

Due date: February 8, 2022 before 2.00 pm

Question 1 (60 Points)

Score.....

Consider the individual's portfolio choice problem given in the below equation:

$$\max_A E[U(\tilde{W})] = \max_A E[U(W_0(1+r_f) + A(\tilde{r} - r_f))]$$

Assume the utility of this investor: $U(W) = \ln(W)$ and the rate of return on the risky asset equals

$$\tilde{r} = \begin{cases} 4r_f & \text{with probability } \frac{1}{2} \\ -r_f & \text{with probability } \frac{1}{2} \end{cases}$$

Solve for the individual's proportion of initial wealth invested in the risky asset, $(\frac{A}{W_0})$.

$$\begin{aligned} & (W_0 - A)(1+r_f) + A(1+\tilde{r}) \\ \text{FOC: } & E[U'(\tilde{W})(\tilde{r} - r_f)] = 0 \\ & \frac{1}{2} U'(W^h)(r^h - r_f) + \frac{1}{2} U'(W^l)(r^l - r_f) = 0 \\ & \frac{1}{2} \left[\frac{1}{W_0(1+r_f) + A(4r_f - r_f)} \right] (4r_f - r_f) + \frac{1}{2} \left[\frac{1}{W_0(1+r_f) + A(-r_f - r_f)} \right] (-r_f - r_f) = 0 \\ & \frac{1}{2} \left[\frac{3r_f}{W_0(1+r_f) + A(3r_f)} \right] + \frac{1}{2} \left[\frac{-2r_f}{W_0(1+r_f) - 2r_f A} \right] = 0 \\ & \frac{3}{2} \left[\frac{r_f}{W_0(1+r_f) + 3r_f A} \right] = \frac{r_f}{W_0(1+r_f) - 2r_f A} \\ & \frac{3}{2} (W_0(1+r_f) - 2r_f A) = (W_0(1+r_f) + 3r_f A) \\ & 3W_0 + 3W_0 r_f - 6r_f A = 2W_0 + 2W_0 r_f + 6r_f A \\ & W_0 + W_0 r_f = 12r_f A \\ & W_0(1+r_f) = 12r_f A \\ & \frac{1+r_f}{12r_f} = \frac{A}{W_0} \end{aligned}$$