

PRODUCTION AND COSTS IN THE LONG RUN

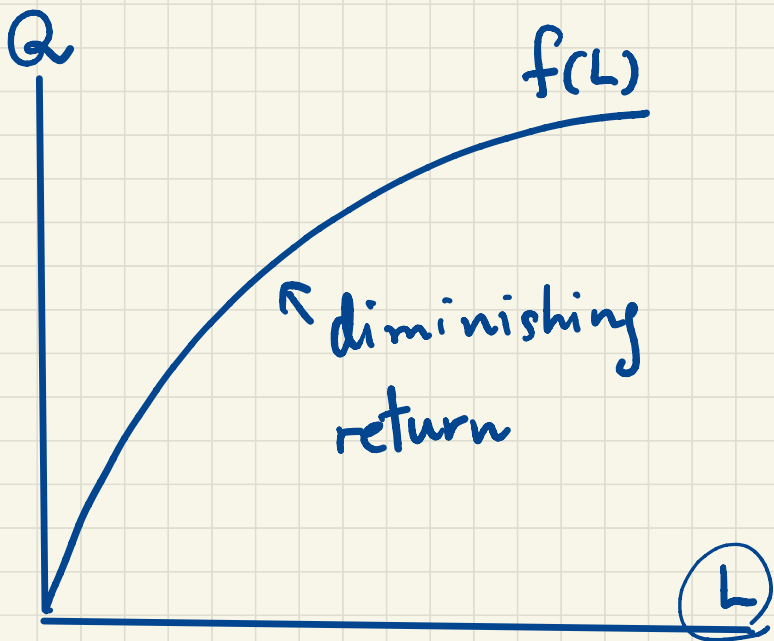
Topics

- Isoquant → "same" quantity
- Isocost → "same" cost
- Production equilibrium and Expansion Path
- Relationship between Expansion Path and LRTC
- Long-run Costs of Production: LRTC, LRAC, LRMC
- Relationship between Long-run and Short-run Costs
- The Meaning of Returns to Scale
- Economies and Diseconomies of Scale

Recall

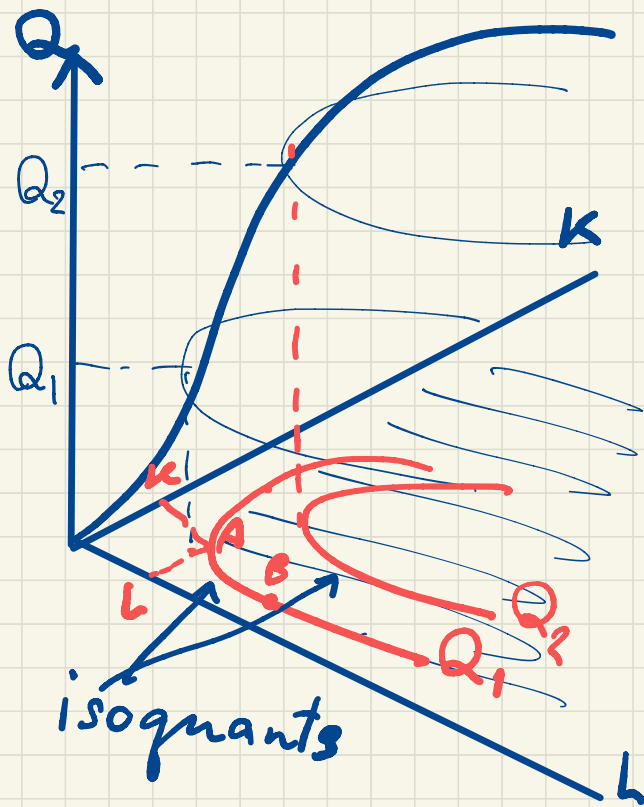
SR Production

$$Q = f(L), \text{ k is fixed}$$



LR Production

$$Q = f(L, K)$$



Production in the Long Run

- In the long run, all inputs are variable. Hence, there is no fixed cost.
- To determine the optimal amount of inputs (say, L & K) for a given price (w & r), the firm face one of the two problems: $w = \text{price of labor (L)}, r = \text{price of K.}$

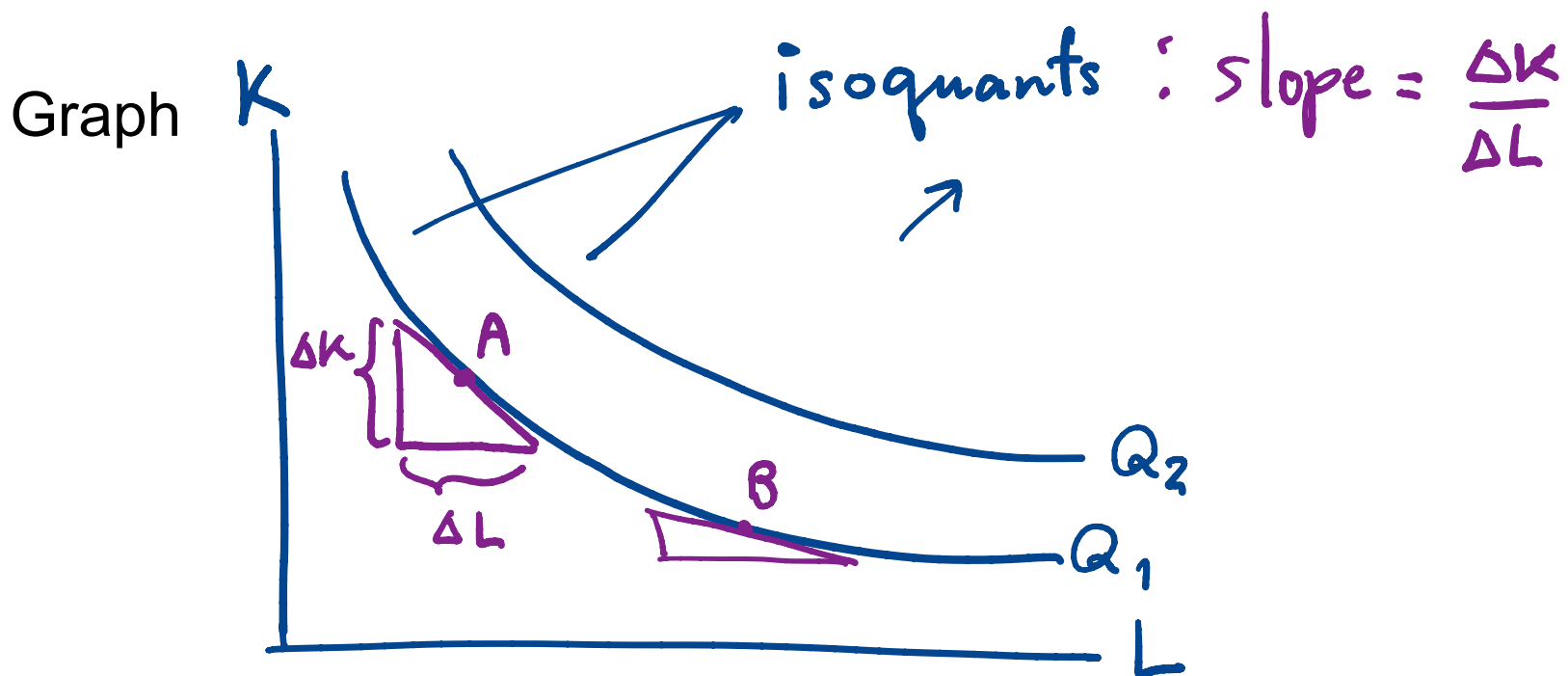
1. **Output maximization** $\text{Max}_{k,L} Q(k,L)$ subject to C_0 .
 ➤ To maximize the output (Q) under the constraint of a given cost (C_0). ↑ fixed

2. **Cost Minimization** $\text{Min}_{k,L} C(k,L) = wL + rK$ subject to fixed Q_0 .
 ➤ To minimize the cost (C) under the constraint of a given output (Q_0).

Long-Run Production Function

- Suppose there are two inputs: L and K.
- Long-run production function can be written as:

$$Q = f(L, K)$$

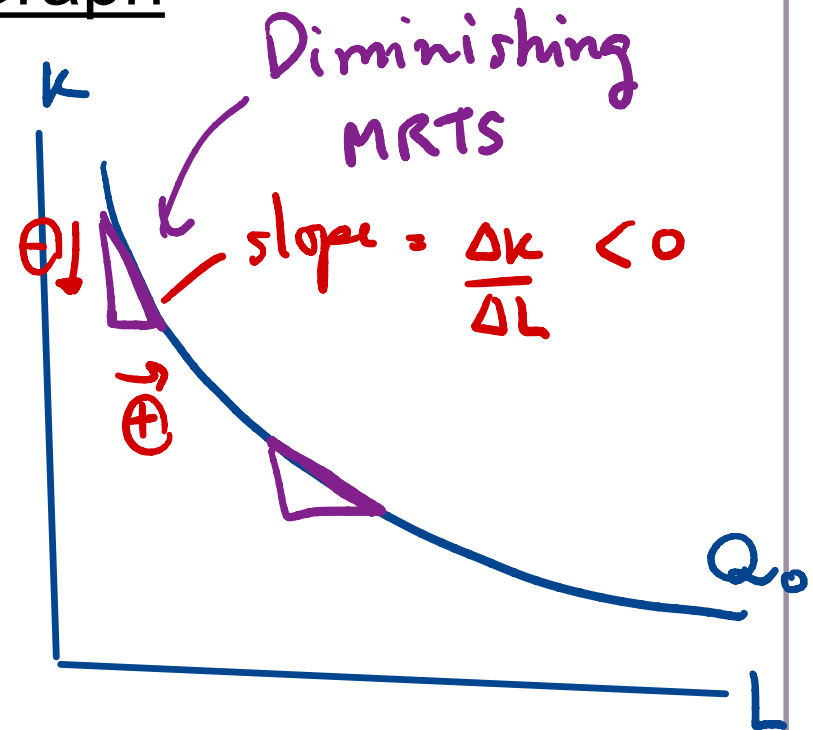


Isoquant

- **Isoquant** illustrates all combinations of inputs that yield the same level of output (Q).
- Slope of isoquant is the **Marginal Rate of Technical Substitution (MRTS)**:

$$\frac{\Delta K}{\Delta L} = MRTS$$

Graph

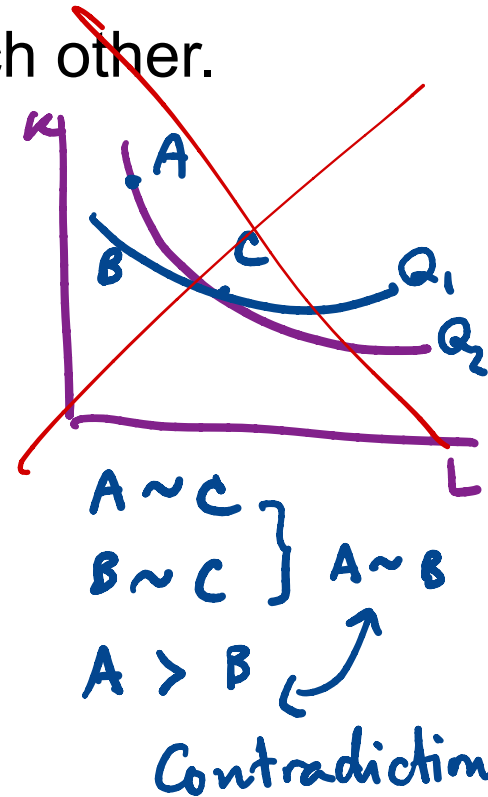


Properties of Isoquants

- Higher isoquants mean higher levels of output.
- Isoquants cannot cross nor be tangent to each other.
- Isoquants always have negative slope.

$$MRTS = \frac{\Delta K}{\Delta L} = - \frac{MP_L}{MP_K}$$

- MRTS is assumed to be diminishing.

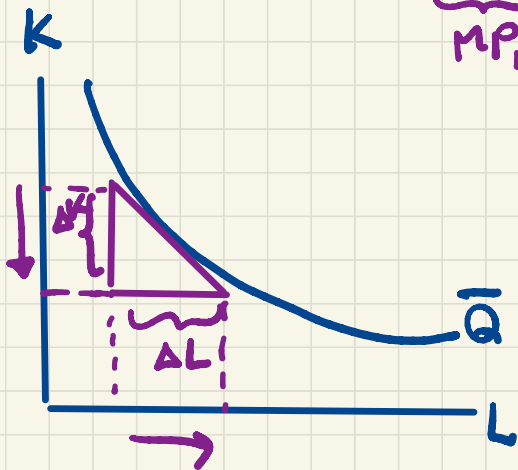


Extra Notes on MRTS

Recall $Q = f(K, L)$. We want to find the slope of an isoquant. We need to derive $\frac{\Delta K}{\Delta L}$ (or $\frac{dk}{dL}$ in continuous term).

If we decompose change in Q , we have

$$\Delta Q = \underbrace{\frac{dQ}{dK}}_{MP_K} \cdot \Delta K \ominus \underbrace{\frac{dQ}{dL}}_{MP_L} \cdot \Delta L \oplus$$



But $\Delta \bar{Q} = 0$ (same quantity),

$$\text{So, } \frac{dQ}{dK} \cdot \Delta K = - \frac{dQ}{dL} \cdot \Delta L$$

$$\therefore \frac{\Delta K}{\Delta L} = \frac{-dQ/dL}{dQ/dK} = \frac{-MP_L}{MP_K} = \text{MRTS}$$

Isocost

- **Isocost** represents all combinations of inputs that can be purchased for the **same cost**, given that the input prices are fixed.

price of L price of K

- Equation: $wL + rK = C_0$

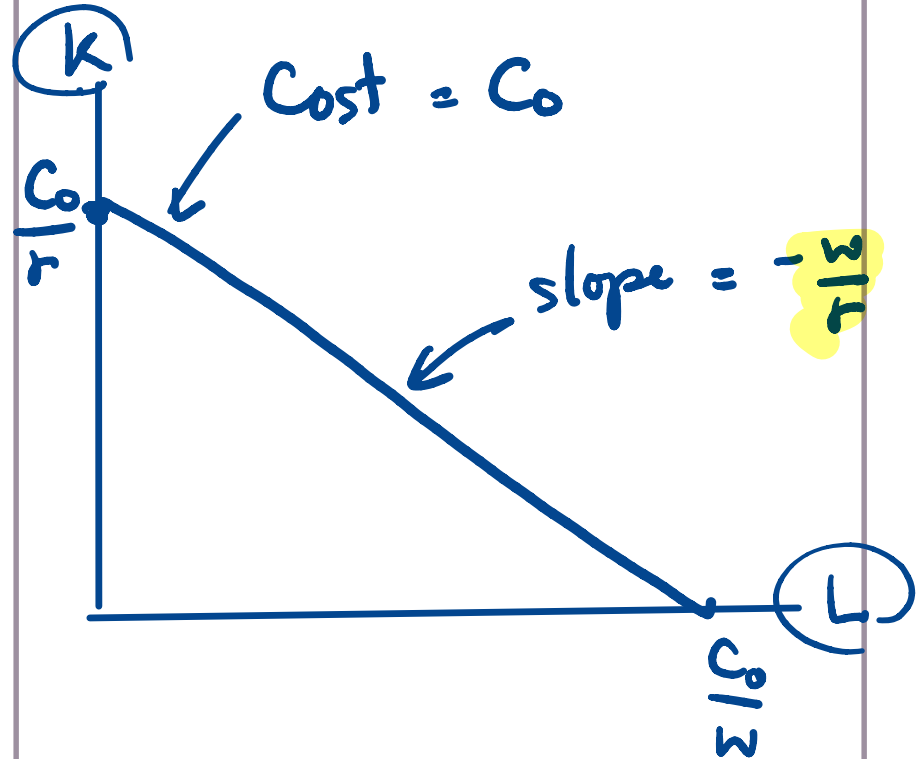
- Slope of isocost = $-\frac{w}{r}$

$$wL + rK = C_0$$

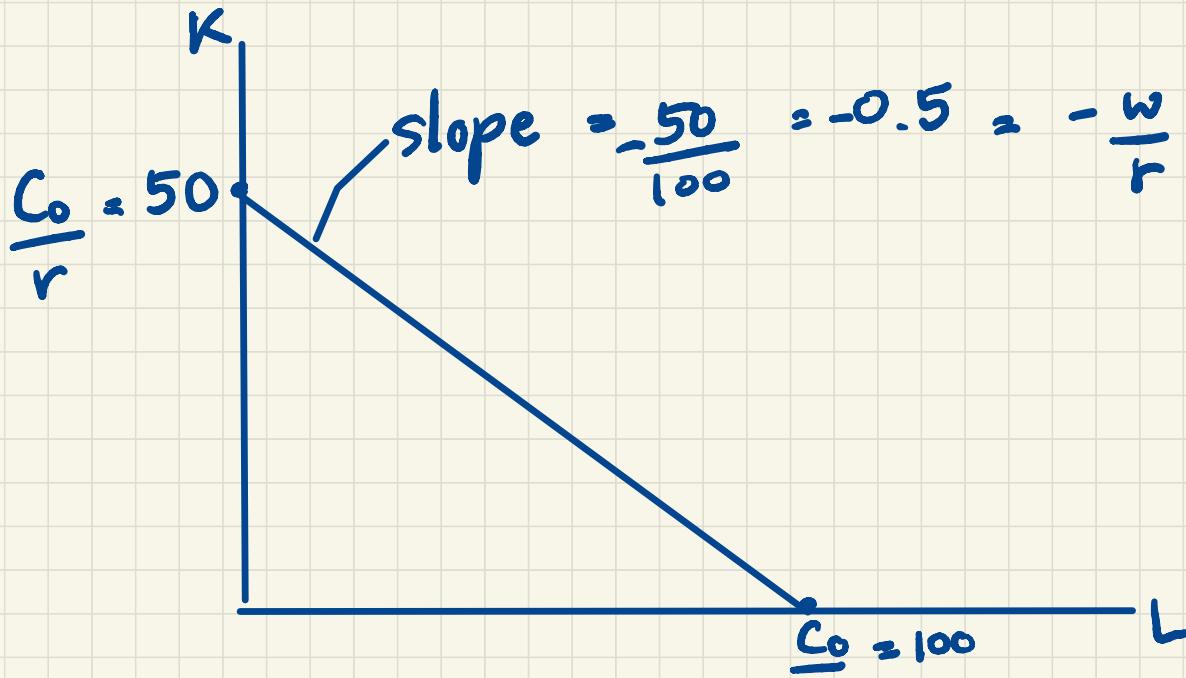
$$rK = C_0 - wL$$

$$K = \frac{C_0}{r} - \frac{w}{r}L$$

Graph



Suppose $C_0 = 5000$, $w = 50$, $r = 100$



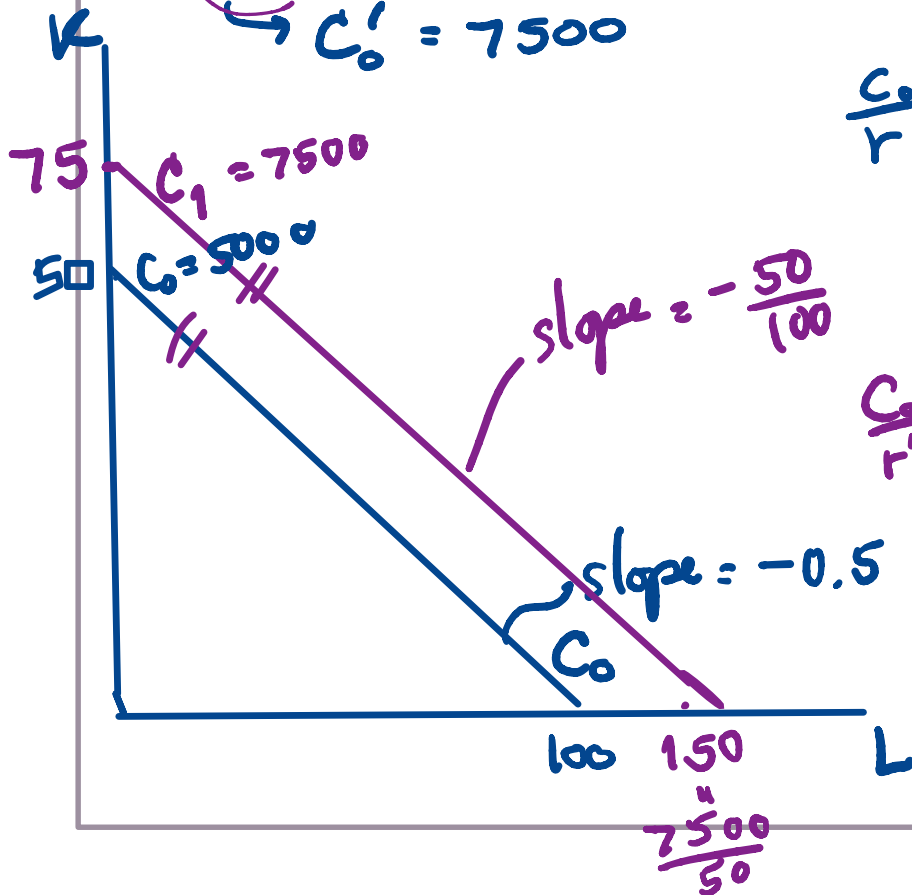
$$C_0 = wL + rK$$
$$5000 = 50L + 100K \quad \rightarrow \quad 100K = 5000 - 50L$$
$$K = 50 - 0.5L$$

Changes in Isocost

- When C_0 increases.

$$C_0 = 5000, w = 50, r = 100$$

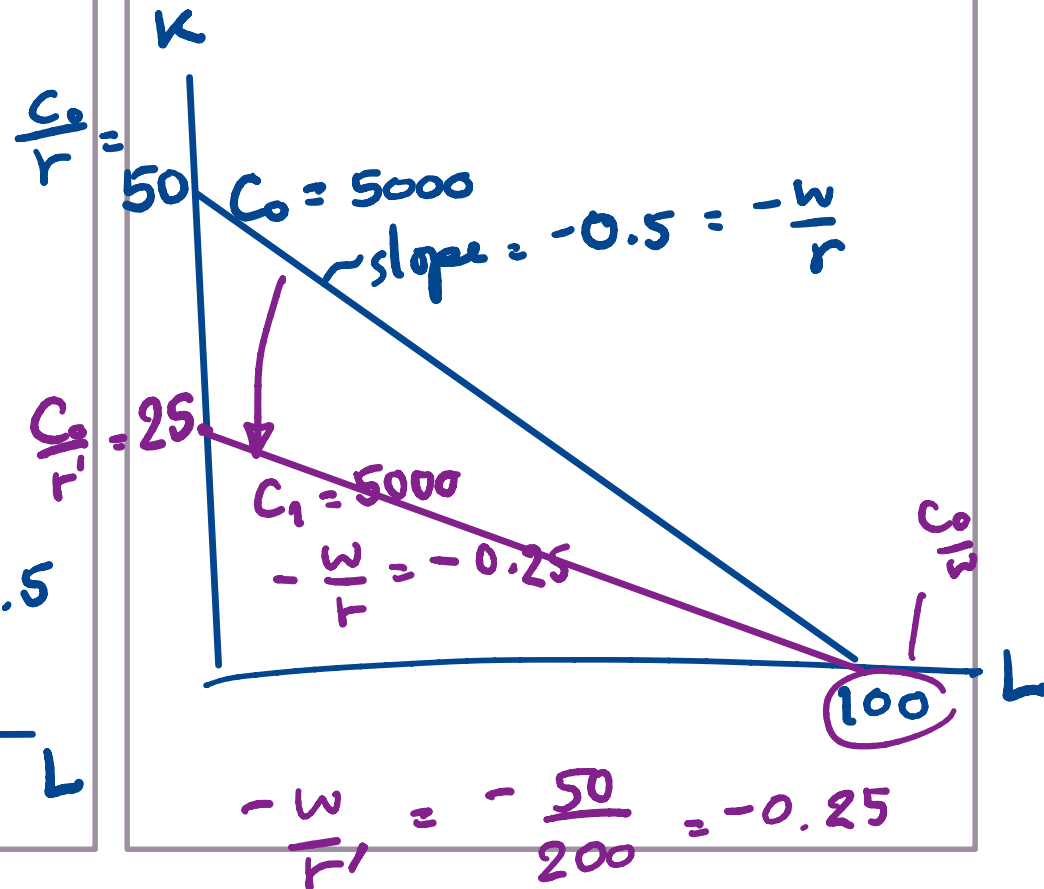
$$\rightarrow C_1 = 7500$$



- When r increases.

$$C_0 = 5000, w = 50, r = 100$$

$$\rightarrow r' = 200$$



Production Equilibrium: Output Maximization

- Firm's objective:

$$\max_{L, K} Q = f(L, K) \quad \text{subject to} \quad wL + rK = C_0$$

choice variables \rightarrow (L, K)

- Equilibrium condition

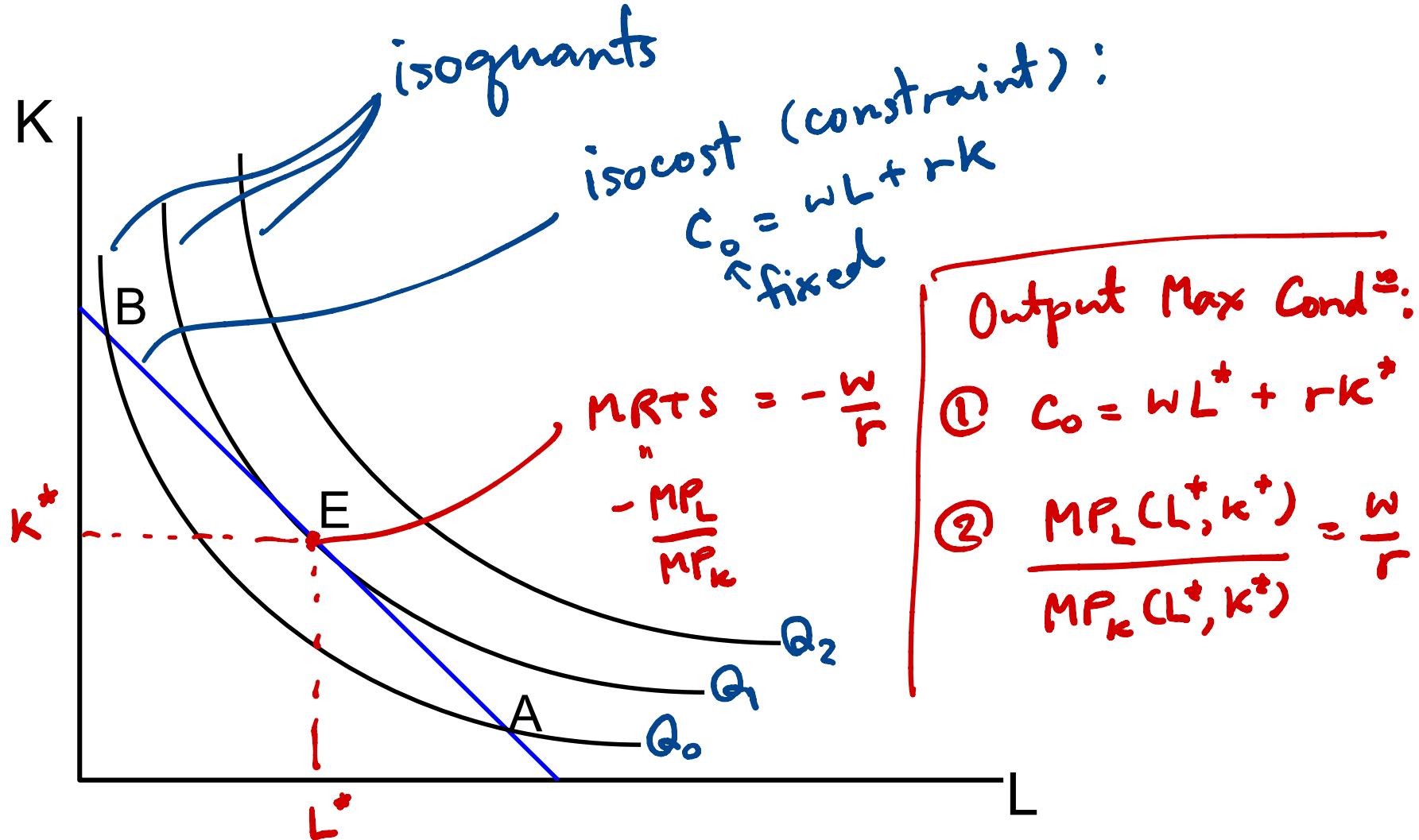
$E = (L^*, K^*)$ is the equilibrium when:

1. $wL^* + rK^* = C_0$ $\leftarrow (L^*, K^*)$ is on the isocost curve.

2. $MRTS = -\frac{w}{r}$ (i.e. $-\frac{MP_L}{MP_K} = -\frac{w}{r}$) $\Rightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$

\rightarrow slope of isoquant slope of iso cost

Graph: Output Maximization



Excercise Given $C_0 = 5000$, $w = 50$, $r = 100$.
If w decreases to 25, illustrate the change
in the optimal input combinations (L^*, k^*) .