

## Assignment 2 Simultaneous Equations Model

Due: 1/9/2020

### Demand and Supply Equations

$$\ln S_t = \beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t} \quad (1)$$

$$\ln D_t = \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t} \quad (2)$$

where:  $S_t$  = Domestic Supply at time  $t$

$D_t$  = Domestic Demand at time  $t$

$P_{Dt}$  = Domestic Price at time  $t = P_{Mt} + T_t$

$T_t$  = Tariff at time  $t$

$P_{X2t}$  = Price of Input 2 at time  $t$

$P_{X3t}$  = Price of Input 3 at time  $t$

$P_{X4t}$  = Price of Input 4 at time  $t$

$GDP_t$  = Gross Domestic Product (Representing Income) at time  $t$

Endogenous variables in this system include  $S_t$ ,  $D_t$ , and  $P_{Dt}$

Exogenous variables in this system include  $P_{X2t}$ ,  $P_{X3t}$ ,  $P_{X4t}$ , and  $GDP_t$

From Data Assignment 2.dta:

1. State reduce form model of these system models.
2. Estimate reduce form model using OLS and prediction of the endogenous variables.
3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.
4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and 4SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?
5. What do  $\beta_{21}$  and  $\beta_{22}$  mean?

# 1. Structural form

$$\ln S_t = \beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t}$$

$$\ln D_t = \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t}$$

## Reduce form

$$\ln S_t = \pi_{10} + \pi_{11} \ln P_{X2t} + \pi_{12} \ln P_{X3t} + \pi_{13} \ln P_{X4t} + \pi_{14} \ln GDP_t + u_{1t}$$

$$\ln D_t = \pi_{20} + \pi_{21} \ln P_{X2t} + \pi_{22} \ln P_{X3t} + \pi_{23} \ln P_{X4t} + \pi_{24} \ln GDP_t + u_{2t}$$

# 2.

. g lndt = ln(dt)

. g pd = pm+t

. g lnpd = ln(pd)

. g lnpx2 = ln(px2)

. g lnpx3 = ln(px3)

. g lnpx4 = ln(px4)

. g lngdp = ln(gdp)

. reg lnst lnpx2 lnpx3 lnpx4 lngdp

Source	SS	df	MS	Number of obs	=	22
Model	4.64569724	4	1.16142431	F(4, 17)	=	37.32
Residual	.529104674	17	.031123804	Prob > F	=	0.0000
Total	5.17480192	21	.246419139	R-squared	=	0.8978
				Adj R-squared	=	0.8737
				Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lnpx2	-.4503744	.1515961	-2.97	0.009	-.7702142    -.1305347
lnpx3	-.9242052	.2783356	-3.32	0.004	-1.511442    -.3369685
lnpx4	-.3883793	.4222332	-0.92	0.371	-1.279214    .5024549
lngdp	.3438812	.1913463	1.80	0.090	-.0598242    .7475865
_cons	24.65741	5.309757	4.64	0.000	13.4548    35.86002

```
. predict lnsthat
(option xb assumed; fitted values)
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```
. reg lndt lnp2 lnp3 lnp4 lngdp
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Source	SS	df	MS	Number of obs	=	22
Model	3.4026552	4	.850663799	F(4, 17)	=	26.43
Residual	.54721789	17	.032189288	Prob > F	=	0.0000
				R-squared	=	0.8615
				Adj R-squared	=	0.8289
Total	3.94987309	21	.188089195	Root MSE	=	.17941

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lnp2	-.4887365	.1541691	-3.17	0.006	-.8140049 -.1634682
lnp3	-.7243134	.2830597	-2.56	0.020	-1.321517 -.1271097
lnp4	-.577921	.4293997	-1.35	0.196	-1.483875 .3280333
lngdp	.1265855	.194594	0.65	0.524	-.2839719 .5371429
_cons	27.18614	5.399879	5.03	0.000	15.79339 38.57889

```
. predict lndthat
(option xb assumed; fitted values)
```

```
. reg lnpd lnp2 lnp3 lnp4 lngdp
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Source	SS	df	MS	Number of obs	=	22
Model	.17707359	4	.044268398	F(4, 17)	=	6.76
Residual	.111247189	17	.006543952	Prob > F	=	0.0019
				R-squared	=	0.6142
				Adj R-squared	=	0.5234
Total	.288320779	21	.013729561	Root MSE	=	.08089

lnpd	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lnp2	.1318015	.0695123	1.90	0.075	-.0148567 .2784596
lnp3	.0939842	.127627	0.74	0.472	-.1752851 .3632535
lnp4	.4939641	.1936093	2.55	0.021	.0854842 .9024439
lngdp	.1632779	.0877392	1.86	0.080	-.0218357 .3483914
_cons	2.87652	2.434717	1.18	0.254	-2.260283 8.013322

```
. predict lnpdhat
(option xb assumed; fitted values)
```

3.

. reg lnst lnpdhat lnpx2 lnpx3 lnpx4

Source	SS	df	MS	Number of obs	=	22
Model	4.64569773	4	1.16142443	F(4, 17)	=	37.32
Residual	.529104183	17	.031123775	Prob > F	=	0.0000
Total	5.17480192	21	.246419139	R-squared	=	0.8978
				Adj R-squared	=	0.8737
				Root MSE	=	.17642

  

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879 4.578612
lnpx2	-.727963	.1840856	-3.95	0.001	-1.11635 -.3395762
lnpx3	-1.122146	.2824139	-3.97	0.001	-1.717988 -.5263052
lnpx4	-1.428722	.4751381	-3.01	0.008	-2.431176 -.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274 36.63092

. reg lndt lnpdhat lngdp

Source	SS	df	MS	Number of obs	=	22
Model	3.26129847	2	1.63064924	F(2, 19)	=	44.99
Residual	.688574614	19	.036240769	Prob > F	=	0.0000
Total	3.94987309	21	.188089195	R-squared	=	0.8257
				Adj R-squared	=	0.8073
				Root MSE	=	.19037

  

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675 -1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194 .802766
_cons	35.93498	7.189835	5.00	0.000	20.88648 50.98347

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```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (ln dt ln pd lngdp), ols inst (lnpx2 lnpx3 lnpx4 lngdp)
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Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
ln dt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
<b>lnst</b>						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549 - .1961207	
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034 - .1286059	
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736 - .4181034	
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344 .1766516	
_cons	41.4946	3.661911	11.33	0.000	34.0679 48.9213	
<b>ln dt</b>						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008 -1.58365	
lngdp	.5776586	.0887536	6.51	0.000	.397658 .7576593	
_cons	31.03578	3.761201	8.25	0.000	23.40771 38.66385	

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (ln dt ln pd lngdp), 2sls inst (lnpx2 lnpx3 lnpx4 lngdp)
```

Two-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	10.67	0.0000
ln dt	22	2	.1454858	0.8982	77.04	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
<b>lnst</b>						
lnpd	2.10611	2.191774	0.96	0.343	-2.339013 6.551233	
lnpx2	-.7279628	.3442892	-2.11	0.041	-1.426214 -.0297119	
lnpx3	-1.122146	.528189	-2.12	0.041	-2.193363 -.0509293	
lnpx4	-1.428722	.8886357	-1.61	0.117	-3.230959 .3735147	
_cons	18.59914	15.98447	1.16	0.252	-13.81886 51.01715	
<b>ln dt</b>						
lnpd	-2.574157	.4354519	-5.91	0.000	-3.457295 -1.69102	
lngdp	.5212921	.1027745	5.07	0.000	.3128558 .7297283	
_cons	35.93499	5.494663	6.54	0.000	24.7913 47.07868	

Endogenous variables: lnst lnpd ln dt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls inst (lnpx2 lnpx3 lnpx4 lngdp)
```

Three-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.2963642	0.6266	57.47	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
lnst					
lnpd	2.171576	1.926095	1.13	0.260	-1.603501 5.946652
lnpx2	-.7990055	.2985983	-2.68	0.007	-1.384247 -.2137635
lnpx3	-1.329743	.4560002	-2.92	0.004	-2.223487 -.4359989
lnpx4	-1.171403	.775654	-1.51	0.131	-2.691657 .348851
_cons	17.84948	14.04122	1.27	0.204	-9.670808 45.36976
lndt					
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304 -1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951 .708489
_cons	35.93499	5.106302	7.04	0.000	25.92682 45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls ireg3 inst(lnpx2 lnpx3 lnpx4 lngdp)
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```
Iteration 1: tolerance = .1059484
Iteration 2: tolerance = .04569793
Iteration 3: tolerance = .01846611
Iteration 4: tolerance = .00725496
Iteration 5: tolerance = .00281814
Iteration 6: tolerance = .00108981
Iteration 7: tolerance = .00042072
Iteration 8: tolerance = .00016231
Iteration 9: tolerance = .0000626
Iteration 10: tolerance = .00002414
Iteration 11: tolerance = 9.310e-06
Iteration 12: tolerance = 3.590e-06
Iteration 13: tolerance = 1.384e-06
Iteration 14: tolerance = 5.339e-07
```

Three-stage least-squares regression, iterated

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
<b>lnst</b>						
lnpd	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
<b>lndt</b>						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt  
 Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

## Hausman Test

. hausman twostage ols

	Coefficients			
	(b) twostage	(B) ols	(b-B) Difference	sqrt(diag(V_b-V_B)) S.E.
lnpd	2.10611	-1.111835	3.217945	2.144762
lnpx2	-.7279628	-.4189546	-.3090082	.3131123
lnpx3	-1.122146	-.9424196	-.1797266	.4605948
lnpx4	-1.428722	-.521346	-.907376	.8192828

b = consistent under Ho and Ha; obtained from reg3  
 B = inconsistent under Ha, efficient under Ho; obtained from reg3

Test: Ho: difference in coefficients not systematic

$$\begin{aligned} \chi^2(4) &= (b-B)' [(V_b-V_B)^{-1}] (b-B) \\ &= 2.25 \\ \text{Prob}>\chi^2 &= 0.6897 > 0.05 \end{aligned}$$

According to 95% confidence level,  $H_0$  is not rejected. There is no endogeneity bias. Therefore, OLS is more appropriated than alternative models.

5.  $\beta_{21}$  is price elasticity of demand.

It means if domestic price increases by 1, quantity demanded will increase by  $\beta_{21}$ .

$\beta_{22}$  is income elasticity of demand.

It means if GDP increases by 1, quantity demanded will increase by  $\beta_{22}$ .