Cost-Volume-Profit Relationships

Chapter 5

Basics of Cost-Volume-Profit Analysis

The contribution income statement is helpful to managers in judging the impact on profits of changes in selling price, cost, or volume. The emphasis is on cost behavior.

Racing Bicycle Company Contribution Income Statement For the Month of June	
Sales (500 bicycles)	$ 250,000
Less: Variable expenses	150,000
Contribution margin	100,000
Less: Fixed expenses	80,000
Net operating income	$ 20,000

Contribution Margin (CM) is the amount remaining from sales revenue after variable expenses have been deducted.

CM is used first to cover fixed expenses. Any remaining CM contributes to net operating income.

The Contribution Approach

Sales, variable expenses, and contribution margin can also be expressed on a per unit basis. If Racing sells an additional bicycle, $200 additional CM will be generated to cover fixed expenses and profit.

Racing Bicycle Company Contribution Income Statement For the Month of June		
	Total	Per Unit
Sales (500 bicycles)	$ 250,000	$ 500
Less: Variable expenses	150,000	300
Contribution margin	100,000	$ 200
Less: Fixed expenses	80,000	
Net operating income	$ 20,000	

The Contribution Approach

Each month, RBC must generate at least $80,000 in total contribution margin to break-even (which is the level of sales at which profit is zero).

Racing Bicycle Company Contribution Income Statement For the Month of June		
	Total	Per Unit
Sales (500 bicycles)	$ 250,000	$ 500
Less: Variable expenses	150,000	300
Contribution margin	100,000	$ 200
Less: Fixed expenses	80,000	
Net operating income	$ 20,000	

The Contribution Approach

If RBC sells 400 units in a month, it will be operating at the *break-even point*.

Racing Bicycle Company Contribution Income Statement For the Month of June	Total	Per Unit
Sales (400 bicycles)	$ 200,000	$ 500
Less: Variable expenses	120,000	300
Contribution margin	80,000	$ 200
Less: Fixed expenses	80,000	
Net operating income	$ -	

The Contribution Approach

If RBC sells one more bike (401 bikes), net operating income will increase by *$200*.

Racing Bicycle Company Contribution Income Statement For the Month of June	Total	Per Unit
Sales (401 bicycles)	$ 200,500	$ 500
Less: Variable expenses	120,300	300
Contribution margin	80,200	$ 200
Less: Fixed expenses	80,000	
Net operating income	$ 200	

The Contribution Approach

We do not need to prepare an income statement to estimate profits at a particular sales volume. Simply multiply the number of units sold above break-even by the contribution margin per unit.

CVP Relationships in Equation Form

The contribution format income statement can be expressed in the following equation:

Profit = (Sales – Variable expenses) – Fixed expenses

Racing Bicycle Company Contribution Income Statement For the Month of June	Total	Per Unit
Sales (401 bicycles)	$ 200,500	$ 500
Less: Variable expenses	120,300	300
Contribution margin	80,200	$ 200
Less: Fixed expenses	80,000	
Net operating income	$ 200	

CVP Relationships in Equation Form

This equation can be used to show the profit RBC earns if it sells 401. Notice, the answer of $200 mirrors our earlier solution.

Profit = (Sales – Variable expenses) – Fixed expenses

401 units × $500

401 units × $300

$80,000

$200 = ($200,500 – $120,300) – $80,000

CVP Relationships in Equation Form

When a company has only one product we can further refine this equation as shown on this slide.

Profit = (Sales – Variable expenses) – Fixed expenses

Quantity sold (Q)
× Selling price per unit (P)
= Sales (Q × P)

Quantity sold (Q)
× Variable expenses per unit (V)
= Variable expenses (Q × V)

Profit = (P × Q – V × Q) – Fixed expenses

CVP Relationships in Equation Form

This equation can also be used to show the $200 profit RBC earns if it sells 401 bikes.

Profit = (Sales – Variable expenses) – Fixed expenses

Profit = (P × Q – V × Q) – Fixed expenses

$200 = ($500 × 401 – $300 × 401) – $80,000

CVP Relationships in Equation Form

It is often useful to express the simple profit equation in terms of the unit contribution margin (Unit CM) as follows:

Unit CM = Selling price per unit – Variable expenses per unit
Unit CM = P – V

Profit = (P × Q – V × Q) – Fixed expenses
Profit = (P – V) × Q – Fixed expenses
Profit = Unit CM × Q – Fixed expenses

CVP Relationships in Equation Form

Profit = (P × Q – V × Q) – Fixed expenses
Profit = (P – V) × Q – Fixed expenses
Profit = Unit CM × Q – Fixed expenses

Profit = ($500 – $300) × 401 – $80,000
Profit = $200 × 401 – $80,000
Profit = $80,200 – $80,000
Profit = $200

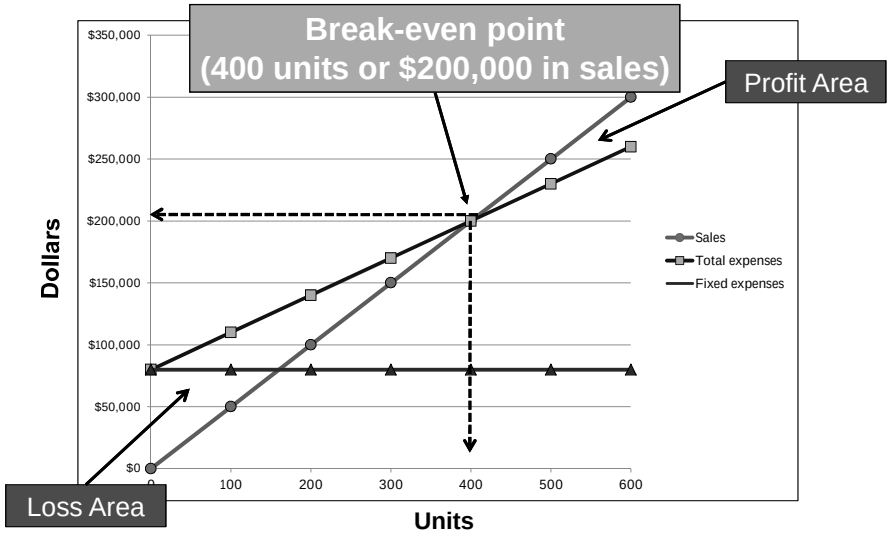
This equation can also be used to compute RBC's $200 profit if it sells 401 bikes.

CVP Relationships in Graphic Form

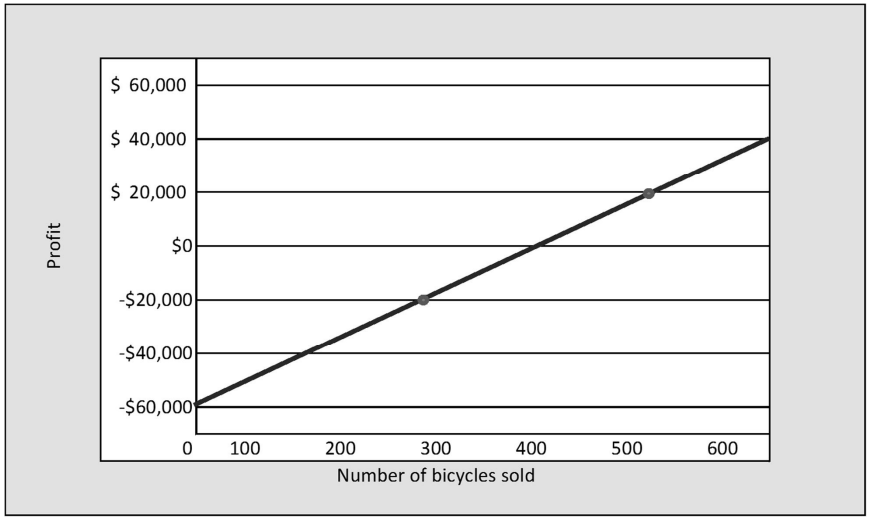
The relationships among revenue, cost, profit, and volume can be expressed graphically by preparing a CVP graph. Racing Bicycle developed contribution margin income statements at 0, 200, 400, and 600 units sold. We will use this information to prepare the CVP graph.

	Units Sold			
	0	200	400	600
Sales	$ -	$ 100,000	$ 200,000	$ 300,000
Total variable expenses	-	60,000	120,000	180,000
Contribution margin	-	40,000	80,000	120,000
Fixed expenses	80,000	80,000	80,000	80,000
Net operating income (loss)	$ (80,000)	$ (40,000)	$ -	$ 40,000

Preparing the CVP Graph

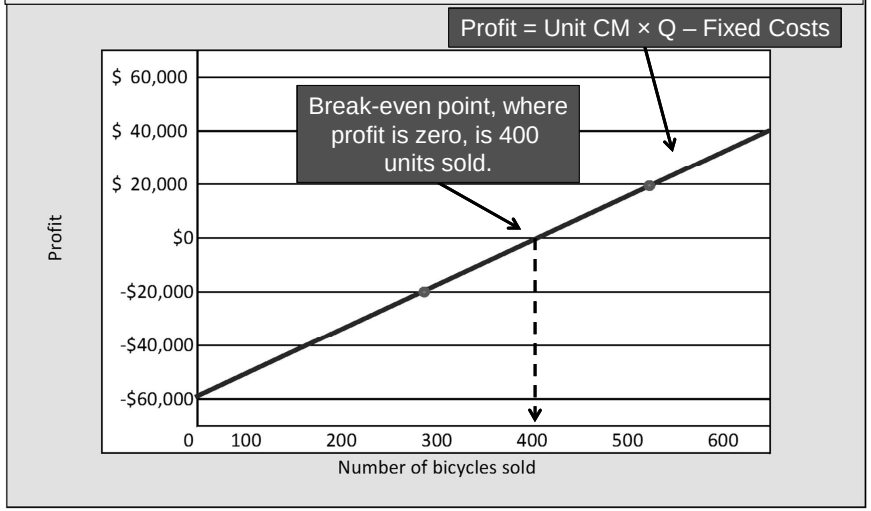


Preparing the CVP Graph



Preparing the CVP Graph

An even simpler form of the CVP graph is called the profit graph.



Contribution Margin Ratio (CM Ratio)

The CM ratio is calculated by dividing the total contribution margin by total sales.

Racing Bicycle Company
Contribution Income Statement
For the Month of June

	Total	Per Unit	CM Ratio
Sales (500 bicycles)	$ 250,000	$ 500	100%
Less: Variable expenses	150,000	300	60%
Contribution margin	100,000	$ 200	40%
Less: Fixed expenses	80,000		
Net operating income	$ 20,000		

$100,000 ÷ $250,000 = 40%

Contribution Margin Ratio (CM Ratio)

The contribution margin ratio at Racing Bicycle is:

$$\text{CM Ratio} = \frac{\text{CM per unit}}{\text{SP per unit}} = \frac{\$200}{\$500} = 40\%$$

The CM ratio can also be calculated by dividing the contribution margin per unit by the selling price per unit.

Contribution Margin Ratio (CM Ratio)

If Racing Bicycle increases sales from 400 to 500 bikes ($50,000), contribution margin will increase by $20,000 ($50,000 × 40%). Here is the proof:

	400 Units	500 Units
Sales	$ 200,000	$ 250,000
Less: variable expenses	120,000	150,000
Contribution margin	80,000	100,000
Less: fixed expenses	80,000	80,000
Net operating income	$ -	$ 20,000

A $50,000 increase in sales revenue results in a $20,000 increase in CM ($50,000 × 40% = $20,000).

Contribution Margin Ratio (CM Ratio)

The relationship between profit and the CM ratio can be expressed using the following equation:

Profit = (CM ratio × Sales) – Fixed expenses

If Racing Bicycle increased its sales volume to 500 bikes, what would management expect profit or net operating income to be?

Profit = (40% × $250,000) – $80,000
Profit = $100,000 – $80,000
Profit = $20,000

The Variable Expense Ratio

The variable expense ratio is the ratio of variable expenses to sales. It can be computed by dividing the total variable expenses by the total sales, or in a single product analysis, it can be computed by dividing the variable expenses per unit by the unit selling price.

Racing Bicycle Company
Contribution Income Statement
For the Month of June

	Total	Per Unit	CM Ratio
Sales (500 bicycles)	$ 250,000	$ 500	100%
Less: Variable expenses	150,000	300	60%
Contribution margin	100,000	$ 200	40%
Less: Fixed expenses	80,000		
Net operating income	$ 20,000		

Changes in Fixed Costs and Sales Volume

What is the profit impact if Racing Bicycle can increase unit sales from 500 to 540 by increasing the monthly advertising budget by $10,000?

Changes in Fixed Costs and Sales Volume

$80,000 + $10,000 advertising = $90,000

	500 units	540 units
Sales	$ 250,000	$ 270,000
Less: Variable expenses	150,000	162,000
Contribution margin	100,000	108,000
Less: Fixed expenses	80,000	90,000
Net operating income	$ 20,000	$ 18,000

Sales *increased* by $20,000, but net operating income *decreased* by $2,000.

Changes in Fixed Costs and Sales Volume

A shortcut solution using incremental analysis

Increase in CM (40 units X $200)	$ 8,000
Increase in advertising expenses	10,000
Decrease in net operating income	$ (2,000)

Change in Variable Costs and Sales Volume

What is the profit impact if Racing Bicycle can use higher quality raw materials, thus increasing variable costs per unit by $10, to generate an increase in unit sales from 500 to 580?

Change in Variable Costs and Sales Volume

580 units × $310 variable cost/unit = $179,800

	500 units	580 units
Sales	$ 250,000	$ 290,000
Less: Variable expenses	150,000	179,800
Contribution margin	100,000	110,200
Less: Fixed expenses	80,000	80,000
Net operating income	$ 20,000	$ 30,200

Sales *increase* by $40,000 and net operating income *increases* by $10,200.

Change in Fixed Cost, Sales Price, and Volume

What is the profit impact if RBC: (1) cuts its selling price $20 per unit, (2) increases its advertising budget by $15,000 per month, and (3) increases sales from 500 to 650 units per month?

Change in Fixed Cost, Sales Price, and Volume

650 units × $480 = $312,000

	500 units	650 units
Sales	$ 250,000	$ 312,000
Less: Variable expenses	150,000	195,000
Contribution margin	100,000	117,000
Less: Fixed expenses	80,000	95,000
Net operating income	$ 20,000	$ 22,000

Sales *increase* by $62,000, fixed costs increase by $15,000, and net operating income *increases* by $2,000.

Change in Variable Cost, Fixed Cost, and Sales Volume

What is the profit impact if RBC: (1) pays a $15 sales commission per bike sold instead of paying salespersons flat salaries that currently total $6,000 per month, and (2) increases unit sales from 500 to 575 bikes?

Change in Variable Cost, Fixed Cost, and Sales Volume

575 units × $315 = $181,125

	500 units	575 units
Sales	$ 250,000	$ 287,500
Less: Variable expenses	150,000	181,125
Contribution margin	100,000	106,375
Less: Fixed expenses	80,000	74,000
Net operating income	$ 20,000	$ 32,375

Sales *increase* by $37,500, fixed expenses *decrease* by $6,000, and net operating income *increases* by $12,375.

Change in Regular Sales Price

If RBC has an opportunity to sell 150 bikes to a wholesaler without disturbing sales to other customers or fixed expenses, what price would it quote to the wholesaler if it wants to increase monthly profits by $3,000?

Change in Regular Sales Price

$ 3,000 ÷ 150 bikes	=	$ 20 per bike
Variable cost per bike	=	300 per bike
Selling price required	=	$ 320 per bike

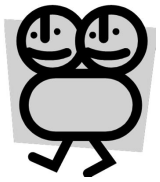
150 bikes × $320 per bike	=	$ 48,000
Total variable costs	=	45,000
Increase in net operating income	=	$ 3,000

Target Profit Analysis

We can compute the number of units that must be sold to attain a target profit using either:

(1) Equation method, or

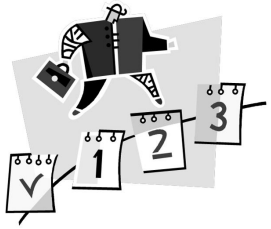
(2) Formula method.



Equation Method

Profit = Unit CM × Q – Fixed expenses

Our goal is to solve for the unknown "Q" which represents the quantity of units that must be sold to attain the target profit.



Target Profit Analysis

Suppose RBC's management wants to know how many bikes must be sold to earn a target profit of $100,000.

Profit = Unit CM × Q – Fixed expenses

$100,000 = $200 × Q – $80,000

$200 × Q = $100,000 – $80,000

Q = ($100,000 + $80,000) ÷ $200

Q = 900

The Formula Method

The formula uses the following equation.

$$\text{Unit sales to attain the target profit} = \frac{\text{Target profit + Fixed expenses}}{\text{CM per unit}}$$

Target Profit Analysis in Terms of Unit Sales

Suppose Racing Bicycle Company wants to know how many bikes must be sold to earn a profit of $100,000.

$$\text{Unit sales to attain the target profit} = \frac{\text{Target profit + Fixed expenses}}{\text{CM per unit}}$$

$$\text{Unit sales} = \frac{\$100{,}000 + \$80{,}000}{\$200}$$

$$\text{Unit sales} = 900$$

Target Profit Analysis

We can also compute the target profit in terms of sales dollars using either the equation method or the formula method.

Equation Method OR Formula Method

Equation Method

Profit = CM ratio × Sales – Fixed expenses

Our goal is to solve for the unknown "Sales," which represents the dollar amount of sales that must be sold to attain the target profit.

Suppose RBC management wants to know the sales volume that must be generated to earn a target profit of $100,000.

$100,000 = 40% × Sales – $80,000
40% × Sales = $100,000 + $80,000
Sales = ($100,000 + $80,000) ÷ 40%
Sales = $450,000

Formula Method

We can calculate the dollar sales needed to attain a target profit (net operating profit) of $100,000 at Racing Bicycle.

$$\text{Dollar sales to attain the target profit} = \frac{\text{Target profit + Fixed expenses}}{\text{CM ratio}}$$

$$\text{Dollar sales} = \frac{\$100{,}000 + \$80{,}000}{40\%}$$

$$\text{Dollar sales} = \$450{,}000$$

Break-even Analysis

The equation and formula methods can be used to determine the unit sales and dollar sales needed to achieve a target profit of zero. Let's use the RBC information to complete the break-even analysis.

Racing Bicycle Company
Contribution Income Statement
For the Month of June

	Total	Per Unit	CM Ratio
Sales (500 bicycles)	$ 250,000	$ 500	100%
Less: Variable expenses	150,000	300	60%
Contribution margin	100,000	$ 200	40%
Less: Fixed expenses	80,000		
Net operating income	$ 20,000		

Break-even in Unit Sales: Equation Method

Profits = Unit CM × Q – Fixed expenses

Suppose RBC wants to know how many bikes must be sold to break-even (earn a target profit of $0).

$0 = $200 × Q + $80,000

Profits are zero at the break-even point.

Break-even in Unit Sales: Equation Method

Profits = Unit CM × Q – Fixed expenses

$0 = $200 × Q + $80,000

$200 × Q = $80,000

Q = 400 bikes

Break-even in Unit Sales: Formula Method

Let's apply the formula method to solve for the break-even point.

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{CM per unit}}$$

$$\text{Unit sales} = \frac{\$80{,}000}{\$200}$$

$$\text{Unit sales} = 400$$

Break-even in Dollar Sales: Equation Method

Suppose Racing Bicycle wants to compute the sales dollars required to break-even (earn a target profit of $0). Let's use the equation method to solve this problem.

Profit = CM ratio × Sales – Fixed expenses

Solve for the unknown "Sales."

Break-even in Dollar Sales: Equation Method

Profit = CM ratio × Sales – Fixed expenses

$ 0 = 40% × Sales – $80,000

40% × Sales = $80,000

Sales = $80,000 ÷ 40%

Sales = $200,000

Break-even in Dollar Sales: Formula Method

Now, let's use the formula method to calculate the dollar sales at the break-even point.

$$\text{Dollar sales to break even} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$\text{Dollar sales} = \frac{\$80{,}000}{40\%}$$

$$\text{Dollar sales} = \$200{,}000$$

The Margin of Safety in Dollars

The margin of safety in dollars is the excess of budgeted (or actual) sales over the break-even volume of sales.

Margin of safety in dollars = Total sales - Break-even sales

Let's look at Racing Bicycle Company and determine the margin of safety.

The Margin of Safety in Dollars

If we assume that RBC has actual sales of $250,000, given that we have already determined the break-even sales to be $200,000, the margin of safety is $50,000 as shown.

	Break-even sales 400 units	Actual sales 500 units
Sales	$ 200,000	$ 250,000
Less: variable expenses	120,000	150,000
Contribution margin	80,000	100,000
Less: fixed expenses	80,000	80,000
Net operating income	$ -	$ 20,000

The Margin of Safety Percentage

RBC's margin of safety can be expressed as **20%** of sales.
($50,000 ÷ $250,000)

	Break-even sales 400 units	Actual sales 500 units
Sales	$ 200,000	$ 250,000
Less: variable expenses	120,000	150,000
Contribution margin	80,000	100,000
Less: fixed expenses	80,000	80,000
Net operating income	$ -	$ 20,000

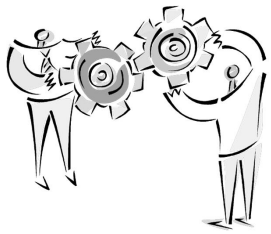
The Margin of Safety

The margin of safety can be expressed in terms of the number of units sold. The margin of safety at RBC is $50,000, and each bike sells for $500; hence, RBC's margin of safety is 100 bikes.

$$\text{Margin of Safety in units} = \frac{\$50{,}000}{\$500} = 100 \text{ bikes}$$

Cost Structure and Profit Stability

Cost structure refers to the relative proportion of fixed and variable costs in an organization. Managers often have some latitude in determining their organization's cost structure.



Cost Structure and Profit Stability

There are advantages and disadvantages to high fixed cost (or low variable cost) and low fixed cost (or high variable cost) structures.

An advantage of a high fixed cost structure is that income will be higher in good years compared to companies with lower proportion of fixed costs.

A disadvantage of a high fixed cost structure is that income will be lower in bad years compared to companies with lower proportion of fixed costs.

Companies with low fixed cost structures enjoy greater stability in income across good and bad years.

Operating Leverage

Operating leverage is a measure of how sensitive net operating income is to percentage changes in sales. It is a measure, at any given level of sales, of how a percentage change in sales volume will affect profits.

$$\text{Degree of operating leverage} = \frac{\text{Contribution margin}}{\text{Net operating income}}$$

Operating Leverage

To illustrate, let's revisit the contribution income statement for RBC.

	Actual sales 500 Bikes
Sales	$ 250,000
Less: variable expenses	150,000
Contribution margin	100,000
Less: fixed expenses	80,000
Net income	$ 20,000

$$\text{Degree of Operating Leverage} = \frac{\$100,000}{\$20,000} = 5$$

Operating Leverage

With an operating leverage of 5, if RBC increases its sales by 10%, net operating income would increase by 50%.

Percent increase in sales		10%
Degree of operating leverage	×	5
Percent increase in profits		50%

Here's the verification!

Operating Leverage

	Actual sales (500)	Increased sales (550)
Sales	$ 250,000	$ 275,000
Less variable expenses	150,000	165,000
Contribution margin	100,000	110,000
Less fixed expenses	80,000	80,000
Net operating income	$ 20,000	$ 30,000

10% increase in sales from $250,000 to $275,000 . . .

. . . results in a 50% increase in income from $20,000 to $30,000.

Verify Increase in Profit

	Actual sales	*Increased sales*
	2,100 cups	*2,520 cups*
Sales	$ 3,129	$ 3,755
Less: Variable expenses	756	907
Contribution margin	2,373	2,848
Less: Fixed expenses	1,300	1,300
Net operating income	$ 1,073	$ 1,548
% change in sales		20.0%
% change in net operating income		44.2%

Structuring Sales Commissions

Companies generally compensate salespeople by paying them either a commission based on sales or a salary plus a sales commission. Commissions based on sales dollars can lead to *lower profits* in a company.

Let's look at an example.

Structuring Sales Commissions

Pipeline Unlimited produces two types of surfboards, the XR7 and the Turbo. The XR7 sells for $100 and generates a contribution margin per unit of $25. The Turbo sells for $150 and earns a contribution margin per unit of $18.

The sales force at Pipeline Unlimited is compensated based on sales commissions.

Structuring Sales Commissions

If you were on the sales force at Pipeline, you would push hard to sell the Turbo even though the XR7 earns a higher contribution margin per unit.

To eliminate this type of conflict, commissions can be based on contribution margin rather than on selling price alone.

The Concept of Sales Mix

- Sales mix is the relative proportion in which a company's products are sold.
- Different products have different selling prices, cost structures, and contribution margins.
- When a company sells more than one product, break-even analysis becomes more complex as the following example illustrates.

Let's assume Racing Bicycle Company sells bikes and carts and that the sales mix between the two products remains the same.

Multi-Product Break-Even Analysis

Bikes comprise 45% of RBC's total sales revenue and the carts comprise the remaining 55%. RBC provides the following information:

	Bicycle		Carts		Total	
Sales	$ 250,000	100%	$ 300,000	100%	$ 550,000	100.0%
Variable expenses	150,000	60%	135,000	45%	285,000	51.8%
Contribution margin	100,000	40.0%	165,000	55%	265,000	48.2%
Fixed expenses					170,000	
Net operating income					$ 95,000	
Sales mix	$ 250,000	45%	$ 300,000	55%	$ 550,000	100%

$$\frac{\$265{,}000}{\$550{,}000} = 48.2\% \text{ (rounded)}$$

Multi-Product Break-Even Analysis

$$\text{Dollar sales to break even} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$\text{Dollar sales to break even} = \frac{\$170{,}000}{48.2\%} = \$352{,}697$$

	Bicycle		Carts		Total	
Sales	$ 158,714	100%	$ 193,983	100%	$ 352,697	100.0%
Variable expenses	95,228	60%	87,293	45%	182,521	51.8%
Contribution margin	63,485	40%	106,691	55%	170,176	48.2%
Fixed expenses					170,000	
Net operating income			Rounding error →		$ 176	
Sales mix	$ 158,714	45%	$ 193,983	55%	$ 352,697	100.0%

Key Assumptions of CVP Analysis

1. Selling price is constant.
2. Costs are linear and can be accurately divided into variable (constant per unit) and fixed (constant in total) elements.
3. In multiproduct companies, the sales mix is constant.
4. In manufacturing companies, inventories do not change (units produced = units sold).

End of Chapter 5