

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^5 (a + bx_i)$
 $\sum_{i=1}^5 a + \sum_{i=1}^5 bx_i = 5a + b \sum_{i=1}^5 x_i = 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$

b. $\sum_{y=0}^5 f(x+y)$
 $= f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

c. $\sum_{i=1}^{10} i^2$
 $= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$
 $= 385$

d. $\sum_{x=1}^2 \sum_{y=2}^3 (2x+y)$
 Σ

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

- a. Find the value of b

$\sum f(x) = 1$; $8b = 1$
 $b = \frac{1}{8} = 0.125$

- b. Find the answer for $P(X \leq 2)$

$= 0.5b + b + 2.25b + 2b + 1.5b$
 $= 7.25(0.125) = 0.90625$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$= 7.75(0.125)$
 $= 0.96875$

- d. Find the answer for $P(X \geq 1)$

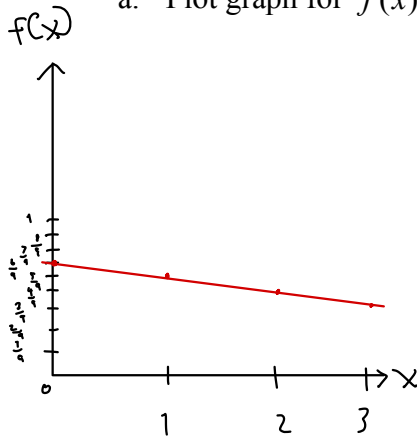
$= 4.25(0.125)$
 $= 0.53125$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

x	0	1	2	3
$f(x)$	$\frac{6}{9}$	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{3}{9}$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} P(1 \leq X \leq 3) &= \int_1^3 \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_1^3 \\ &= \left(-\frac{9}{18} + 2 \right) - \left(-\frac{1}{18} + \frac{6}{9} \right) \\ &= \frac{27}{18} - \frac{11}{18} = \frac{16}{18} = \frac{8}{9} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} P(X \geq 2) &= \int_2^3 \left(-\frac{1}{9}x + \frac{6}{9} \right) dx + \int_3^{\infty} f(x) dx \\ &= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_2^3 + 0 \\ &= \left(-\frac{9}{18} + 2 \right) - \left(-\frac{4}{18} + \frac{12}{9} \right) \\ &= \frac{27}{18} - \frac{20}{18} \\ &= \frac{7}{18} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_0^3 x f(x) dx = \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left[-\frac{1}{27}x^3 + \frac{6}{18}x^2 \right]_0^3 \\ &= (-1 + 6) - 0 \\ &= 5 \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

		Dice (X)					
		1	2	3	4	5	6
Coin (Y)	0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
	1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

- b. Find the marginal probability distribution function (PDF) of X

$$\begin{aligned}
 P(X=1) &= \frac{1}{6} & P(X=2) &= \frac{1}{6} & P(X=3) &= \frac{1}{6} \\
 P(X=4) &= \frac{1}{6} & P(X=5) &= \frac{1}{6} & P(X=6) &= \frac{1}{6}
 \end{aligned}$$

- c. Find the marginal probability distribution function (PDF) of Y

$$P(Y=0) = \frac{1}{2} \quad P(Y=1) = \frac{1}{2}$$

- d. Find the conditional probability distribution function (PDF) of

X given Y is equal to 1

$$P(X|Y=1) = \frac{P(X, Y=1)}{P(Y=1)} = \frac{P(X, Y=1)}{\frac{1}{2}} = 2(P(X, Y=1))$$

- e. Find the expected value of X given Y is equal to 1

$$\begin{aligned}
 E(X|Y=1) &= \sum X_i P(X=X_i | Y=1) \\
 &= \sum X_i (2 P(X, Y=1)) \\
 &= 2 \left[1\left(\frac{1}{12}\right) + 2\left(\frac{1}{12}\right) + 3\left(\frac{1}{12}\right) + 4\left(\frac{1}{12}\right) + 5\left(\frac{1}{12}\right) + 6\left(\frac{1}{12}\right) \right]
 \end{aligned}$$

- f. Find the variance of X given Y is equal to 1

$$\begin{aligned}
 \text{Var}(X|Y=1) &= E[X^2|Y=1] - [E(X|Y=1)]^2 = 2 \left(\frac{21}{12} \right) - \left(\frac{21}{6} \right)^2 = 3.5 \\
 &= \sum X_i^2 P(X_i | Y=1) - (3.5)^2 \\
 &= (1^2)(2)\left(\frac{1}{12}\right) + 2^2(2)\left(\frac{1}{12}\right) + \dots + 6^2(2)\left(\frac{1}{12}\right) - (3.5)^2 \\
 &= \frac{1}{6} (91) - (3.5)^2 = 2.9667
 \end{aligned}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{3}(X_1 + X_2 + X_3)\right) \\ &= \frac{1}{3} [E(X_1) + E(X_2) + E(X_3)] \\ &= \frac{1}{3} (3\mu_X) \end{aligned}$$

$$\therefore E(\bar{X}) = \mu_X$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left[\frac{1}{3}(X_1 + X_2 + X_3)\right] \\ &= \left(\frac{1}{3}\right)^2 \text{Var}(X_1 + X_2 + X_3) \\ &= \frac{1}{9} [\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + 2\text{Cov}(X_1, X_2) + 2\text{Cov}(X_1, X_3) + 2\text{Cov}(X_2, X_3)] \\ &= \frac{1}{9} \left[3\sigma^2 + 2\left(\frac{3\sigma^2}{2}\right)\right] \\ &= \frac{1}{9} \left(\frac{9}{2}\sigma^2\right) \\ \therefore \text{Var}(\bar{X}) &= \frac{\sigma^2}{2} \end{aligned}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

- a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned} E(\bar{X}) &= E\left[\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right] \\ &= \frac{1}{4} [E(X_1) + E(X_2) + E(X_3) + E(X_4)] \\ &= \frac{1}{4} (4\mu_X) \end{aligned}$$

$$\therefore E(\bar{X}) = \mu_X$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left[\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right] \\ &= \left(\frac{1}{4}\right)^2 \text{Var}(X_1 + X_2 + X_3 + X_4) \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{16} [\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + \text{Var}(X_4)] \\
 &= \frac{1}{16} (4\sigma^2) \\
 \therefore \text{Var}(\bar{X}) &= \frac{\sigma^2}{4}
 \end{aligned}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ

$$\begin{aligned}
 E(\tilde{X}) &= E\left[\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right] \\
 &= E\left(\frac{1}{8}X_1\right) + E\left(\frac{1}{4}X_2\right) + E\left(\frac{1}{8}X_3\right) + E\left(\frac{1}{2}X_4\right) \\
 &= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4) \\
 &= \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{2}\mu \\
 \therefore E(\tilde{X}) &= \mu \text{ which indicates that } \tilde{X} \text{ is an unbiased estimator of } \mu
 \end{aligned}$$

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$$\begin{aligned}
 \text{Var}(\tilde{X}) &= \text{Var}\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right) \\
 &= \left(\frac{1}{8}\right)^2 \text{Var}(X_1) + \left(\frac{1}{4}\right)^2 \text{Var}(X_2) + \left(\frac{1}{8}\right)^2 \text{Var}(X_3) + \left(\frac{1}{2}\right)^2 \text{Var}(X_4) \\
 &= \frac{1}{64}\sigma^2 + \frac{1}{16}\sigma^2 + \frac{1}{64}\sigma^2 + \frac{1}{4}\sigma^2 \\
 \text{Var}(\tilde{X}) &= \frac{11}{32}\sigma^2
 \end{aligned}$$

Since $\text{Var}(\bar{X}) < \text{Var}(\tilde{X})$, $\bar{X}$ is better estimator for μ