# H.W.

(at same time)

1) Find cournot equlibrium when there are 3 firms in the market

$P = a - bQ$ ; $Q = q_1 + q_2 + q_3$ ⟶ $P = a - bq_1 - bq_2 - bq_3$

$C_1 = C_2 = C_3 = C$ ⟶ $C = C_1 = C_2 = C_3 = \text{constant}$

what is equlibrium price? : $P^*$

what are firms' profit? : $\Pi_1 = \Pi_2 = \Pi_3 = ?$

$\Pi_1 = TR_1 - TC_1 = (a - bq_1 - bq_2 - bq_3)q_1 - C_1$

$\frac{\partial \Pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 = 0$

$a - bq_2 - bq_3 = 2bq_1$

$\frac{a}{2b} - 0.5q_2 - 0.5q_3 = q_1$ : Best Reponse fw(1)

$\Pi_2 = TR_2 - TC_2 = (a - bq_1 - bq_2 - bq_3)q_2 - C_2$

$\frac{\partial \Pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - bq_3 = 0$

$a - bq_1 - bq_3 = 2bq_2$

$\frac{a}{2b} - 0.5q_1 - 0.5q_3 = q_2$ : Best Reponse fw(2)

$\Pi = TR_3 - TC_3 = (a - bq_1 - bq_2 - bq_3)q_3 - C_3$

$\frac{\partial \Pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 = 0$

$a - bq_1 - bq_2 = 2bq_3$

$\frac{a}{2b} - 0.5q_1 - 0.5q_2 = q_3$ : Best Reponse fw(3)

$q_3$ in $q_1$

$q_1 = \frac{a}{2b} - 0.5q_2 - 0.5\left(\frac{a}{2b} - 0.5q_1 - 0.5q_2\right)$

$q_1 = \frac{a}{4b} + 0.25q_1 - 0.25q_2$

$0.75q_1 = \frac{a}{4b} - 0.25q_2 = 0.25\frac{a}{b} - 0.25q_2$

$q_1 = \frac{1}{3}\left(\frac{a}{b} - q_2\right)$

$q_3$ in $q_2$

$q_2 = \frac{a}{2b} - 0.5q_1 - 0.5\left(\frac{a}{2b} - 0.5q_1 - 0.5q_2\right)$

$q_2 = 0.25\frac{a}{b} - 0.25q_1 + 0.25q_2$

$0.75q_2 = 0.25\left(\frac{a}{b} - q_1\right)$

$q_2 = \frac{1}{3}\left(\frac{a}{b} - q_1\right)$

$q_2$ in $q_1$

$$q_1 = \frac{1}{3}\left\{\frac{a}{b} - \left[\frac{1}{3}\left(\frac{a}{b} - q_1\right)\right]\right\} = \frac{1}{3}\left(\frac{a}{b} - \frac{a}{3b} + \frac{q_1}{3}\right)$$

$$q_1 = \frac{a}{3b} - \frac{a}{9b} + \frac{q_1}{9}$$

$$9q_1 = \frac{3a - a}{b} + q_1$$

$$8q_1 = \frac{2a}{b}$$

$$q_1 = \frac{a}{4b} \quad //$$

$$q_2 = \frac{a}{3b} - \frac{a}{12b}$$

$$12q_2 = \frac{4a}{b} - \frac{a}{b}$$

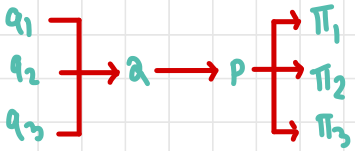
$$q_2 = \frac{a}{4b} \quad //$$

$$q_3 = \frac{a}{2b} - 0.5\frac{a}{4b} - 0.5\frac{a}{4b}$$

$$q_3 = \frac{a}{4b} \quad //$$

$$\therefore P = a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$$

$$P = a - \frac{3}{4}a$$




$$P = 0.25a$$

$$\pi_1 = P \cdot q_1 - C_1 = 0.25a \cdot \frac{a}{4b} - C_1 = \frac{a^2}{16b} - C_1$$

$$\pi_2 = P \cdot q_2 - C_2 = 0.25a \cdot \frac{a}{4b} - C_2 = \frac{a^2}{16b} - C_2 \quad //$$

$$\pi_3 = P \cdot q_3 - C_3 = 0.25a \cdot \frac{a}{4b} - C_3 = \frac{a^2}{16b} - C_3 \quad //$$

2. If there are N firm

$q_i^* = f(N)$, $P = (N)$, $\pi_i = f(N)$

$P = a - bq$ ; $q = q_N$

$C_N = C_N = C_M = C$

$\pi_i = P(Q)q_i - C_i q_i$

$= (a - b(q_N))q_i - C_i q_i$

Assume : $q_1 + q_2 + q_3 + \dots + q_N = A$

$P = a - b(q_1 + q_2 + \dots q_N)$

$P = a - bq_1 - bq_2 - \dots - bq_N$

$\pi_1 = (a - bq_1 - bq_2 - \dots - bq_n) \cdot q_1 - C_1$

$\pi_N = (a - bq_1 - bq_2 - \dots bq_N) \cdot q_N - C_n$

$\frac{\partial \pi_i}{\partial \pi_N} = a - 2bq_1 - bq_2 - \dots - bq_N = 0$

$\frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_N) = q_1$

$\frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_{N-1}) = q_N$

rearrange

$q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_N)$

$0.5q_1 = \frac{a}{2b} - 0.5A$

$q_1 = \frac{a}{b} - A$

$q_2 = \frac{a}{b} - A$

$\vdots$

$q_n = \frac{a}{b} - A$

$q_1 + q_2 + \dots q_N \Rightarrow A = n\left(\frac{a}{b} - A\right)$

$A = n\frac{a}{b} - nA$

$(n+1)A = n\frac{a}{b}$

$* A = Nq$

$A = \frac{nu}{q(1+u)}$

$q_i = \frac{q(1+u)}{nu}$ //

$\therefore P = a - b(A) = a - b\left(\frac{q(1+u)}{eu}\right)$

$P = a - \frac{1+u}{n+1}a = \frac{eu - e(1+u)}{n+1} = \frac{eu - e + eu}{n+1}$

$P = \frac{a}{n+1}$ //

$\pi_i = P \cdot q_i - C_i = \frac{a}{n+1} \cdot \frac{q(1+u)}{eu} - C_i$

$\pi_i = \frac{a^2}{(n+1)^2 b} - C_i$ //

3. From $q_2$, what happen if $N \to \infty$, $N = 1$

if $n \to \infty$ : $q_i = \frac{a}{(n+1)b} \to 0$ # each firm will near 0 unit

: $A = nq_i \longrightarrow \infty$ # every firm combine at A and very close to $\infty$ unit

: $P = \frac{a}{n+1} \longrightarrow 0$ # supply ↑ ⇒ P↓ near 0

: $\frac{a^2}{(n+1)^2 b} - C_i \longrightarrow C_i$ # profit in each term will loss and negative
fixed cost = C

if $n \to 1$ : $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ # monopoly will decrease $a = \frac{a}{2b} < a = \frac{m}{(n+1)b}$

: $A = nq_i = a$ # $n = 1$ ∴ firm is monopoly

: $P = \frac{a}{n+1} = \frac{a}{2}$ # $P_m = \frac{a}{2} > P = \frac{a}{n+1}$

: $\frac{a^2}{(n+1)^2 b} - C_i = \frac{a^2}{4b} - C_i$ # $\pi_m > \pi_i = \frac{a^2}{(n+1)^2 b} - C_i$

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