A.1) first rearrange to be demand equation.

$$P = a - bQ \qquad Q = \frac{a - P_0}{b}$$

$$P = a - cQ \qquad Q = \frac{a - P_0}{c}$$

to find PED of black line at P_0

$$PED = \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$$

$$= \frac{1}{b} \frac{P_0}{Q_0}$$

$$\therefore PED = \frac{1}{\cancel{b}} \cdot \frac{\cancel{b} P_0}{a - P_0} = \boxed{\frac{P_0}{a - P_0}}$$

For Red line,

$$PED = \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$$

$$= \frac{1}{c} \cdot \frac{P_0}{Q_0}$$

$$= \frac{1}{c} \cdot \frac{c P_0}{a - P_0} = \boxed{\frac{P_0}{a - P_0}}$$

equal!!

2.A.2) $P = c + dQ \quad , d \geq 0$

$$Q_0 = \frac{P_0 - c}{d}$$

i) $c > 0$

$$\frac{\% \Delta Q_s}{\% \Delta P} = \frac{\Delta Q_s / Q_0}{\Delta P / P_0} = \frac{\Delta Q_s}{\Delta P} \cdot \frac{P_0}{Q_0}$$

$$= \frac{1}{d} \times \frac{P_0}{Q_0} = \frac{1}{d} \times \frac{P_0}{\frac{P_0 - c}{d}}$$

$$= \frac{1}{\cancel{d}} \times \frac{\cancel{d} P_0}{P_0 - c}$$

$$= \frac{P_0}{P_0 - c} = \frac{c + dQ}{\cancel{c} + dQ - \cancel{c}} > 1.$$

ii) $c = 0$

$$\frac{\% \Delta Q_s}{\% \Delta P} = \frac{P_0}{P - \cancel{c}} = 1$$

1

iii) $c < 0$

$$\frac{\% \Delta Q_s}{\% \Delta P} = \frac{P_0}{P - C} = \frac{c + dQ}{\cancel{c} + dQ - \cancel{c}} < 1.$$

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$. total cost fn

- What is the revenue-maximizing level of output?

Revenue function: $TR(Q) = P(Q) \cdot Q = (10 - Q)Q$

$$= 10Q - Q^2$$

$$\frac{dTR}{dQ} = 10 - 2Q = 0 \quad \rightarrow \text{maximum occur when } \frac{dTR}{dQ} = 0$$

$$10 = 2Q$$

$$Q = 5 \quad \rightarrow \text{At } Q = 5, TR \text{ is max; } TR = 25.$$

- What is the break-even output?

$$TR = TC$$

$$TC = C(Q) = 4Q$$

$$10Q - Q^2 = 4Q$$

$$6Q = Q^2$$

$$\therefore Q = 6$$

- What is the profit-maximizing level of output?

① $MC = MR$

$$\frac{dTR}{dQ} = \frac{dTR}{dQ}$$

$$4 = 10 - 2Q$$

$$Q = 3$$

\#

② $\pi = TR - TC$

$$\pi = 10Q - Q^2 - 4Q$$

$$= 6Q - Q^2$$

$$\frac{\partial \pi}{\partial Q} = 6 - 2Q = 0$$

$$= 3 \quad \#$$

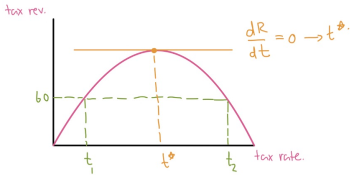
Laffer Curve.

Exrcise 2B. Consider a function that relates tax revenues R, in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion? $t_1 \& t_2$

(b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion? same process.

(c) What tax rate is consistent with the maximum tax revenue? t^*



a) $60 = 350t - 500t^2$

$0 = -500t^2 + 350 - 60$

$0 = 10(-50t^2 + 35t - 6)$

$0 = 10(5t-2)(-10t+3)$

$5t - 2 = 0$ and $-10t + 3 = 0$

$t = \frac{2}{5} = 40\%$ $\quad t = \frac{3}{10} = 30\%$

$\therefore$ 30% and 40% tax rates are consistent w/ 60 billion tax revenue.

b) $61.25 = 350t - 500t^2$

$0 = -500t^2 + 350 - 61.25$

$0 = \frac{-5}{4}(20t-7)^2$

$(20t - 7^2) = 0$

$20t - 7 = 0$

$t = \frac{7}{20} = 35\%$

$\therefore$ 35% tax rate is consistent w/ 61.25 billion tax revenue

c) $\frac{dR}{dt} = 350 - 1000t = 0$ → max revenue.

$350 = 10000t$

$t = 0.35 = 35\%$

$\therefore$ 35% tax rate is consistent w/ the maximum tax revenue.