FN 312 Investment Study Guide

This study guide is designed to help you review the material but is by no means exhaustive. Please review all notes and material on your own for complete coverage of all topics.

Chapter 1&2: Introduction

- **Real versus Financial Assets**

1. The material wealth of a society is determined ultimately by the productive capacity of its economy, which is a function of the **real assets** of the economy: the land, buildings, knowledge, and machines that are used to produce goods and the workers whose skills are necessary to use those resources.
2. **Financial assets,** like stocks or bonds, contribute to the productive capacity of the economy indirectly, because they allow for separation of the ownership and management of the firm and facilitate the transfer of funds to enterprise with attractive investment opportunities. Financial assets are claims to the income generated by real assets.
3. **Real vs. Financial assets**:

a. Real assets produce goods and services, whereas financial assets define the allocation of income or wealth among investors.

b. They are distinguished operationally by the balance sheets of individuals and firms in the economy. Whereas **real assets appear only on the asset side** of the balance sheet, **financial assets always appear on both sides of the balance sheet.** Your financial claim on a firm is an asset, but the firm's issuance of that claim is the firm's liability. When we aggregate over all balance sheets, financial assets will cancel out, leaving only the sum of real assets as the net wealth of the aggregate economy.

c. Financial assets are created and destroyed in the ordinary course of doing business. E.g. when a loan is paid off, both the creditor's claim and the debtor's obligation cease to exist. In contrast, real assets are destroyed only by accident or by wearing out over time.

- **Financial Assets**

Financial markets are segmented into **money markets and capital markets**.

1. **Money market instruments** (they are called cash equivalents, or just cash for short) include short-term, marketable, liquid, low-risk debt securities.
2. **Capital markets** include longer-term and riskier securities. We subdivide the capital market into **four segments**: **longer-term bond markets, equity markets, and the derivative markets for options and futures.**

Money Market instruments

1. **T-bills**: Investors buy the bills at a discount from the stated maturity value and get the face value at the bill's maturity. T-bills with initial maturities of 91 days or 182 days are issued weekly. Offerings of 52-week bills are made monthly. Sales of bills are conducted via auction, at which investors can submit competitive or noncompetitive bids. T-bills sell in minimum denominations of only $10,000. The income earned on T-bills is **tax-free**.
2. **CD**: certicficates of deposit is a time deposit with a bank. CDs issued in denominations greater than $100,000 are usually negotiable. Short-term CDs are highly marketable.
3. **CP:** commerical paper, large companies often issue their own short-term **unsecured** debt notes rather than borrow directly from banks. Very often, CP is backed by a bank line of credit, which gives the borrower access to cash that can be used to pay off the paper at maturity. CP maturities range up to 270 days. Usually, it is issued in multiples of $100,000. So, small investors can invest in CP only indirectly, via money market mutual funds.
4. **Bankers' acceptances**: starts as an order to a bank by a bank's customer to pay a sum of money at a future date, typically within six months. At this stage, it is similar to a postdated check. When the bank endorses the order for payment as "accepted", it assumes responsibility for ultimate payment to the holder of the acceptance. Bas are considered very safe assets because **traders can substitute the bank's credit standing for their own**.
5. **Eurodollars**: are dollar-denomined deposits at foreign banks or foreign branches of American banks. These banks **escape regulation by FED**.
6. **Repos**: Dealers in government securities use Repos as a form of short-term, usually overnight, borrowing. The dealer thus takes out a one-day loan from the investor, and the securities serve as collateral.
7. **Federal funds**: Banks maintain deposits of their own at FED. Funds in the bank's reserve account are called federal funds
8. **LIBOR: London Interbank Offered Rate** is the rate at which large banks in London are willing to lend money among themselves. This rate, which is quoted on dollar-denominated loans, has become the **premier short-term rate** quoted in the European money market, and it serves as a reference rate for a wide range of transactions.

Capital Markets

Bond market (fixed income capital market): it is composed of **longer-term** borrowing instruments than those that trade in the money market, including **Treasury notes and bonds**, corporate bonds, municipal bonds, mortage securities, and federal agency debt.

1. **Treasury notes and bonds:** T-note maturities range up to 10 years, whereas bonds are issued with maturities ranging from 10 to 30 years. Both are issued in denominations of $1000 or more. Both make semiannual interest payments called coupon payments. The only major distinction between T-notes and T-bonds is that **T-bonds may be callable during a given period**, usually the last five years of the bond's life.
2. **International bonds:**
    1. **Eurobond is** a bond denominated in a currency other than that of the country in which it is issued. Eg, a dollar-denominated bond sold in Britain would be called a Eurodollar bond.
    2. **Foreign bonds**: bonds issued and denominated in the currency of a country other than the one in which the issuer is primarily located. A Yankee bond is a dollar-denominated bond sold in the US by a non-US issurer. Samurai bonds are yen denomiated bonds sold within Japan.
3. **Municipal bonds** are issued by state and local governments. They are similar to Treasury and corporate bonds except that their interest income is exempt from federal income taxation.
    1. two types of municipal bonds: general obligation bonds, which are backed by the "full faith and credit" of the issuer, and revenue bonds, which are issued to finance particular projects and are backed either by the revenues from that project or by the particular municipal agency operating the project..
    2. the key feature of municipal bonds is "**Tax-exempt status**". Because investors pay neither federal nor state taxes on the interest proceeds, they are willing to accept lower yields on these securities. These lower yields represent a huge savings to state and local governments.
    3. Equivalent taxable yield: the rate that a taxable bond must offer to match the after-tax yield on the tax-free municipal.

    $R(1\text{-}t) = r_m$ → municipal bond yield.

    $R = r_m/(1 - t)$
4. **Corporate bonds**: they often come with options attached.
    1. Callable bonds give the firm the option to repurchase the bond from the holder at a stipulated call price.
    2. Convertible bonds give the bondholder the option to convert each bond into a stipulated number of shares of stock.

5. **Mortgages and Mortgage-backed securities** are either an ownership claim in a pool of mortgages or an obligation that is secured by such a pool. Mortgage lenders originate loans and then sell packages of these loans in the secondary market. Speicifically, they sell their claim to the cash inflows from the mortgages as those loans are paid off. The mortgage originator continues to serve the loan, collecting principal and interest payments, and passes these payments along to the purchaser of the mortgage. These securities are called **pass-throughs**. Although pass-through securities often guarantee payment of interest and principal, they do not guarantee the rate of return. Intestors can be hurt in years when interest rates drop significantly. This is because homeowners usually have an option to prepay, or pay ahead of schedule, the remaining principal outstanding on their mortgages.

Equity securities:

1. **Common stock:** also known as equity securities or equities, represent ownership shares in a corporation. Each share of common stock entitles its owner to one vote on any matters of corporate governance that are put to a vote at the corporation's annual meeting and to a share in the financial benefits of ownership.
2. **Characteristics of Common stock:**
    1. **Residual claim:** stockholders are the last in line of all those who have a claim on the assets and income of the corporation. In a liquidation of the firm's assets the shareholders have a claim to what is left after all other claimants such as the tax authorities, employees, suppliers, bondholders, and other creditors have been paid. For a firm not in liquidation, shareholders have claim to the part of operating income left over after interest and taxes have been paid. Management can either pay this residual as cash dividends to shareholders or reinvest it in the business to increase the value of the shares.
    2. **Limited liability:** the most shareholders can lose in the event of failure of the corporation is their original investment.
3. **Preferred stock:** has features similar to both equity and debt.

    a. Like a bond, it promises to pay to its holder a fixed amount of income each year and it does not convey voting power regarding the management of the firm.

    b. It is an equity investment, however. The firm retains discretion to make the dividend payments to the preferred stockholders; it has no contractual obligation to make those dividends. Instead, preferred dividends are usually cumulative; that is, unpaid dividends cumulative and must be paid in full before any dividends may be paid to holders of common stock. In contrast, the firm does have a **contractual obligation** to make the interest payments on the debt. Failure to make these payments sets off corporate bankruptcy proceedings.

c. Unlike interest payment of bonds, dividends on preferred stock are not considered tax-deductible expenses to the firm. This reduces their attractiveness as a source of capital to issuing firms. However, there is an offsetting tax advantage to preferred stock. **When one corporation buys the preferred stock of another corporation, it pays taxes on only 30% of the dividends received.** Given this tax rule, it is not surprising that most preferred stock is held by corporations.

d. Preferred stock rarely gives its holders full voting privileges in the firm. However, if the preferred dividend is skipped, the preferred stockholders will then be provided some voting power.

Stock and bond indexes

1. Dow Jones Averages: **Price-weighted average**, measures the return (excluding dividends) on a portfolio that holds one share of each stock. It gives higher-priced shares more weight in determining performance of the index. Divisor, d now is .146, in stead of 30, because of splits or dividents. The averaging procedure is adjusted whenever a stock splits or pays a stock dividend of more than 10%, or there is a company changes.
2. S&P 500: **market-value-weighted index**. The split irrelevnat to the performance of the index.
3. **Equally weighted indexes**: an averaging technique, by placing equal weight on each return, corresponds to an implicit portfolio strategy that **places equal dollar values on each stock**. This is in contrast to both price weighting (which requires equal numbers of shares of each stock) and market value weighting (which requires investments in proportion to outstanding value). Unlike price- or market-value-weighted indexes, equally weighted indexes do not correspond to buy-and-hold portfolio strategies.

Derivative markets:

1. Derivatives are instruments that provide payoffs that depend on the values of other assets such as commodity prices, bond and stock prices, or market index values.
2. **Options**: a **call (put) option** gives its holder the right to purchase (sell) an asset for a specified price, called the **exercise or strike price**, on or before a specified expiration date. The prices of call (put) options decrease (increase) as the exercise price increases. The purchase price of option is called the premium.
3. **Futures contracts**: call for delivery of an asset at a specified delivery or maturity date for an agreed-upon price, called the futures price, to be paid at contract maturity. The long position is held by the trader who commits to purchasing the asset on the delivery date. The trader who takes the short position commits to delivering the asset at contract maturity.

Chapter 5 & 6: Risk and Return, Risk Aversion and Capital Allocation to Risky Assets

I. **The investment process consists of two broad tasks**. One task is **security and market analysis**, by which we assess the risk and expected-return attributes of the entire set of possible investment vehicles. The second task is the **formation of an optimal portfolio of assets.** This task involves the determination of the best risk-return opportunities available from feasible investment portfolios and the choice of the best portfolio from the feasible set. The formal analysis of investments with the latter task is called portfolio theory. This chapter introduces **three themes in portfolio theory, all centering on risk**.

a. **The first** is the basic tenet that **investors avoid risk and demand a reward for engaging in risky investments.** The reward is taken as a risk premium, the difference between the expected rate of return and that available on alternative risk-free investments.

b. **The second theme** allows us to **quantify investors' personal trade-offs between portfolio risk and expected return**. To do this we introduce the utility function, which assumes that investors can assign a welfare or "utility" score to any investment portfolio depending on its risk and return.

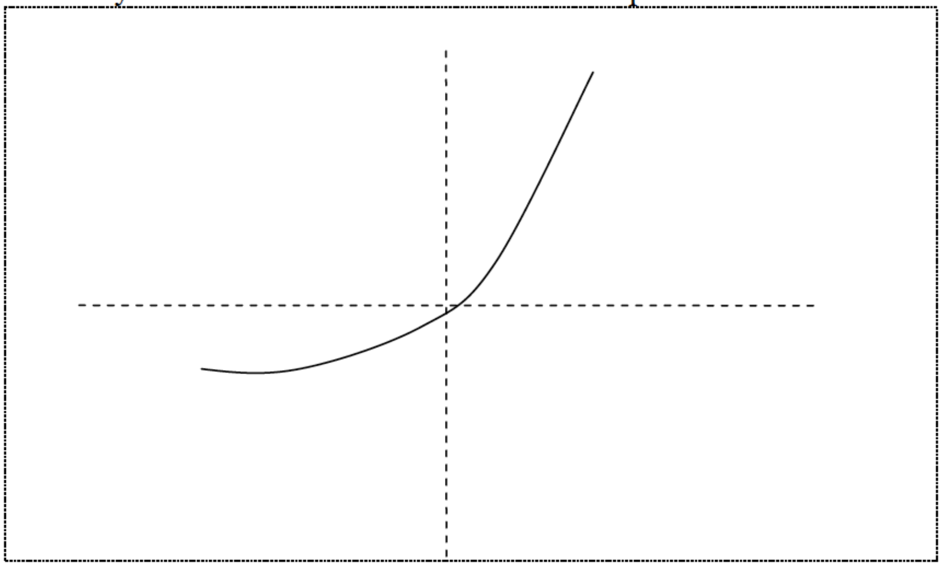
c. **The third theme** is that **we cannot evaluate the risk of an asset separate from the portfolio of which it is a part**; that is, the proper way to measure the risk of an individual asset is to assess its impact on the volatility of the entire portfolio of investments. Taking this approach, we find that seemingly risky securities may be portfolio stabilizers and actually low-risk assets.

II. Risk and risk aversion

1. **Risk:** The chance that an investment's actual return will be different than expected. This includes the possibility of losing some or all of the original investment. It is usually measured using the historical returns or average returns for a specific investment. Higher risk means a greater opportunity for high returns... and a higher potential for loss.
2. **Risk Premium**: The extra return that a risky investment provides over the risk free rate to compensate for the risk of the investment. **A higher rate of return is required to entice investors into a riskier investment**.
3. **Risk Averse:** Describes an investor who, when faced with two investments with a similar expected return (but different risks), will prefer the one with the lower risk. **A risk averse person dislikes risk.**
4. **Utility:** Assume each investor can assign a welfare, or utility, score to competing investment portfolios based on the expected return and risk of those portfolios. The utility score may be viewed as a means of ranking portfolios. Portfolios receive higher utility scores for higher expected returns and lower scores for higher volatility. Many particular "scoring" systems or utility functions are legitimate. We choose the one that assigns a portfolio with expected return E(r) and variance of returns σ^2 the following utility score:

$U = E(r) - 0.005A\sigma^2$. where U is the utility value and A is an index of the investor's risk aversion.

8. **<u>Indifference curve</u>**: the curve that connects all portfolio points with the same utility value in the mean-standard deviation plan.



III. Key statistical concepts

1. **<u>Expected return:</u>**
   a. Definition: a probability-weighted average of an asset's return in all states (scenarios).
   b. Formula: $E(R) = \sum_{i=1}^{n} P_i R_i$,

   Where
   i: states of nature
   Pi: probability of that the event will happen in state i.
   Ri: the return of the asset if the event happens in state i.

   c. Example: suppose an asset is priced at $100. There are three possible outcomes in one year: good, soso and bad.

Three possible outcomes (i):		**<u>Good</u>**	**<u>Soso</u>**	**<u>Bad</u>**
Probability (Pi)	**25%**	**50%**	**25%**	
Prices	**$ 120**	**$ 110**	**90**	
Return (Ri)	**20%**	**10%**	**-10%**	

How do you calculate the average of a probability distribution? Simply take the probability of each possible return outcome and multiply it by the return outcome itself.
E(R) = (0.5) (0.1) + (0.25) (0.2) + (0.25) (-0.1) = 0.075= 7.5%
Although this is what you expect the return to be, there is no guarantee that it will be the actual return.

   d. the **rate of return on a portfolio** is a weighted average of the return of each asset comprising the portfolio, with portfolio proportions as weights. This implies that the **expected return on a portfolio** is a weighted average of the expected return on each component asset.

   For example, a portfolio with value of $200, which consists of $100 stock A and $100 stock B. E(rA) = 20%, E(rB) = 10%,
   E(expected return of portfolio) = (100/200)*20% + (100/200)*10% = 15%

2. **<u>Variance (the second central moment)</u>**:
   a. Definition: A measure of the dispersion of a set of data points around their mean value. It is the expected value of the squared deviations from the expected return.
   b. Formula: $\sigma^2 = Var(R) = \sum_{i=1}^{n} P_i[R_i - E(R)]^2$ Variance measures the variability (volatility) from an average. **Volatility is a measure of risk, so this statistic can help determine the risk an investor might take on when purchasing a specific security**.
   c. Example: Var(R) = $0.25(0.2-0.075)^2 + 0.5(0.1-0.075)^2 + 0.25(-0.1-0.075)^2$ = 1.19%
3. **Standard deviation:**
   a. **Definition: The square root of the variance**. A measure of the dispersion of a set of data from its mean. The more spread apart the data is, the higher the deviation. In finance, standard deviation is applied to the annual rate of return of an investment to measure the investment's volatility (risk).
   b. Formula: $\sigma = \sqrt{Var(R)}$
   c. Example: Standard deviation = $\sqrt{Var(R)} = \sqrt{1.19\%}$ =10.90%
4. **Covariance:**
   a. **Definition: A measure of the degree to which returns on two risky assets move in tandem.** A positive covariance means that asset returns move together. A negative covariance means returns vary inversely.
   b. Formula: $Cov(R_A, R_B) = \sum_{i=1}^{n} P_i[R_{iA} - E(R_A)][R_{iB} - E(R_B)]$, where

   A: Asset A.
   B: Asset B.

If you owned one asset that had a high covariance with another asset that you didn't own, then you would receive very little increased diversification by adding the second asset. Of course, the opposite is true as well, adding assets with low covariance to your portfolio lowers overall portfolio risk.

c. example: suppose two risky assets A and B:

Three possible outcomes (i):	**Good**	**Soso**	**Bad**
Probability (P_i)	**25%**	**50%**	**25%**
Return of Asset A (R_{iA})	**20%**	**10%**	**-10%**
Return of Asset B (R_{iB})	**30%**	**0%**	**-30%**

$E(R_A) = 7.5\%$

$E(R_B) = 0\%$

$Cov(R_A, R_B) = 0.25(0.2-0.075)(0.3-0) + 0.5(0.1-0.075)(0-0) + 0.25(-0.1-0.075)(-0.3-0) = 2.25\%$

5. **Portfolio's variance:** when two risky assets with variance var1 and var2, respectively, are combined into a portfolio with portfolio weights w1 and w2, respectively, the portfolio variance σ_p^2 is given by $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_1 \operatorname{cov}(r_1, r_1)$

A positive covariance increases portfolio variance, and a negative covariance acts to reduce portfolio variance.

If var1 = var2 = - covariance and w1 and w2, then portfolio's variance is zero (prove it!)

6. **Correlation coefficient: a statistical measure of how two securities move in relation to each other. It ranges between -1 and +1.**
   a. Formula: $\operatorname{corr}(R_A, R_B) = \dfrac{\operatorname{Cov}(R_A, R_B)}{\sigma_A \sigma_B}$
   b. **Perfect positive correlation** (a correlation co-efficient of +1) implies that as one security moves, either up or down, the other security will move in lockstep, in the same direction. Alternatively, **perfect negative correlation** means that if one security moves in either direction the security that is perfectly negatively correlated will move by an equal amount in the opposite direction. **If the correlation is 0,** the movements of the securities is said to have no correlation, it is completely random. If one security moves up or down there is as good a chance that the other will move either up or down, the way in which they move is totally random.
      In real life however you likely will not find perfectly correlated securities, rather you will find securities with some degree of correlation. For example, the performance of two stocks within the same industry is strongly positively correlated although it may not be exactly +1.

   Correlation V.S.Covariance:

   Basically, Correlation is better than covariance:

   1 -- Because correlation removes the effect of the variance of the variables, it provides a standardized, absolute measure of the strength of the relationship, bounded by -1.0 and 1.0. This is good because it makes it possible to compare any correlation to any other correlation and see which is stronger. You cannot do this with covariance.

   2 -- The squared correlation (r2) is a measure of how much of the variance in one variable is explained by the other variable. This measure, the coefficient of determination, ranges from 0.0 to 1.0. You cannot do this with covariance.

7. Skewness (the third central moment): A statistical term used to describe a situation's asymmetry in relation to a normal distribution. A positive skew describes a distribution favoring the right tail, whereas a negative skew describes a distribution favoring the left tail.

Formula: $M_3 = \sum_{i=1}^{n} P_i[R_i - E(R)]^3$

8. Kurtosis: A statistical measure used to describe the distribution of observed data around the mean. Used generally in the statistical field, it describes trends in charts. A high kurtosis portrays a chart with fat tails and a low even distribution, whereas a low kurtosis portrays a chart with skinny tails and a distribution concentrated towards the mean. It is sometimes referred to as the "volatility of volatility."

Chapter 7: Optimal Risky Portfolios

I. Most institutional investors follow a top-down approach. Capital allocation and asset allocation decisions will be made at a high organizational level, with the choice of the specific securities to hold within each asset class delegated to particular portfolio managers.

1. **Capital allocation decision** is the choice of the proportion of the overall portfolio to place in safe but low-return money market securities versus risky but higher return securities like stocks.
2. **Asset allocation decision** describes the distribution of risky investments across broad asset classes – stocks, bonds, real estate, foreign assets, and so on.
3. **Security selection decision** describes the choice of which particular securities to hold within each asset class.

II. The risk-free asset

1. **Treasury bills**: Their short-term nature makes their values insensitive to interest rate fluctuation. Moreover, inflation uncertainty over the course of a few weeks, or even months, is negligible compared with the uncertainty of stock market returns.
2. In practice, most investors use a broader range of money market instruments as a risk-free asset. All the money market instruments are virtually free of interest rate risk because of their short maturities and are fairly safe in terms of default or credit risk, e.g. bank certificates of deposits (CDs), and commercial paper (CP)

III. Portfolio of one risky asset and one risk-free asset

1. Assumptions:
   a. Suppose a risky portfolio, P, consists of different risky assets, which are hold in fixed proportions. In this case, we can actually treat P as one single risky asset.
   b. We allocate y proportion of our investment budget to P. The remaining proportion, 1 – y, is to be invested in the risk-free asset, F.
   c. r_p, the return of P, its expected return $E(r_p)$, and its standard deviation by σ_p.
   d. R_f, the rate of return on the risk-free asset. Of course, the standard deviation of F is 0 and $r_f = E(r_f)$.

2. return on the complete portfolio, C is r_C where

$r_c = yr_p + (1-y)r_f$

$E(r_c) = yE(r_p) + (1-y)r_f. = r_f + y(E(r_p) - r_f)$ -- (7.1)

The base return for any portfolio s the risk-free rate. In addition, the portfolio is expected to earn a risk premium that depends on the risk premium of the risky portfolio, $E(r_p) - r_f$, and the investor's position in the risky asset, y.

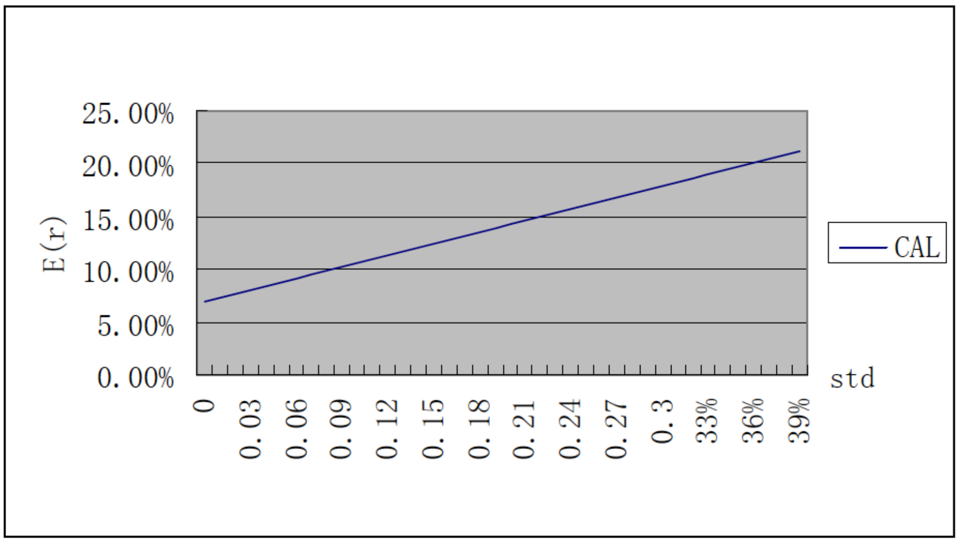
3. Standard deviation of C, $\sigma_c = y\,\sigma_p$ -- (7.2)

4. $E(r_C) = r_f + y[E(r_P) - r_f] = r_f + \frac{\sigma_C}{\sigma_P}[E(r_P) - r_f]$ ------------------------ (7.3)

Equation 7.3 describes the expected return – standard deviation trade-off. The **<u>expected return of the complete portfolio as a function of its standard deviation is a straight line</u>**, with intercept rf and slope as follows:

$$S = \frac{E(r_P) - r_f}{\sigma_P} = \frac{E(r_C) - r_f}{\sigma_C}$$

The straight line in the following figure is called the **<u>capital allocation line (CAL)</u>**. It depicts all the risk-return combinations available to investors. The slope of the CAL, denoted S, equals the increase in the expected return of the complete portfolio per unit of additional standard deviation – in other words, incremental return per incremental risk. Thus, the slope is also called the reward-to-variability ratio.



5. the range of y – y can be greater than 1 (short risk-free asset (leveraged position in the risky asset)).
   a. If investors can borrow at the risk-free rate, the CAL is the same as before.
   b. If investors must have to borrow at a higher rate, the CAL is kinked at y=1. When y<1, investors are lending at r_f; when y>1, the investors are borrowing at $r > r_f$.

IV. **<u>Risk tolerance and asset allocation</u>**

1. The investor confronting the CAL now must choose one optimal portfolio, C, from the set of feasible choices. This choice entails a trade-of between risk and return. **Individual investor differences in risk aversion imply that, given an identical opportunity set (that is, a risk-free rate and a reward-to-variability ratio), different investors will choose different positions in the risky asset. In particular, the more risk-averse investors will choose to hold less of the risky asset and more of the risk-free asset.**
2. The utility that an investor derives from a portfolio can be described by:

$U = E(r) - 0.005A\,\sigma^2$. --- (7.4).

3. By equations 7.1 and 7.2, we maximize the investor's utility:

$$\underset{y}{Max}\,U = E(r_C) - .005A\sigma_C^2 = r_f + y[E(r_P) - r_f] - 0.005Ay^2\sigma_P^2$$

, Solve this maximization problem: we get the optimal position for risk-averse investors in the risky asset, y*, as follows:

$$y^* = \frac{E(r_P) - r_f}{0.01A\sigma_P^2} \quad \text{---} (7.5)$$

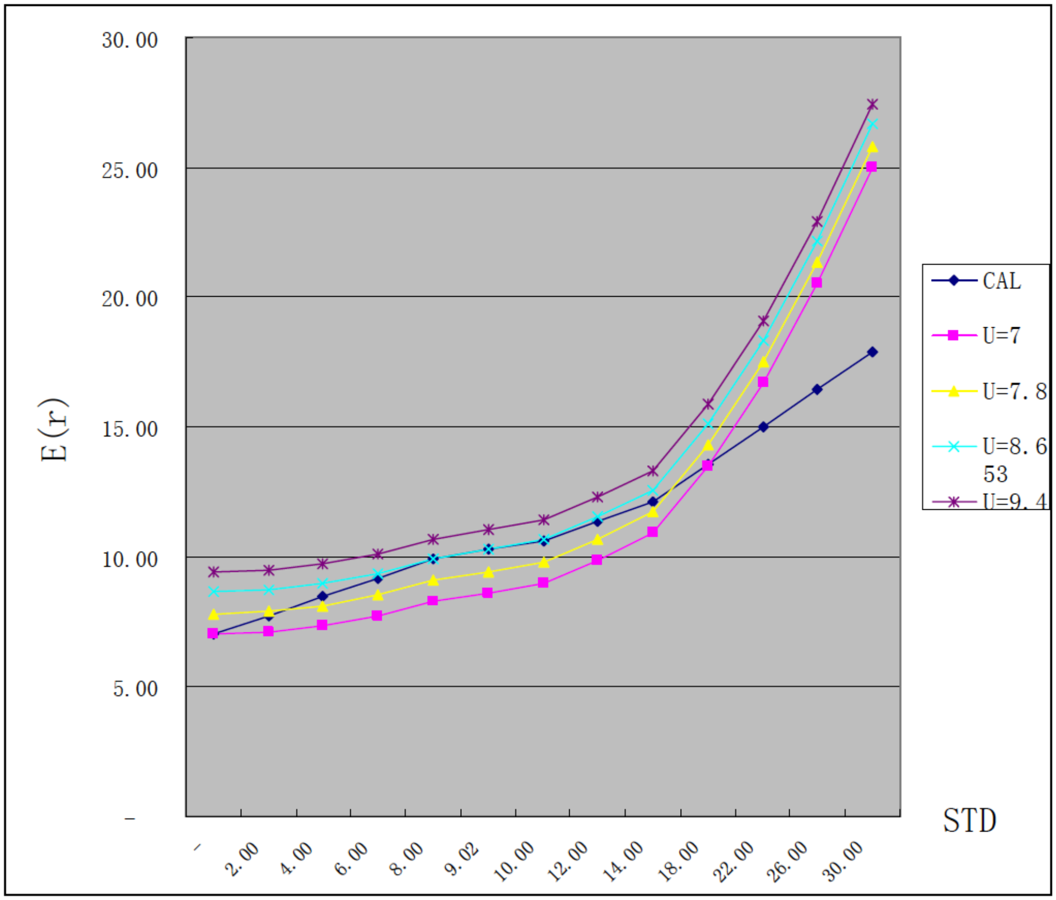
This solution shows that the **optimal position in the risky asset is inversely proportional to the level of risk aversion and the level of risk (as measured by the variance) and directly proportional to the risk premium offered by the risky asset**.

4. A graphical way of presenting this decision problem is to use **<u>indifference curve analysis</u>**.

a. an indifference curve is a graph in the expected return-standard deviation plane of all points that result in a given level of utility. The curve displays the investor's required trade-off between expected return and standard deviation.

b. The more risk-averse investor has steeper indifference curves than the less risk-averse investor. Steeper curves mean that the investor requires a greater increase in expected return to compensate for an increase in portfolio risk.

c. Higher indifference curves correspond to higher levels of utility. The investor thus attempts to find the complete portfolio on the highest possible indifference curve.



V. Passive strategies: the capital Market Line: A passive investment strategy disregards security analysis, targeting instead the risk-free asset and a broad portfolio of risky assets such as the S&P 500 stock portfolio.

Diversification and Portfolio Risk

1. Diversification can reduce portfolio risk.
2. **The risk that remains even after extensive diversification is called market risk**, risk that is attributable to marketwide risk sources. Such risk is also called systematic risk or nondiversifiable risk.
3. A **risk that can be eliminated by diversification is called unique risk**, **firm-specific risk, nonsystematic risk or diversifiable risk.**

II. Portfolios of two risky assets:

Consider a portfolio comprised of two risky assets: D, long-term bond and E, stock. A proportion of w_D is invested in the bond and the remainder, $1 - w_D = w_E$, is invested in the stock. The rate of return on this portfolio, r_p, will be $r_p = w_D r_D + w_E r_E$ and the expected return on the portfolio is:

$E(r_p) = w_D E(r_D) + w_E E(r_E)$ --- (8.1)

The portfolio variance is: $\sigma_p^2 = w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E Cov(r_D, r_E)$ ------ (8.2)

1. Equation 8.2 reveals that **variance is reduced if the covariance term is negative**. However, **even if the covariance term is positive, the portfolio standard deviation still is less than the weighted average of the individual security standard deviations**, unless the two securities are perfectly positively correlated.

Proof:

Since $Cov(r_D, r_E) = \rho_{DE}\sigma_D\sigma_E$, thus $\sigma_p^2 = w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E \rho_{DE}\sigma_D\sigma_E$

because $\rho_{DE} \leqslant 1$ → $\sigma_p^2 \le w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E\sigma_D\sigma_E = (w_D\sigma_D + w_E\sigma_E)^2$, Thus

$\sigma_P \le w_D\sigma_D + w_E\sigma_E$

□

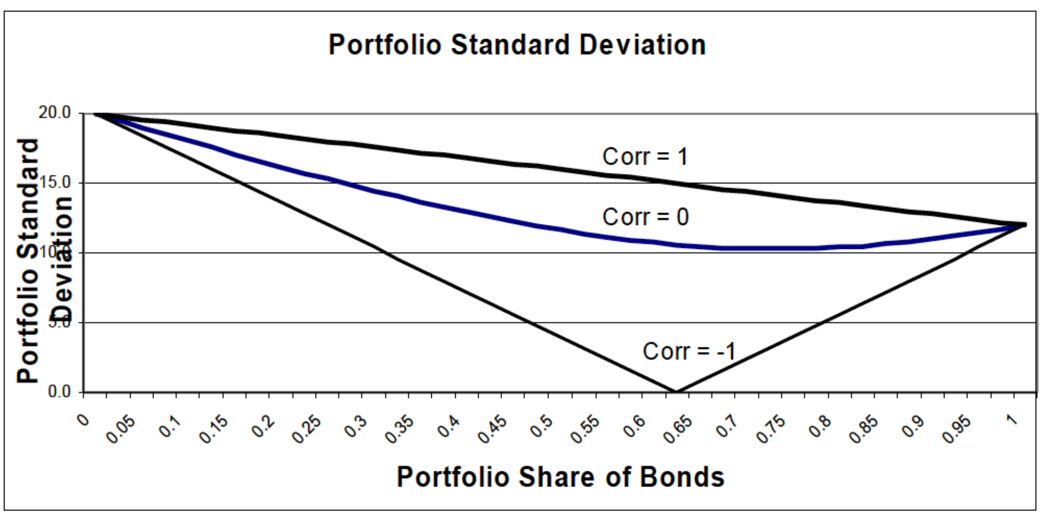
In words, the standard deviation of the portfolio with perfect positive correlation is just the weighted average of the component standard deviations. In all other cases, the correlation coefficient is less than 1, making the portfolio standard deviation less than the weighted average of the component standard deviations.

2. **Because the portfolio's expected return is the weighted average of its component expected returns, whereas its standard deviation is less than the weighted average of the component standard deviations, portfolios of less than perfectly correlated assets always offer better risk-return opportunities than the individual component securities on their own. The lower the correlation between the assets, the greater the gain in efficiency.**

3. The lowest possible standard deviation = 0. Let correlation = -1, the equation 8.5 simplifies to

$\sigma_p^2 = (w_D\sigma_D - w_E\sigma_E)^2$, let $w_D\sigma_D - w_E\sigma_E = 0$ → $w_D = \dfrac{\sigma_E}{\sigma_D + \sigma_E}$, $w_E = \dfrac{\sigma_D}{\sigma_D + \sigma_E} = 1 - w_D$

4. Minimum-variance portfolio: Maximize equation 8.2 relative to w_D (replace w_E with 1- w_D), we can get minimum-variance portfolio. $w_{min}(D) = \dfrac{\sigma_E^2 - cov(r_D, r_E)}{\sigma_D^2 + \sigma_E^2 - 2cov(r_D, r_E)}$



5. **Expected return – standard deviation**: By equation 8.1 expected return, $E(r_p)$ has one-for-one relationship with w_D, substitute $E(r_p)$ for w_D in equation 8.2, we can get the relationship between σ_p^2 and $E(r_p)$
6. **Portfolio opportunity set:** points satisfy the relationship between expected return and standard deviation.

III. Asset Allocation with stocks, bonds and bills

Now consider three assets: two risky assets: bond (D) and stock (E) and one risk-free asset: T-bill (T).

1. **Optimal risky portfolio**: the objective is to find the weights w_D and w_E that result in the highest slope of the CAL (i.e., the weights that result in the risky portfolio with the highest reward-to-variability ratio). Therefore, the objective is to maximize the slope of the CAL for any possible portfolio, P.

   Max $S = \frac{E(r_P) - r_f}{\sigma_p}$, subject to equation 8.1, 8.2 and $1 - w_D = w_E$, → get optimal risky

portfolio.

2. Optimal complete portfolio: use equation 7.5 to get y – the proportion taken in risky portfolio P.
3. Steps we followed to arrive at the complete portfolio:

a. Specify the return characteristics of all securities (expected returns, variances, covariances).

b. Establish the risky portfolio:

(1) Calculate the optimal risk portfolio, P.

(2) calculate the properties of Portfolio P using the weights determined in step (a) and equations 8.1 and 8.2

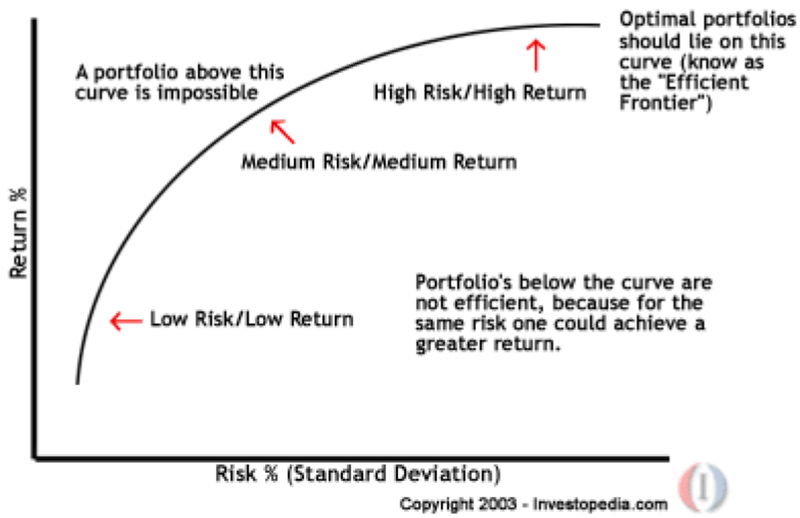
c. allocate funds between the risky portfolio and the risk-free asset:

(1) calculate the fraction of the complete portfolio allocated to Portfolio P (the risky portfolio) and to T-bills (the risk-free asset)

(2) Calculate the share of the complete portfolio invested in each asset and in T-bills.

The Markowitz Portfolio Selection Problem

1. **Minimum-variance frontier** of risky assets: it is a graph of the lowest possible variance that can be attained for a given portfolio expected return.
2. **Efficient frontier:** the part of the frontier that lies above the global minimum-variance portfolio. For any portfolio on the lower portion of the minimum-variance frontier, there is a portfolio with the same standard deviation and a greater expected return positioned directly above it. Hence the bottom part of the minimum-variance frontier is inefficient.



3. **add in the risk-**free asset: the CAL that is supported by the optimal portfolio, P, is tangent to the efficient frontier. This CAL dominates all alternative feasible lines.
4. **Finally, the individual investor chooses the appropriate mix between the optimal risky portfolio P and T-**bills.
5. **Harry Markowitz'**s model is precisely step one of portfolio management: the identification of the efficient set of portfolios, or the efficient frontier of risky assets. The principal idea behind the frontier set of risky portfolios it that, for any risk level, we are interested only in that portfolio with the highest expected return. Alternatively, the frontier is the set of portfolios that minimize the variance for any target expected return.
6. **The optimal risky portfolio is the same for all investors!!!**
7. **Separation property:** the portfolio choice problem may be separated into two independent tasks. The first task, determination of the optimal risky portfolio – the best risky portfolio is the same for all investors, regardless of risk aversion. The second task, however, allocation of the complete portfolio to risk-free asset versus the risky portfolio, depends on personal preference. Here the investor is the decision maker. This analysis suggests that a limited number of portfolios may be sufficient to serve the demands of a wide range of investors. This is the theoretical basis of the mutual fund industry.
8. **Asset allocation and security selection:** the theories of security selection and asset allocation are identical. Both all for the construction of an efficient frontier, and the choice of a particular portfolio from along that frontier. In reality, top management of an investment firm continually updates the asset allocation of the organization, adjusting the investment budget allotted to each asset-class portfolio. The next level managers then optimize the security selection of each asset-class portfolio independently.

Chapter 9: The Capital Asset Pricing Model

I. The capital asset pricing model, CAPM is a centerpiece of modern financial economics. The model gives a precise prediction of the relationship that we should observe between the risk of an asset and its expected return. This relationship serves two vital functions. First, it provides a benchmark rate of return for evaluating possible investments. Second, the model helps us to make an educated return on assets that have not yet been traded in the marketplace.

Intuition: Risk-averse investors measure the risk of the optimal risky portfolio by its variance. In this world, we would expect the reward, or the risk premium on individual assets, to depend on the contribution of the individual asset to the risk of the portfolio. The beta of a stock measures the stock's contribution to the variance of the market portfolio. Hence we expect, for any asset or portfolio, the required risk premium to be a function of beta.

II. The Capital Asset Pricing Model:

1. Assumptions:

A1. Investors are price-takers, in that act as though security prices are unaffected by their own trades.

A2. All investors plan for one identical holding period. This behavior is myopic in that it ignores everything that might happen after the end of the single-period horizon.

A3. Investments are limited to a universe of publicly traded financial assets, such as stocks and bonds, and to risk-free borrowing or lending arrangement. This assumption rules out investment in nontraded assets such education, private enterprises, etc. It is assumed also that investors may borrow or lend any amount at a fixed, risk-free rate.

A4. Investors pay no taxes on returns and no transaction costs on trades in securities.

A5. All investors are rational mean-variance optimizers, meaning that they all use the Markowitz portfolio selection model.

A6. Homogeneous expectations: all investors have the same beliefs concerning returns, variances, and covariances. But all investors may have different aversion to risk. All investors use the same expected returns and covariance matrix of security returns to generate the efficient frontier and the unique optimal risky portfolio.

2. Results under these assumptions:

a. All investors will choose to hold the same **market portfolio** (M), which is a market-value-weighted portfolio of all existing securities

b. Not only will the market portfolio be on the efficient frontier, but it also will be the tangency portfolio to the optimal **capital allocation line (CAL)** derived by each and every investor. As a result, the **capital market line (CML),** the line from the risk-free rate through the market portfolio, M, is also the best attainable capital allocation line. All investors hold M as their optimal risky portfolio, differing only in the amount invested in it versus in the risk-free asset.

c. The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the repensentative investor.

d. The risk premium on individual assets will be proportional the risk premium on the market portfolio, M, and the beta coefficient of the security relative to the market portfolio. Beta masure the extent to which returns on the stock and the market move together. Beta is defined as $\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}$

And the risk premium on individual securities is

$$E(r_i) - r_f = \frac{Cov(r_i, r_M)}{\sigma_M^2}[E(r_M) - r_f] = \beta_i[E(r_M) - r_f]$$

3. Why do all investors hold the market portfolio?
a. When we aggregate the portfolios of all investors, lending and borrowing will cancel out, and the value of the aggregate risky portfolio will equal the entire wealth of the economy. This is the market portfolio, M. The proportion of each stock in this portfolio equals the market value of the stock (price per share times number of shares outstanding) divided by the sum of the market value of all stocks.
b. If all investors use identical Markowitz analysis (A5) applied to the same universe of securities (A3) for the same time horizon (A2) and use the same input list (A6), they all must arrive at the same determination of the optimal risky portfolio, the portfolio on the efficient frontier identified by the tangency line from risk-free asset to that frontier.
c. Suppose that the optimal portfolio of investors does not include stock AB. When all investors avoid AB, the demand is zero, and AB's price takes a free fall. As AB stock gets progressively cheaper, it becomes ever more attractive and other stocks look relative less attractive. Ultimately, AB reaches a price where it is attractive enough to include in the optimal stock portfolio. Thus, all assets have to be included in the market portfolio.
4. The risk premium of the market portfolio:
a. Recall the each individual investor chooses a proportion y, allocated to the optimal portfolio M, such that: $y = \frac{E(r_M) - r_f}{.001A\sigma_M^2}$ --- (9.1)
b. In the simplified CAPM economy, risk-free investments involve borrowing and lending among investors. This means that net borrowing and lending across all investors must be zero, and in consequence the average position in the risky portfolio is 100%, or y = 1, Setting y = 1 in 9.1 and rearranging, we get that the risk premium on the market portfolio is related to its variance by the average degree of risk aversion:

$$E(r_M) - r_f = 0.01 \times A\sigma_M^2$$

5. Expected returns on individual securities:
a. The CAPM is built on the insight that the appropriate risk premium on an asset will be determined by its contribution to the risk of investors' overall portfolios. Portfolio risk is what matters to investors and is what governs the risk premiums they demand.
b. Suppose, we want to gauge the portfolio risk of GM stock. We measure the contribution to the risk of the overall portfolio from holding GM stock by its covariance with the market portfolio.

Portfolio	W_1	W_2		W_{GM}		W_n

Weights						
W_1	$Cov(r_1,r_1)$	$Cov(r_1,r_2)$	…	$Cov(r_1,r_{GM})$	…	$Cov(r_1,r_n)$
W_2	$Cov(r_2,r_1)$	$Cov(r_2,r_2)$	…	$Cov(r_2,r_{GM})$	…	$Cov(r_2,r_n)$
:	:	:	:	:	:	:
W_{GM}	$Cov(r_{GM},r_1)$	$Cov(r_{GM},r_2)$	…	$Cov(r_{GM},r_{GM})$	…	$Cov(r_{GM},r_n)$
:	:	:	:	:	:	:
Wn	$Cov(r_n,r_1)$	$Cov(r_n,r_2)$	…	$Cov(r_n,r_{GM})$	…	$Cov(r_n,r_n)$

The contribution of GM's stock to the variance of the market portfolio is

$W_{GM}\,[W_1\,Cov(r_{GM},r_1)+ W_2\,Cov(r_{GM},r_2)+\ldots + W_{GM}\,Cov(r_{GM},r_{GM})+ W_n\,Cov(r_{GM},r_n)]$

Therefore, GM's contribution to variance = $W_{GM}\,Cov(r_{GM},r_M)$

c. If the covariance between GM and the rest of the market is negative, then GM makes a "negative contribution" to portfolio risk: by providing returns that move inversely with the rest of the market, GM stabilizes the return on the overall portfolio. If the covariance is positive, GM makes a positive contribution to overall portfolio risk because its returns amplify swings in the rest of the portfolio.

d. The reward-to-risk ratio for investments in GM can be expressed as:

GM's contribution to risk premium/ GM's contribution to variance =

$$\frac{W_{GM}[E(r_{GM})-r_f]}{W_{GM}Cov(r_{GM},r_M)}=\frac{E(r_{GM})-r_f}{Cov(r_{GM},r_M)}$$

e. The market portfolio is the tangency portfolio. The reward-to-risk ratio for investment in the market portfolio is:

Market risk premium/market variance = $\dfrac{E(r_M)-r_f}{\sigma_M^2}$ ------------------ (9.5)

Notice that for components of the efficient portfolio, such as shares of GM, we measure risk as the contribution to portfolio variance. In contrast, for the efficient portfolio itself, its variance is the appropriate measure of risk.

f. A basic principle of equilibrium is that all investments should offer the same reward-to-risk ration. If the ratio were better for one investment than another, investors would rearrange their portfolios, tilting toward the alternative with the better trade-off and shying away from the other. Such activity would impart pressure on security prices until the ratios were equalized. Therefore:

$$\frac{E(r_M)-r_f}{\sigma_M^2}=\frac{E(r_{GM})-r_f}{Cov(r_{GM},r_M)}$$

g. CAPM – expected return-best relationship:

$$E(r_i)-r_f=\frac{Cov(r_i,r_M)}{\sigma_M^2}[E(r_M)-r_f]=\beta_i[E(r_M)-r_f]$$

An expected return consists of two components: the risk-free rate, which is compensation for the time value of money, and a risk premium, determined by multiplying a benchmark risk premium (i.e., the risk premium offered by the market portfolio) times the relative measure of risk, beta.

h. if the expected return-beta relationship holds for any individual asset, it must hold for any combination of assets. Let $E(r_P)=\sum_k w_k E(r_k)$, and $\beta_P=\sum_k w_k\beta_k$ is the portfolio beta, then CAPM has to hold for a portfolio: $E(r_p)-r_f=\beta_P[E(r_M)-r_f]$

i. In a rational market investors receive high expected returns only if they are willing to bear risk.

6. The security market line:

a. The expected return-beat relationship can be portrayed graphically as the **security market line (SML).**

b. **SML VS. CML**: the **CML** graphs the risk premiums of efficient portfolios as a function of **portfolio standard deviation**. This is appropriate because standard deviation is a valid measure of risk for efficiently diversified portfolios that are candidates for an investor's overall portfolio. The **SML** graphs individual asset risk premiums as a function of asset risk. The relevant measure of risk for individual asset held as parts of well-diversified portfolios is not the asst's standard deviation or variance; it is, instead, the contribution of the asset to the portfolio variance, which we measure by the **asset's beta.** The SML is valid for both efficient portfolios and individual assets.

c. CAPM can be **used in the money-management industry**: suppose that the SML relation is used as a benchmark to asset the fair expected return on a risky asset. The difference between the expected returns indicated in SML and the actual return is called the stock's **alpha**. Alpha is one measure of professional money managers' performance.

d. CAPM can be **used in capital budgeting decisions**: for a firm considering a new project, the CAPM can provide the required rate of return that the project needs to yield, based on its beta, to be acceptable to investors. Managers can use the CAPM to obtain this cutoff internal rate of return (IRR) for the project.

Chapter 8: Index Models and CAPM

I. **Markowtiza prodecure** requires a huge number of estimates of covariances between all pairs of available securities and also a huge optimization program. This is burdensome and sometimes, impossible. If we plan to analyze n stocks, we have to estimate n (estimates of expected returns) + n (estimates of variances) + $(n^2 - n)/2$ (estimates of covariances) = $(n^2+3n)/2$. If n =50, we will estimate 1325 estimates. If n = 3,000, we need more than 4.5 million estimates!!! We need to find one way out of this:

II. A single-index security market:

1. Assumptions:

A1: we summarize all relevant economic factors by one macroeconomic indicator and it moves the security market as a whole.

A2: beyond this common effect, all remaining uncertainty in stock returns is firm specific – there is no other source of correlation between securities.

A3: firm-specific events would include new inventions, deaths of key employees, and other factors that affect the fortune of the individual firm without affecting the broad economy in a measurable way.

2. The model:

a. the holding-period return on security I is: $r_i = E(r_i) + m_i + e_i$, ---------- (10.1)

where $E(r_i)$ is the expected return on the security as of the beginning of the holding period, m_i is the impact of unanticipated macro events on I's return during the period, and e_i is the impact of unanticipated firm-specific events.

b. Both **m_i and e_i have zero expected values** because each represents the impact of unanticipated events, which by definition must average out to zero.

c. **Single-factor model:** Different firms have different sensitivities to macroeconomic events. Thus if we denote the unanticipated components of the macro factor by F, and denote the responsiveness of security I to macro events by beta, β_i, then the macro component of security I is $m_i = \beta_i F$,

$$r_i = E(r_i) + \beta_i F + e_i \quad \text{-- (10.2)}$$

d. If we future assume that the rate of return on a broad index of securities such as the S&P 500 is a valid proxy for the common macro factor, then we get a **single-index model** because it uses the market index to proxy for the common or systematic factor.

3. Decomposition of the return on i: We can separate the actual or realized return on I into macro (systematic) and micro (firm-specific) components in a manner similar to that in equation 10.2. We write the rate of return on each security as a sum of three components:

a. α_i: The stock's expected return if the market is neutral, that is, if the market's excess return, $r_M - r_f$, is zero;

b. $\beta_i(r_M - r_f)$: The component of return due to movements in the overall market: is I's responsiveness to market movements.

c. e_i: the unexpected component due to unexpected events that are relevant only to I (firm specific)

$$r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + e_i$$

4. Excess return: let us denote excess returns over the risk-free rate by capital R, then $R_i = r_i - r_f$

$$R_i = \alpha_i + \beta_i R_M + e_i \quad \text{-- (10.3)}$$

5. risk of security I:

Variance of return (excess return) of i: $\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$ (the covariance between RM and ei is zero because ei is defined as firm specific, that is, independent of movements in the market. The variance has two components: the uncertainty of the common macroeconomic factor ($\beta_i^2 \sigma_M^2$) and the firm-specific uncertainty ($\sigma^2(e_i)$).

6. Covariance between returns on two stocks:

$$\text{cov}(R_i, R_j) = \text{cov}(\alpha_i + \beta_i R_M + e_i, \alpha_j + \beta_j R_M + e_j)$$

Since α_i and α_j are constants, their covariance with any variables are zero. Further, the firm-specific terms (ei, ej) are assumed uncorrelated with the market and with each other. Therefore, the only source of covariance in the returns between the two stocks derives from their common dependence on the common factor, RM.

$$\text{cov}(R_i, R_j) = \text{cov}(\beta_i R_M, \beta_j R_M) = \beta_i \beta_j \sigma_M^2$$

$$\text{cov}(R_i, R_M) = \text{cov}(\beta_i R_M + e_i, R_M) = \beta_i \sigma_M^2$$

$$\beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2}$$

7. The magic of simplification: if we plan to analyze n stocks, we have to estimate n (estimates of expected returns) + n (estimates of sensitivity coefficients, β i) + n (estimates of firm-specific variances var(ei)) + 1 estimate for the variance of the common macroeconic factor, σ_M^2 = 3n +1. If n =50, we will estimate 151 estimates.
8. The cost of the model: the classification of uncertainty into a simple dichotomy – macro versus micro risk – oversimplifies sources of real-world uncertainty and misses some important sources of dependence in stock returns. For example, this dichotomy rules out industry events, events that may affect many firms within an industry without substantially affecting the broad macroeconomy.
9. The index model and diversification:

Let $R_P = \sum_{i=1}^{n} w_i R_i$, and assume w, = 1/n, plug 10.3 into it:

$$R_P = \frac{1}{n}\sum_{i=1}^{n}(\alpha_i + \beta_i R_M + e_i)$$

$$\sum w_i = 1$$

$$r_P = E(r_P) + \beta_P F + e_P$$

$$\beta_P = \sum w_i \beta_i$$

Let $\alpha_P = \frac{1}{n}\sum_{i=1}^{n}\alpha_i$, $\beta_P = \frac{1}{n}\sum_{i=1}^{n}\beta_i$, $e_P = \frac{1}{n}\sum_{i=1}^{n} e_i$, hence the portfolio's variance is

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma^2(e_P)$$

- The systematic risk depends on portfolio beta and variance of market portfolio and will persist regardless of the extent of portfolio diversification. No matter how many stocks are held, their common exposure to the market will be reflected in portfolio systematic risk.
- The nonsystematic risk is $\sigma^2(e_P)$. Because e_is are independent with zero expected values, the law of averages can be applied to conclude that as more and more stocks are added to the portfolio, the firm-specific components tend to cancel out, resulting in ever-smaller nonmarket risk. Such risk is thus termed **diversifiable**.

III. The CAPM and the index model:

1. The CAPM is a statement about ex ante or expected returns, whereas in practice all we can observe directly are ex post or realized return. To make the leap from expected to realized returns, we can employ the index model:

$$R_i = \alpha_i + \beta_i R_M + e_i$$

2. since $\text{cov}(R_i, R_M) = \text{cov}(\beta_i R_M + e_i, R_M) = \beta_i \sigma_M^2$ →

$\beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2}$ → the index model beta coefficient turns out to be the same beta as that of the CAPM expected return-beta relationship, except that we replace the (theoretical) market portfolio of the CAPM with the well-specified and observable market index.

3. The index model and the expected return-beat relationship:

Since $r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + e_i$, if the index M represents the true market portfolio, we can take the expectation of each side of the equation to show that the index model specification is:

$E(r_i) - r_f = \alpha_i + \beta_i[E(r_M) - r_f]$, compare this with CAPM:

$$E(r_i) - r_f = \beta_i[E(r_M) - r_f]$$

$E(r_P) = r_f + \beta_P[E(r_M) - r_f]$ → CAPM predicts that alpha should be zero for all assets. The alpha of a stock is its expected return in excess of (or below) the fair expected return as predicted by the CAPM. If the stock is fairly priced, its alpha must be zero. The CAPM states that the expected value of alpha is zero for all securities, whereas the index model representation of the CAPM holds that the realized value of alpha should averaged out to zero for a sample of historical observed returns. The sample alphas should be unpredictable, that is, independent from one sample period to the next.

IV. The industry versions of the index model: Practitioners routinely estimate the index model using total rather excess returns. This makes their estimate of alpha equal to

$\alpha + r_f(1 - \beta)$.

Also read relevant sectors of Ch 13: Empirical Evidence on Security Returns