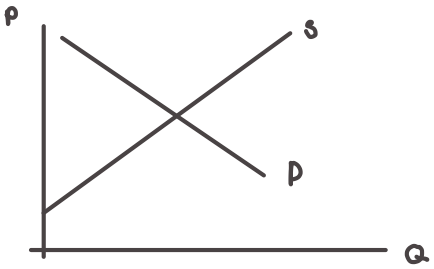
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

(1)



$D = P = 10 - Q^2$

$S = P = Q - a$

$= Q - 14$

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$R(Q)' = \left(\ln(Q^2+1) + \frac{3Q}{Q+1}\right)$$

$$= \frac{d}{dQ}\left(\ln(Q^2+1)\right) + \frac{d}{dQ}\left(\frac{3Q}{Q+1}\right)$$

$$= \frac{1}{Q^2+1} \times 2Q + \frac{3(Q-1) - 3Q}{(Q+1)^2}$$

$$= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$\Pi'(Q)' = -1Q^2 - 2Q + 8$

$\Pi'(Q)'' = -2Q - 2$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

NO ANS.

4.2 A*B

$$\begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 8+36 & 16+45 & 24+54 \\ 10+44 & 20+55 & 30+66 \end{bmatrix}$$

$$= \begin{bmatrix} 42 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3 det(A)

$$\begin{vmatrix} 8 & 9 \\ 10 & 11 \end{vmatrix} = 88 - 90 = -2$$

4.4 det(B)

NO ANS.

4.5 det(C)

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$$

$$= (45 + 84 + 96) - (105 + 48 + 72)$$

$$= 225 - 225$$

$$= 0$$

5. Suppose that $U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{d\Pi(R)}{dQ} = -Q^2 - 2Q + 8 = 0$$

$$= -(Q^2 + 2Q - 8) = 0$$

$$(Q + 4)(Q - 2) = 0$$

$$Q = 2, -4 \times$$

$$Q = 2 \; ; \quad \Pi = \frac{25}{3}$$

$$\frac{d\Pi'(Q)}{dQ} \; ; \quad 2Q + 2 = 0$$

$$Q = -1 < 0$$