Example of OLS Estimation

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Example of Estimation by OLS

Relationship between output (Y) and fertilizer (X)

Observation	Y	X
1.	5	0
2.	10	1
3.	12	2
4.	14	3
5.	20	4
6.	23	5

$$\sum_{i=1}^{6} Y_i = 84 \qquad \sum_{i=1}^{6} X_i = 15$$

$$\bar{Y} = 14 \qquad \bar{X} = 2.5$$

Model $Y_i = a + bX_i$

1. Free-hand estimation : Pick any 2 points

1.1 Suppose we pick the 1st and the 6th observations.

$$\left.\begin{array}{ll} 1^{st} & 5 = a + b(0) \\ 6^{th} & 23 = a + b(5) \end{array}\right\} \Rightarrow \begin{array}{l} \hat{a} = 5 \\ \hat{b} = 3.6 \end{array}$$

1.2 Suppose we pick the 2nd and the 5th observations

$$\left.\begin{array}{ll} 2^{nd} & 10 = a + b(1) \\ 5^{th} & 20 = a + b(4) \end{array}\right\} \Rightarrow \begin{array}{l} \hat{a} = 6.667 \\ \hat{b} = 3.333 \end{array}$$

Estimated model (1.1) $\hat{Y}_i = 5 + 3.6 X_i$

Estimated model (1.2) $\hat{Y}_i = 6.667 + 3.333 X_i$

2. Estimation by OLS method

OLS formula: $\hat{a} = \bar{Y} - \hat{b}\bar{X}$

$$\hat{b} = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

x_i	x_i^2	y_i	$x_i y_i$
$X_1 - \bar{X} = -2.5$	6.25	$Y_1 - \bar{Y} = -9$	$x_1 y_1 = 22.5$
$X_2 - \bar{X} = -1.5$	2.25	$Y_2 - \bar{Y} = -4$	$x_2 y_2 = 6$
$X_3 - \bar{X} = -0.5$	0.25	$Y_3 - \bar{Y} = -2$	$x_3 y_3 = 1$
$X_4 - \bar{X} = 0.5$	0.25	$Y_4 - \bar{Y} = 0$	$x_4 y_4 = 0$
$X_5 - \bar{X} = 1.5$	2.25	$Y_5 - \bar{Y} = 6$	$x_5 y_5 = 9$
$X_6 - \bar{X} = 2.5$	6.25	$Y_6 - \bar{Y} = 9$	$x_6 y_6 = 22.5$

$\sum x_i^2 = 17.5$ $\quad\quad$ $\sum x_i y_i = 61$

$$\hat{b} = \frac{61}{17.5} = 3.49$$

$$\hat{a} = 14 - 3.49(2.5) = 5.275$$

The estimated model by OLS method is

$$\hat{Y}_i = 5.275 + 3.49 X_i$$