FN312 Investment Lecture 5 Risk Aversion and Capital Allocation to Risky Assets

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Reading:
Chapter 6

Outline

- **Risk aversion and its estimation**
- **Two-step process of portfolio construction**
  - Composition of risky portfolio
  - Capital allocation between risky and risk-free assets
- *Passive strategies and the capital market line (CML)*

Risk aversion and its estimation

Risk and Risk Aversion

Speculation	Gambling
• Taking considerable risk for a commensurate gain • Parties have heterogeneous expectations	• Bet on an uncertain outcome for enjoyment • Parties assign the same probabilities to the possible outcomes

Risk and Risk Aversion

- Utility Values
  - **Investors are willing to consider:**
    - Risk-free assets
    - Speculative positions with positive risk premiums
  - **Portfolio attractiveness**
    - Increases with expected return
    - Decreases with risk
    - What happens when return increases with risk?

- Utility Function
  - *U* = Utility
  - *E*(*r*) = Expected return on the asset or portfolio
  - *A* = Coefficient of risk aversion
  - σ^2 = Variance of returns
  - ½ = A scaling factor

$$U = E(r) - \frac{1}{2} A\sigma^2$$

Investor Types

- Utility Function $$U = E(r) - \frac{1}{2} A\sigma^2$$

  - ***Risk Averse:*** Rejects investment portfolios that are fair games or worse
    $$A > 0$$
  - ***Risk-neutral:*** Judges risky prospects solely by their expected returns
    $$A = 0$$
  - ***Risk Lover:*** Accepts a fair game or gamble; the investor adjusts the expected return upward to take into account the "fun" of confronting the prospect's risk $A < 0$

Indifference curve

Equally preferred portfolios will lie in the mean–standard deviation plane on an **indifference curve,** which connects all portfolio points with the same utility value

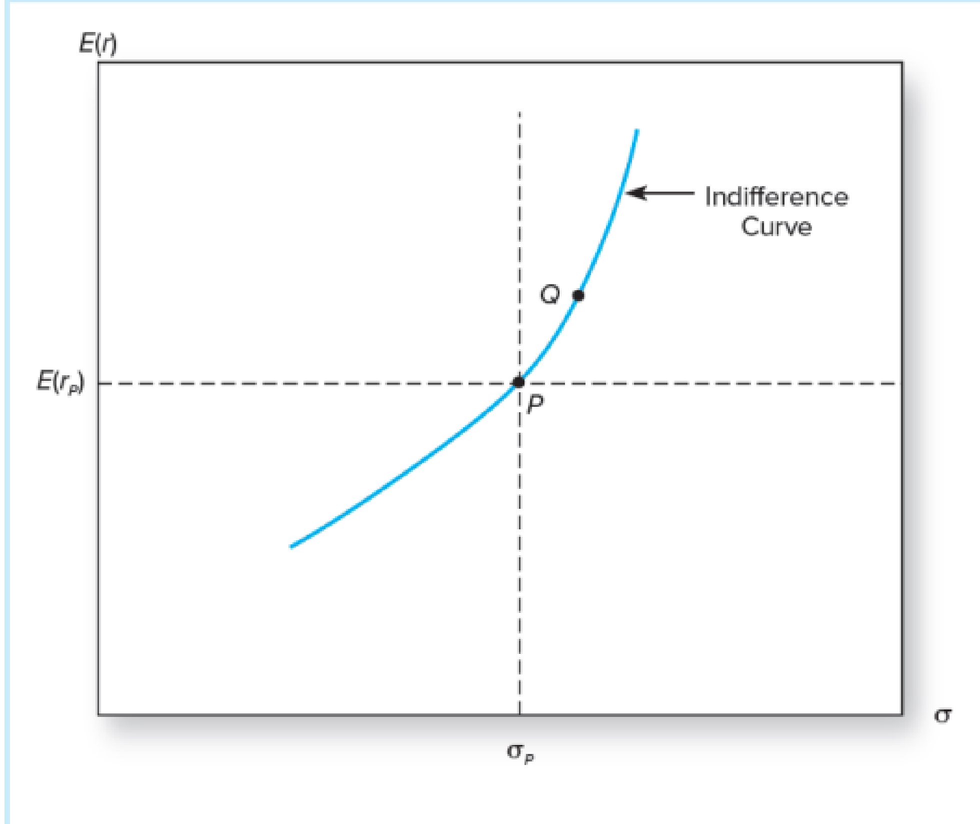


Figure 6.2 The indifference curve

If Portfolio X dominates Portfolio Y, where would X lie with respect to Y (and its indifference curve)?

- Mean-Variance (M-V) Criterion
  - Portfolio X dominates portfolio Y if:

$$E(r_X) \geq E(r_Y)$$

and

$$\sigma_X \leq \sigma_Y$$

and at least one inequality is strict

Two-step process of portfolio construction

Capital Allocation Across Risky and Risk-Free Portfolios

- ***Asset Allocation:*** The choice among broad asset classes that represents a very important part of portfolio construction
- Simplest way to control risk is to manipulate the ratio of risky assets to risk-free assets

Basic Asset Allocation Example

Total market value	$300,000
Risk-free money market fund	$90,000
Total risk assets	$210,000
Equities	$113,400
Bonds (long-term)	$96,600

<u>Risk assets</u>

$$W_E = \frac{\$113{,}400}{\$210{,}000} = 0.54 \qquad W_B = \frac{\$96{,}600}{\$210{,}00} = 0.46$$

Basic Asset Allocation Example

Let

- y = Weight of the RISKY PORTFOLIO, P, in the complete portfolio
- (1-y) = Weight of RISK-FREE ASSETS

$$y = \frac{\$210{,}000}{\$300{,}000} = 0.7$$

$$1 - y = \frac{\$90{,}000}{\$300{,}000} = 0.3$$

$$E : \frac{\$113{,}400}{\$300{,}000} = .378$$

$$B : \frac{\$96{,}600}{\$300{,}000} = .322$$

Note: The Risk-Free Asset

- Only the government can issue **default-free securities**
  - A security is risk-free in real terms only if
    - Its price is indexed
    - **Maturity is equal to investor's holding period**
- T-bills viewed as "the" risk-free asset
- Money market funds are also considered risk-free in practice

Portfolios: Risky Asset and Risk-Free Asset

- It's possible to create a complete portfolio by splitting investment funds between safe and risky assets

Let

- *y* = Weight of the <u>RISKY PORTFOLIO</u>, *P*, in the complete portfolio
- (1-*y*) = Weight of <u>RISK-FREE ASSETS</u>

- **Recall:**
  - Expected return of the complete portfolio:

$$E(r_c) = r_f + y \times \left[E(r_p) - r_f \right]$$

  - Variance:

$$\sigma_c^2 = y^2 \times \sigma_p^2$$

Portfolios: Risky Asset and Risk-Free Asset

- Define complete portfolio consists of risky asset and risk-free asset

$E(r_p) = 15\%$ $\sigma_p = 22\%$

$r_f = 7\%$ $\sigma_{rf} = 0\%$

- What is “Expected Return”?

$$E\left(r_c\right) = 7 + y \times \left(15 - 7\right)$$

- What is “Risk”?

$$\sigma_C = y \times \sigma_P = 22 \times y$$

Portfolios: Risky Asset and Risk-Free Asset

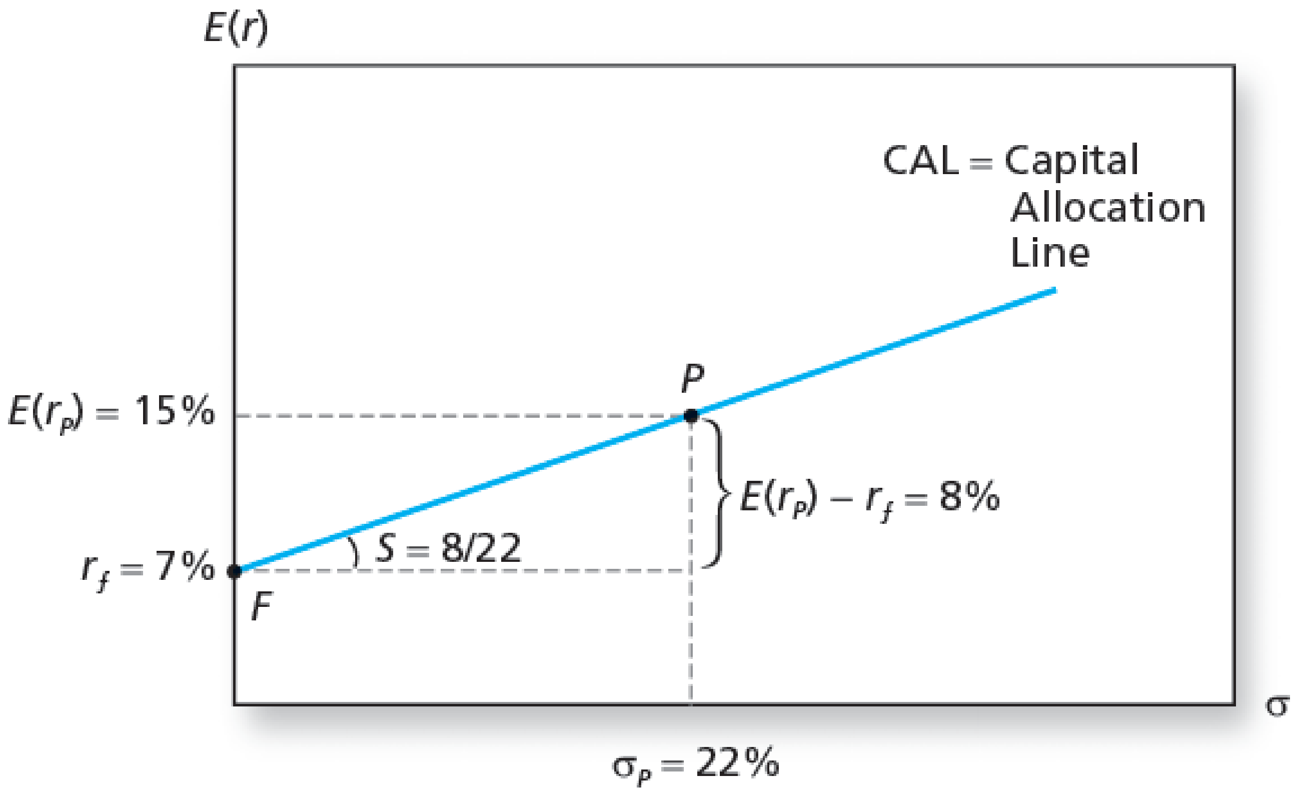
- Rearrange and substitute y = σ_C/σ_P:

$$E(r_C) = r_f + \frac{\sigma_C}{\sigma_P} \times \left[E(r_P) - r_f \right] = 7 + \frac{8}{22} \times \sigma_C$$

$$\text{Slope} = \frac{E(r_P) - r_f}{\sigma_P} = \frac{8}{22}$$

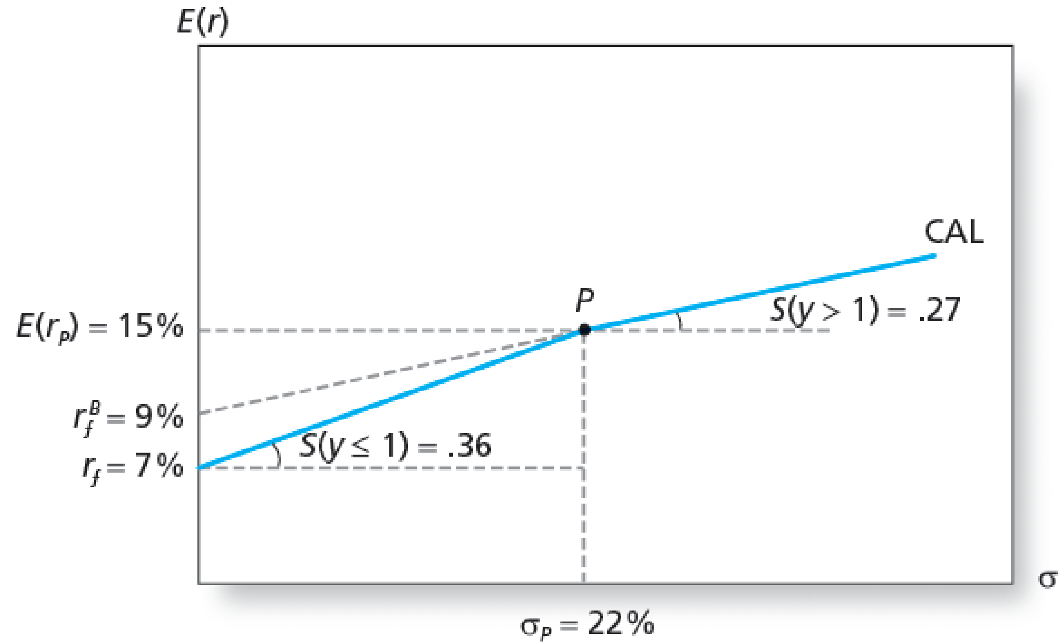
Portfolios: Risky Asset and Risk-Free Asset

The Investment Opportunity Set: $E\left(r_C\right)=7+\dfrac{8}{22}\times\sigma_C$



Note: Can “y” be greater than 1 ?

- Capital allocation line with leverage
  - **Lend at r_f = 7%** and **borrow at r_f = 9%**
    - Lending range slope = 8/22 = 0.36
    - Borrowing range slope = 6/22 = 0.27
    - CAL kinks at *P*



Risk Tolerance and Asset Allocation

- The investor must choose one optimal portfolio, *C*, from the set of feasible choices

From $U = E(r) - \frac{1}{2} A\sigma^2$ ➡ Indifference Curve

$E(r_C) = 7 + \frac{8}{22} \times \sigma_C$ ➡ CAL

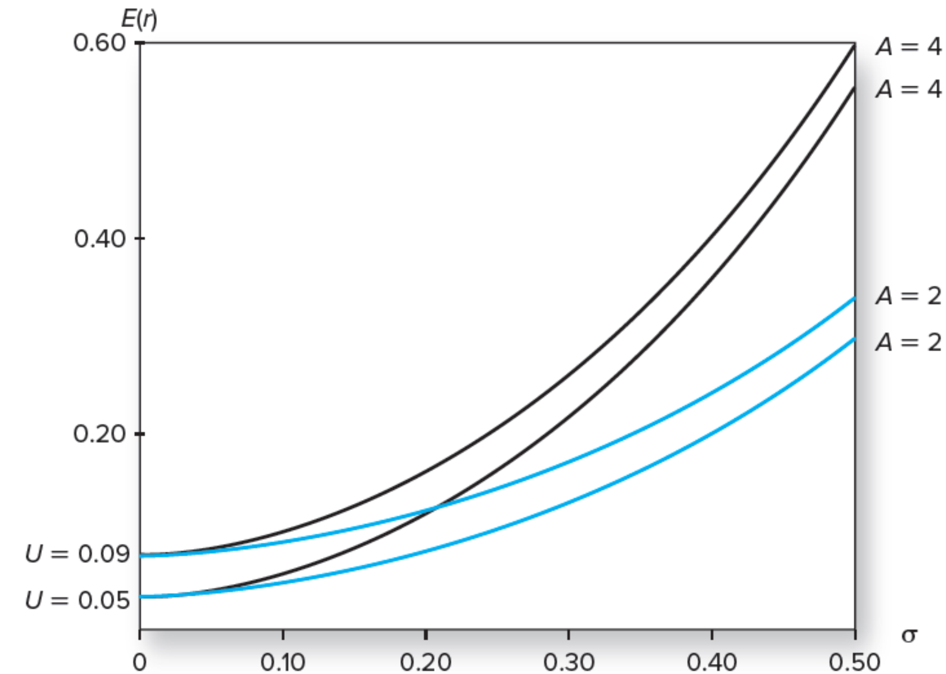
Therefore, objective function:

$$\underset{y}{Max}\, U = \boxed{r_f + y[E(r_P) - r_f]} - \frac{1}{2} Ay^2\sigma_P^2$$

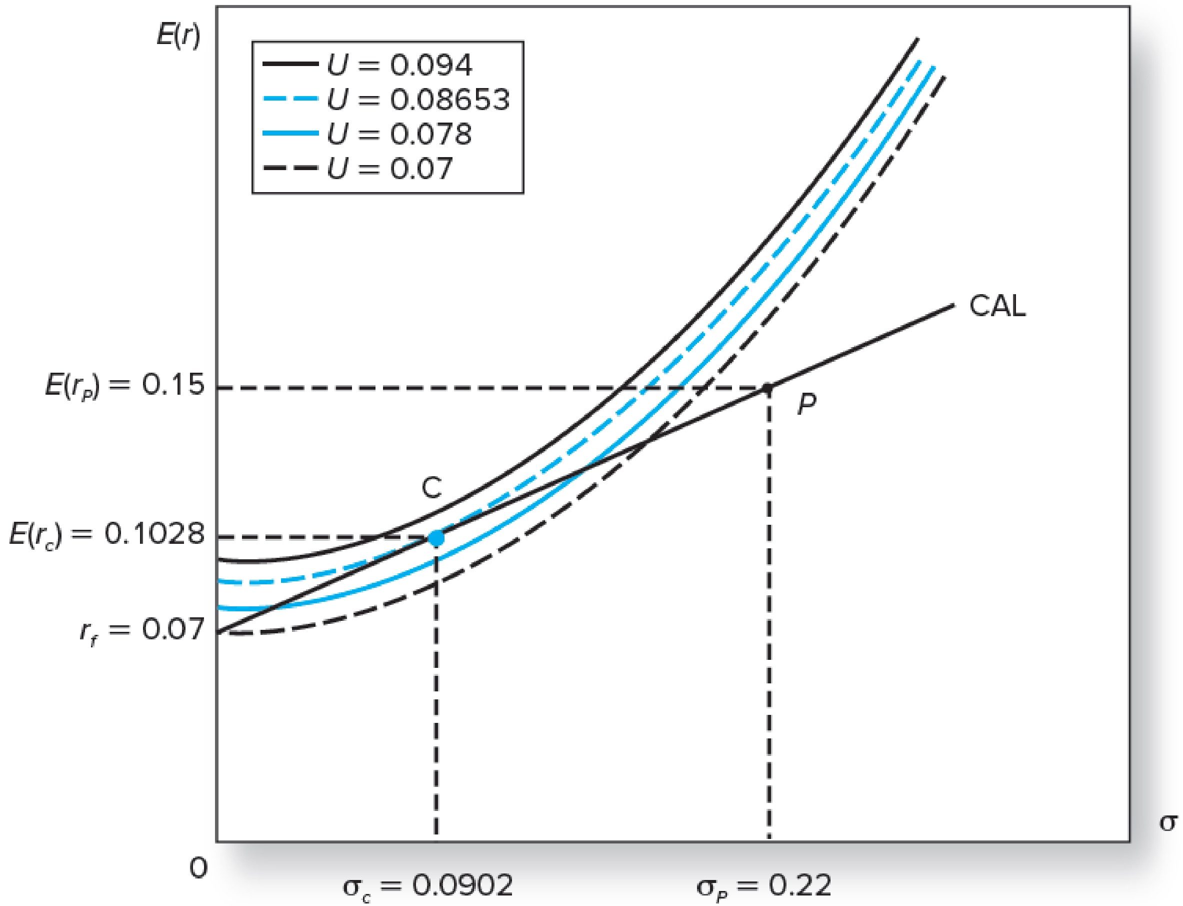
*Solve for y**

Calculations of Indifference Curves

σ	A = 2		A = 4	
	U = 0.05	U = 0.09	U = 0.05	U = 0.09
0	0.0500	0.0900	0.050	0.090
0.05	0.0525	0.0925	0.055	0.095
0.10	0.0600	0.1000	0.070	0.110
0.15	0.0725	0.1125	0.095	0.135
0.20	0.0900	0.1300	0.130	0.170
0.25	0.1125	0.1525	0.175	0.215
0.30	0.1400	0.1800	0.230	0.270
0.35	0.1725	0.2125	0.295	0.335
0.40	0.2100	0.2500	0.370	0.410
0.45	0.2525	0.2925	0.455	0.495
0.50	0.3000	0.3400	0.550	0.590



Finding the Optimal Complete Portfolio



Expected Returns on Four Indifference Curves and the CAL

σ	$U = 0.07$	$U = 0.078$	$U = 0.08653$	$U = 0.094$	CAL
0	0.0700	0.0780	0.0865	0.0940	0.0700
0.02	0.0708	0.0788	0.0873	0.0948	0.0773
0.04	0.0732	0.0812	0.0897	0.0972	0.0845
0.06	0.0772	0.0852	0.0937	0.1012	0.0918
0.08	0.0828	0.0908	0.0993	0.1068	0.0991
0.0902	0.0863	0.0943	0.1028	0.1103	0.1028
0.10	0.0900	0.0980	0.1065	0.1140	0.1064
0.12	0.0988	0.1068	0.1153	0.1228	0.1136
0.14	0.1092	0.1172	0.1257	0.1332	0.1209
0.18	0.1348	0.1428	0.1513	0.1588	0.1355
0.22	0.1668	0.1748	0.1833	0.1908	0.1500
0.26	0.2052	0.2132	0.2217	0.2292	0.1645
0.30	0.2500	0.2580	0.2665	0.2740	0.1791

Table 6.6

Expected returns on four indifference curves and the CAL (Investor's risk aversion is $A = 4$.)

Note to the model

- Above analysis implicitly assumes normality
- Passive Strategies: The Capital Market Line (CAL)
    - Supply/demand forces may make this strategy reasonable for many investors
    - A natural candidate for a passively held risky asset would be the capital market index
    - Is a capital allocation line formed investment in two passive portfolios:
        1. Virtually risk-free short-term T-bills (or a money market fund)
        2. Fund of common stocks that mimics a broad market index

Use these inputs for Problems 13 through 19: You manage a risky portfolio with an expected rate of return of 18% and a standard deviation of 28%. The T-bill rate is 8%.

13. Your client chooses to invest 70% of a portfolio in your fund and 30% in an essentially risk-free money market fund. What is the expected value and standard deviation of the rate of return on his portfolio?

14. Suppose that your risky portfolio includes the following investments in the given proportions:

Stock A	25%
Stock B	32%
Stock C	43%

What are the investment proportions of your client's overall portfolio, including the position in T-bills?

15. What is the reward-to-volatility (Sharpe) ratio (S) of your risky portfolio? Your client's?

16. Draw the CAL of your portfolio on an expected return–standard deviation diagram. What is the slope of the CAL? Show the position of your client on your fund's CAL.

17. Suppose that your client decides to invest in your portfolio a proportion y of the total investment budget so that the overall portfolio will have an expected rate of return of 16%.

    a. What is the proportion y?
    b. What are your client's investment proportions in your three stocks and the T-bill fund?
    c. What is the standard deviation of the rate of return on your client's portfolio?

Use these inputs for Problems 13 through 19: You manage a risky portfolio with an expected rate of return of 18% and a standard deviation of 28%. The T-bill rate is 8%.

18. Suppose that your client prefers to invest in your fund a proportion y that maximizes the expected return on the complete portfolio subject to the constraint that the complete portfolio's standard deviation will not exceed 18%.

    a. What is the investment proportion, y?

    b. What is the expected rate of return on the complete portfolio?

19. Your client's degree of risk aversion is $A = 3.5$.

    a. What proportion, y, of the total investment should be invested in your fund?

    b. What is the expected value and standard deviation of the rate of return on your client's optimized portfolio?

Question?