Lecture 2: Interest Rates

INTEREST RATES

Present Value

- The concept of **present value** (or **present discounted value**) is based on the commonsense notion that a dollar of cash flow paid to you one year from now is less valuable to you than a dollar paid to you today.
- This notion is true because you could invest the dollar in a savings account that earns interest and have more than a dollar in one year.
- The term present value (PV) can be extended to mean the PV of a single cash flow or the *sum* of a sequence or group of cash flows.

Present Value Applications

There are four basic types of credit instruments which incorporate present value concepts:

1. Simple Loan
2. Fixed Payment Loan
3. Coupon Bond
4. Discount Bond

Present Value Concept: Simple Loan Terms

- *Loan Principal:* the amount of funds the lender provides to the borrower.
- *Maturity Date:* the date the loan must be repaid; the *Loan Term* is from initiation to maturity date.
- *Interest Payment:* the cash amount that the borrower must pay the lender for the use of the loan principal.
- *Simple Interest Rate:* the interest payment divided by the loan principal; the percentage of principal that must be paid as interest to the lender. **Convention is to express on an annual basis**, irrespective of the loan term.

Present Value Concept: Simple Loan

Simple loan of $100

Year: 0	1	2	3	n
\$100	\$110	\$121	\$133	$100\times(1+i)^n$

$$\text{PV of future \$1} = \frac{\$1}{(1+i)^n}$$

Present Value Concept: Simple Loan (cont.)

- The previous example reinforces the concept that $100 today is preferable to $100 a year from now since today's $100 could be lent out (or deposited) at 10% interest to be worth $110 one year from now, or $121 in two years or $133 in three years.

Yield to Maturity: Loans

Yield to maturity = interest rate that equates today's value with present value of all future payments

1.Simple Loan Interest Rate ($i = 10\%$)

$$\$100 = \frac{\$110}{(1+i)} = \quad +$$

$$i = \frac{\$110 - \$100}{\$100} = \frac{\$10}{\$100} = .10 = 10\%$$

Present Value of Cash Flows: Example

EXAMPLE 3.1 Simple Present Value

What is the present value of $250 to be paid in two years if the interest rate is 15%?

Solution

The present value would be $189.04. Using Equation 1:

$$PV = \frac{CF}{(1+i)^n}$$

where

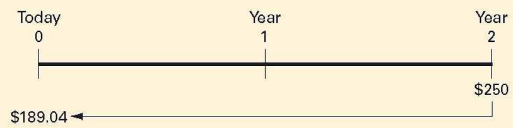
CF = cash flow in two years = \$250

i = annual interest rate = 0.15

n = number of years = 2

Thus,

$$PV = \frac{\$250}{(1+0.15)^2} = \frac{\$250}{1.3225} = \$189.04$$



Present Value Concept: Fixed-Payment Loan Terms

- *Fixed-Payment Loans* are loans where the loan principal and interest are repaid in several payments, often monthly, in equal dollar amounts over the loan term.
  - *Installment Loans*, such as auto loans and home mortgages are frequently of the fixed-payment type.

Yield to Maturity: Loans

2. Fixed Payment Loan (i = 12%)

$$\$1000 = \frac{\$126}{(1+i)} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + \ldots + \frac{\$126}{(1+i)^{25}}$$

$$LV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}$$

Yield to Maturity: Bonds

3. Coupon Bond (Coupon rate = 10% = C/F)

$$P = \frac{\$100}{(1+i)} + \frac{\$100}{(1+i)^2} + \frac{\$100}{(1+i)^3} + \ldots + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}$$

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Consol (short for consolidated annuities or perpetual bonds): Fixed coupon payments of $C forever

$$P = \frac{C}{i} \qquad i = \frac{C}{P}$$

Yield to Maturity: Bonds

4. One-Year Discount Bond
   (P = \$900, F = \$1000)

$$\$900 = \frac{\$1000}{(1+i)} =$$

$$i = \frac{\$1000 - \$900}{\$900} = .111 = 11.1\%$$

$$i = \frac{F - P}{P}$$

Relationship Between Price and Yield to Maturity

TABLE 3.1 Yields to Maturity on a 10% Coupon Rate Bond Maturing in 10 Years (Face Value = \$1,000)

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- Three interesting facts in Table 3.1
  1. When bond is at par, yield equals coupon rate
  2. Price and yield are negatively related
  3. Yield greater than coupon rate when bond price is below par value

Current Yield

$$i_c = \frac{C}{P}$$

- Current yield (CY) is just an approximation for YTM—easier to calculate. However, we should be aware of its properties:
  1. If a bond's price is near par and has a long maturity, then CY is a good approximation.
  2. A change in the current yield always signals change in same direction as yield to maturity

Yield on a Discount Basis: Less Than 1 Year

$$i_{db} = \frac{(F - P)}{F} \times \frac{360}{(\text{number of days to maturity})}$$

- One-Year Bill (P = \$900, F = \$1000)

$$i_{db} = \frac{\$1000 - \$900}{\$1000} \times \frac{360}{365} = .099 = 9.9\%$$

- **Two Characteristics**
  1. **Understates yield to maturity; longer the maturity, greater is understatement**
  2. **Change in discount yield always signals change in same direction as yield to maturity**

Distinction Between Interest Rates and Returns

- Rate of Return: we can decompose returns into two pieces:

$$\text{Return} = \frac{C + P_{t+1} - P_t}{P_t} = i_c + g$$

$$\text{where } i_c = \frac{C}{P_t} = \text{current yield, and}$$

$$g = \frac{P_{t+1} - P_t}{P_t} - \text{capital gains}$$

Key Facts about the Relationship Between Rates and Returns

TABLE 3.2 One-Year Returns on Different-Maturity 10% Coupon Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	–49.7	–39.7
20	10	1,000	516	–48.4	–38.4
10	10	1,000	597	–40.3	–30.3
5	10	1,000	741	–25.9	–15.9
2	10	1,000	917	–8.3	+ 1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator using Equation 3.

Sample of current coupon rates and yields on government bonds
http://www.bloomberg.com/markets/iyc.html

Maturity and the Volatility of Bond Returns

- Key findings from Table 3.2
  1. Only bond whose return = yield is one with maturity = holding period
  2. For bonds with maturity > holding period, $i\uparrow P\downarrow$ implying capital loss
  3. Longer is maturity, greater is price change associated with interest rate change
  4. Longer is maturity, more return changes with change in interest rate
  5. Bond with high initial interest rate can still have negative return if $i\uparrow$

Maturity and the Volatility of Bond Returns (cont.)

- Conclusion from Table 3.2 analysis
  1. Prices and returns more volatile for long-term bonds because have higher interest-rate risk
  2. No interest-rate risk for any bond whose maturity equals holding period
- Reinvestment risk occurs if hold series of short bonds over long holding period
  – *i* at which reinvest uncertain
  – Gain from $i\uparrow$, lose when $i\downarrow$

Formula for Duration

- Duration is a measurement of how long, in years, it takes for the price of a bond to be repaid by its internal cash flows.
- Bonds with higher durations carry more risk and have higher price volatility than bonds with lower durations.

$$DUR = \sum_{t=1}^{n} t\frac{CP_t}{(1+i)^t} \Bigg/ \sum_{t=1}^{n} \frac{CP_t}{(1+i)^t}$$

- Key facts about duration
  1. All else equal, when the maturity of a bond lengthens, the duration rises as well
  2. All else equal, when interest rates rise, the duration of a coupon bond fall

Calculating Duration *i* = 10%, 10-Year 10% Coupon Bond

TABLE 3.3 Calculating Duration on a $1,000 Ten-Year 10% Coupon Bond When Its Interest Rate Is 10%

(1) Year	(2) Cash Payments (Zero-Coupon Bonds) ($)	(3) Present Value (*PV*) of Cash Payments (*i* = 10%) ($)	(4) Weights (% of total *PV* = *PV*/$1,000) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	90.91	9.091	0.09091
2	100	82.64	8.264	0.16528
3	100	75.13	7.513	0.22539
4	100	68.30	6.830	0.27320
5	100	62.09	6.209	0.31045
6	100	56.44	5.644	0.33864
7	100	51.32	5.132	0.35924
8	100	46.65	4.665	0.37320
9	100	42.41	4.241	0.38169
10	100	38.55	3.855	0.38550
10	1,000	385.54	38.554	3.85500
Total		1,000.00	100.000	6.75850

Calculating Duration *i* = 20%, 10-Year 10% Coupon Bond

TABLE 3.4 Calculating Duration on a $1,000 Ten-Year 10% Coupon Bond When Its Interest Rate Is 20%

(1) Year	(2) Cash Payments (Zero-Coupon Bonds) ($)	(3) Present Value (*PV*) of Cash Payments (*i* = 20%) ($)	(4) Weights (% of total *PV* = *PV*/$580.76) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	83.33	14.348	0.14348
2	100	69.44	11.957	0.23914
3	100	57.87	9.965	0.29895
4	100	48.23	8.305	0.33220
5	100	40.19	6.920	0.34600
6	100	33.49	5.767	0.34602
7	100	27.91	4.806	0.33642
8	100	23.26	4.005	0.32040
9	100	19.38	3.337	0.30033
10	100	16.15	2.781	0.27810
10	$1,000	161.51	27.808	2.78100
Total		580.76	100.000	5.72204

Formula for Duration

1. The higher is the coupon rate on the bond, the shorter is the duration of the bond
2. Duration is additive: the duration of a portfolio of securities is the weighted-average of the durations of the individual securities, with the weights equaling the proportion of the portfolio invested in each

Duration and Interest-Rate Risk

$$\% \Delta P \approx -DUR \times \frac{\Delta i}{1+i}$$

- $i \uparrow$ 10% to 11%:
  - Table 3.4, –10% coupon bond

$$\% \Delta P \approx -6.76 \times \frac{0.01}{1+0.10}$$

$$\% \Delta P \approx -0.0615 = -6.15\%$$

Duration and Interest-Rate Risk (cont.)

- $i \uparrow$ 10% to 11%:
  - 20% coupon bond, *DUR* = 5.72 years

$$\% \Delta P \approx -5.72 \times \frac{0.01}{1+0.10}$$

$$\% \Delta P \approx -0.0520 = -5.20\%$$

Duration and Interest-Rate Risk (cont.)

- The greater is the duration of a security, the greater is the percentage change in the market value of the security for a given change in interest rates
- Therefore, the greater is the duration of a security, the greater is its interest-rate risk

DEMAND AND SUPPLY OF ASSETS

EXAMPLE 1: Expected Return

EXAMPLE 4.1 Expected Return

What is the expected return on the Exxon-Mobil bond if the return is 12% two-thirds of the time and 8% one-third of the time?

Solution

The expected return is 10.68%.

$$R^e = p_1R_1 + p_2R_2$$

where

p_1 = probability of occurrence of return 1 $= \frac{2}{3} = 0.67$

R_1 = return in state 1 $= 12\% = 0.12$

p_2 = probability of occurrence return 2 $= \frac{1}{3} = 0.33$

R_2 = return in state 2 $= 8\% = 0.08$

Thus,

$$R^e = (.67)(0.12) + (.33)(0.08) = 0.1068 = 10.68\%$$

EXAMPLE 2: ~~Standard Deviation (a)~~

Consider the following two companies and their forecasted returns for the upcoming year:

		Fly-by-Night	Feet-on-the-Ground
Outcome 1	Probability	50%	100%
	Return	15%	10%
Outcome 2	Probability	50%	
	Return	5%	

- Of these two stocks, which is riskier?

EXAMPLE 2: ~~Standard Deviation~~

Solution

Fly-by-Night Airlines has a standard deviation of returns of 5%.

$$\sigma = \sqrt{p_1(R_1 - R^e)^2 + p_2(R_2 - R^e)^2}$$
$$R^e = p_1R_1 + p_2R_2$$

where

p_1 = probability of occurrence of return 1 $= \frac{1}{2} = 0.50$

R_1 = return in state 1 $= 15\% = 0.15$

p_2 = probability of occurrence of return 2 $= \frac{1}{2} = 0.50$

R_2 = return in state 2 $= 5\% = 0.05$

R^e = expected return $= (.50)(0.15) + (.50)(0.05) = 0.10$

Thus,

$$\sigma = \sqrt{(.50)(0.15 - 0.10)^2 + (.50)(0.05 - 0.10)^2}$$
$$\sigma = \sqrt{(.50)(0.0025) + (.50)0.0025)} = \sqrt{0.0025} = 0.05 = 5\%$$

EXAMPLE 2: Standard Deviation

Feet-on-the-Ground Bus Company has a standard deviation of returns of 0%.

$$\sigma = \sqrt{p_1(R_1 - R^e)^2}$$

$$R^e = p_1 R_1$$

where

p_1 = probability of occurrence of return 1 = 1.0

R_1 = return in state 1 = 10% = 0.10

R^e = expected return = (1.0)(0.10) = 0.10

Thus,

$$\sigma = \sqrt{(1.0)(0.10 - 0.10)^2} = \sqrt{0} = 0 = 0\%$$

The Demand Curve

Let's start with the demand curve.

Let's consider a one-year discount bond with a face value of $1,000. In this case, the return on this bond is entirely determined by its price. The return is, then, the bond's yield to maturity.

Derivation of Demand Curve

$$i = R^e = \frac{(F - P)}{P}$$

- Point A: if the bond was selling for $950.

$$P = \$950$$

$$i = \frac{(\$1000 - \$950)}{\$950} = .053 = 5.3\%$$

$$B^d = 100$$

Derivation of Demand Curve (cont.)

- Point B: if the bond was selling for $900.

$$P = \$900$$

$$i = \frac{(\$1000 - \$900)}{\$900} = .111 = 11.1\%$$

$$B^d = 200$$

Supply and Demand for Bonds

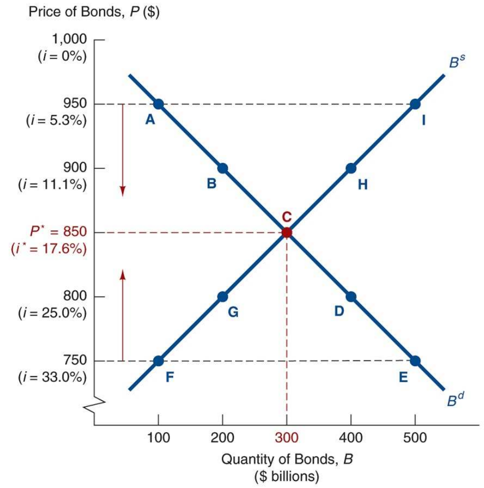


FIGURE 4.1 Supply and Demand for Bonds

Equilibrium in the bond market occurs at point C, the intersection of the demand curve B^d and the bond supply curve B^s. The equilibrium price is $P^* = \$850$, and the equilibrium interest rate is $i^* = 17.6\%$.

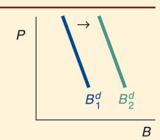
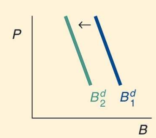
Market Equilibrium

The equilibrium follows what we know from supply-demand analysis:

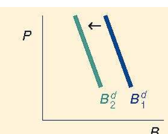
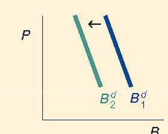
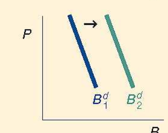
- Occurs when $B^d = B^s$, at $P^* = 850$, $i^* = 17.6\%$
- When $P = \$950$, $i = 5.3\%$, $B^s > B^d$ (excess supply): $P \downarrow$ to P^*, $i \uparrow$ to i^*
- When $P = \$750$, $i = 33.0$, $B^d > B^s$ (excess demand): $P \uparrow$ to P^*, $i \downarrow$ to i^*

Factors That Shift Demand Curve (a)

TABLE 4.2 SUMMARY Summary Factors That Shift the Demand Curve for Bonds

Variable	Change in Variable	Change in Quantity Demanded at Each Bond Price	Shift in Demand Curve
Wealth	↑	↑	
Expected interest rate	↑	↓	

Factors That Shift Demand Curve (b)

Expected inflation	↑	↓	
Riskiness of bonds relative to other assets	↑	↓	
Liquidity of bonds relative to other assets	↑	↑	

Note: Only increases in the variables are shown. The effect of decreases in the variables on the change in demand would be the opposite of those indicated in the remaining columns.

Shifts in the Demand Curve

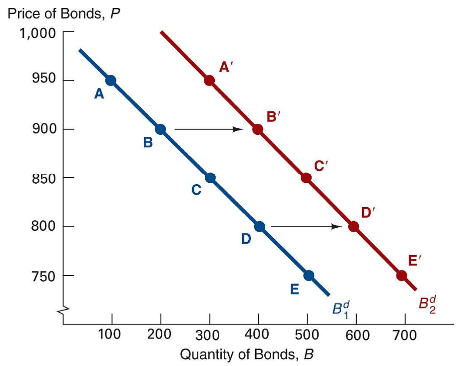


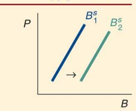
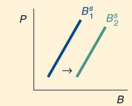
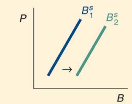
FIGURE 4.2 Shift in the Demand Curve for Bonds

When the demand for bonds increases, the demand curve shifts to the right as shown.

Factors That Shift Supply Curve

We now turn to the supply curve. We summarize the effects in this table:

TABLE 4.3 Summary Factors That Shift the Supply of Bonds
SUMMARY

Variable	Change in Variable	Change in Quantity Supplied at Each Bond Price	Shift in Supply Curve
Profitability of investments	↑	↑	
Expected inflation	↑	↑	
Government deficit	↑	↑	

Note: Only increases in the variables are shown. The effect of decreases in the variables on the change in supply would be the opposite of those indicated in the remaining columns.

Shifts in the Supply Curve

1. Profitability of Investment Opportunities
   - Business cycle expansion,
   - investment opportunities ↑, B^s ↑,
   - B^s shifts out to right
2. Expected Inflation
   - π^e ↑, B^s ↑
   - B^s shifts out to right
3. Government Activities
   - Deficits ↑, B^s ↑
   - B^s shifts out to right

Shifts in the Supply Curve

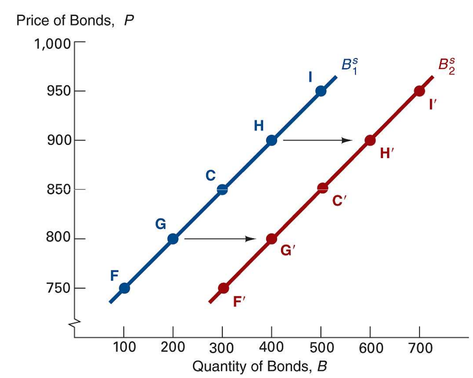


FIGURE 4.3 Shift in the Supply Curve for Bonds

When the supply of bonds increases, the supply curve shifts to the right.

TERM STRUCTURE AND RISKS

Risk Structure of Long Bonds in the U.S.

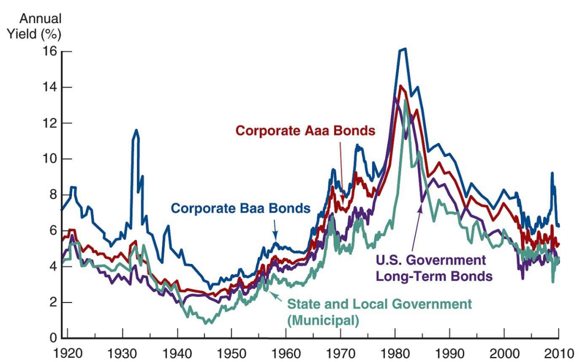


FIGURE 5.1 Long-Term Bond Yields, 1919–2010

Sources: Board of Governors of the Federal Reserve System, *Banking and Monetary Statistics, 1941–1970;* Federal Reserve: www.federalreserve.gov/releases/h15/data.htm.

Risk Structure of Long Bonds in the U.S.

The figure shows two important features of the interest-rate behavior of bonds.

- Rates on different bond categories change from one year to the next.
- Spreads on different bond categories change from one year to the next.

Factors Affecting Risk Structure of Interest Rates

To further examine these features, we will look at two specific risk factors.

- Default Risk
- Liquidity

Default Risk Factor

- The spread between the interest rates on bonds with default risk and default-free bonds, called the **risk premium,** indicates how much additional interest people must earn in order to be willing to hold that risky bond.
- A bond with default risk will always have a positive risk premium, and an increase in its default risk will raise the risk premium.

Increase in Default Risk on Corporate Bonds

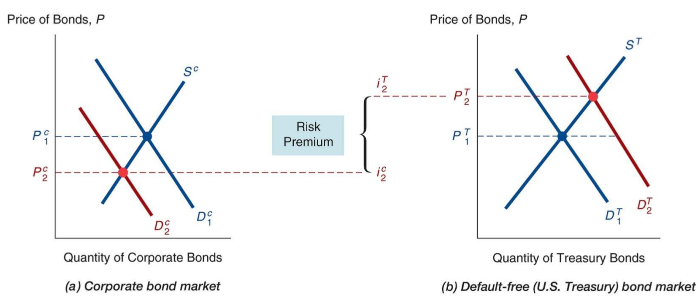


FIGURE 5.2 Response to an Increase in Default Risk on Corporate Bonds

Initially $P_1^c = P_1^T$ and the risk premium is zero. An increase in default risk on corporate bonds shifts the demand curve from D_1^c to D_2^c. Simultaneously, it shifts the demand curve for Treasury bonds from D_1^T to D_2^T. The equilibrium price for corporate bonds falls from P_1^c to P_2^c, and the equilibrium interest rate on corporate bonds rises to i_2^c. In the Treasury market, the equilibrium bond price rises from P_1^T to P_2^T and the equilibrium interest rate falls to i_2^T. The brace indicates the difference between i_2^c and i_2^T, the risk premium on corporate bonds. (Note that because P_2^c is lower than P_2^c, i_2^c is greater than i_2^T.)

Analysis of Figure 5.2: Increase in Default on Corporate Bonds

- Corporate Bond Market
  1. R^e on corporate bonds ↓, D^c ↓, D^c shifts left
  2. Risk of corporate bonds ↑, D^c ↓, D^c shifts left
  3. P^c ↓, i^c ↑
- Treasury Bond Market
  1. Relative R^e on Treasury bonds ↑, D^T ↑, D^T shifts right
  2. Relative risk of Treasury bonds ↓, D^T ↑, D^T shifts right P^T ↑, i^T ↓
- Outcome
  – Risk premium, i^c - i^T, rises

Bond Ratings

TABLE 5.1 Bond Ratings by Moody's and Standard and Poor's

Rating: Moody's	Rating: Standard and Poor's	Descriptions	Examples of Corporations with Bonds Outstanding in 2010
Aaa	AAA	Highest quality (lowest default risk)	Microsoft, Johnson & Johnson, Mobil Corp.
Aa	AA	High quality	Shell Oil, Abbott Laboratories, General Electric
A	A	Upper-medium grade	Bank of America, Hewlett-Packard, McDonald's, Inc.
Baa	BBB	Medium grade	Best Buy, FedEx, Harley Davidson
Ba	BB	Lower-medium grade	Charter Communications, Colonial Penn, US Steel Corp.
B	B	Speculative	Rite Aid, Ford Motors, Delta
Caa	CCC, CC	Poor (high default risk)	Blockbuster, Century Indemnity, Everspan Financial Guarantee
C	D	Highly speculative	Citation Corp.

Liquidity Factor

- The differences between interest rates on corporate bonds and Treasury bonds (that is, the risk premiums) reflect not only the corporate bond's default risk but its liquidity too. This is why a risk premium is sometimes called a *risk and liquidity premium.*

Liquidity Factor

- Another attribute of a bond that influences its interest rate is its liquidity
- A liquid asset is one that can be quickly and cheaply converted into cash if the need arises.
- The more liquid an asset is, the more desirable it is (higher demand), holding everything else constant. Why?

Corporate Bond Becomes Less Liquid

- Corporate Bond Market
  1. Liquidity of corporate bonds ↓, D^c ↓, D^c shifts left
  2. P^c ↓, i^c ↑
- Treasury Bond Market
  1. Relatively more liquid Treasury bonds, D^T ↑, D^T shifts right
  2. P^T ↑, i^T ↓
- Outcome
  - Risk premium, $i^c - i^T$, rises
- Risk premium reflects not only corporate bonds' default risk but also lower liquidity

Term Structure of Interest Rates

- Now that we understand risk, liquidity, and taxes, we turn to another important influence on interest rates—maturity.
- Bonds with different maturities tend to have different required rates, all else equal.

Interest Rates on Different Maturity Bonds Move Together

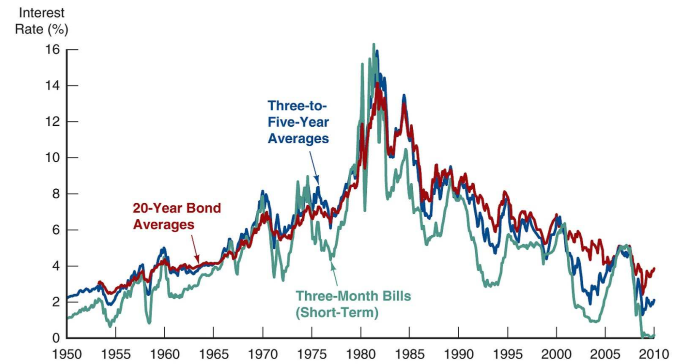


FIGURE 5.4 Movements over Time of Interest Rates on U.S. Government Bonds with Different Maturities

Source: Federal Reserve: www.federalreserve.gov/releases/h15/data.htm.

Term Structure Facts to Be Explained

Besides explaining the shape of the yield curve, a good theory must explain why:

1. Interest rates for different maturities move together.
2. Yield curves tend to have steep upward slope when short rates are low and downward slope when short rates are high.
3. Yield curve is typically upward sloping.

Three Theories of Term Structure

1. Expectations Theory
   - Pure Expectations Theory explains 1 and 2, but not 3
2. Market Segmentation Theory
   - Market Segmentation Theory explains 3, but not 1 and 2
3. Liquidity Premium Theory
   - Solution: Combine features of both Pure Expectations Theory and Market Segmentation Theory to get Liquidity Premium Theory and explain all facts

Expectations Theory

- **Key Assumption:** Bonds of different maturities are perfect substitutes
- **Implication:** R^e on bonds of different maturities are equal

Expectations Theory

To illustrate what this means, consider two alternative investment strategies for a two-year time horizon**.**

1. Buy $1 of one-year bond, and when it matures, buy another one-year bond with your money.
2. Buy $1 of two-year bond and hold it.

Expectations Theory

The important point of this theory is that if the Expectations Theory is correct, your *expected* wealth is the same (at the start) for both strategies. Of course, your actual wealth may differ, if rates change *unexpectedly* after a year.

We show the details of this in the next few slides.

Expectations Theory

- Expected return from strategy 1

$$(1 + i_2)(1 + i^e_{t+1}) - 1 = 1 + i_t + i^e_{t+1} + i_t(i^e_{t+1}) - 1$$

- Since $i_t(i^e_{t+1})$ is also extremely small, expected return is approximately

$$i_t + i^e_{t+1}$$

Expectations Theory

- Expected return from strategy 2

$$(1 + i_{2t})(1 + i_{2t}) - 1 = 1 + 2(i_{2t}) + (i_{2t})^2 - 1$$

- Since $(i_{2t})^2$ is extremely small, expected return is approximately $2(i_{2t})$

Expectations Theory

- From implication above expected returns of two strategies are equal
- Therefore

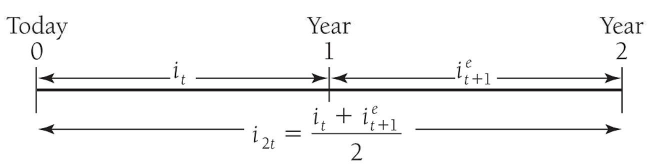
$$2(i_{2t}) = i_2 + i^e_{t+1}$$

Solving for i_{2t}

$$i_{2t} = \frac{i_t + i^e_{t+1}}{2}$$

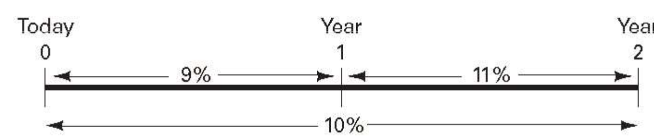
Expectations Theory

- To help see this, here's a picture that describes the same information:



Example 5.2: Expectations Theory

- This is an example, with actual #'s:



More generally for *n*-period bond…

$$i_{nt} = \frac{i_t + i_{t+1} + i_{t+2} + \ldots + i_{t+(n-1)}}{n}$$

- Don't let this seem complicated. Equation 2 simply states that the interest rate on a long-term bond equals the average of short rates expected to occur over life of the long-term bond.

More generally for *n*-period bond…

- Numerical example
  - One-year interest rate over the next five years are expected to be 5%, 6%, 7%, 8%, and 9%
- Interest rate on two-year bond today:
  (5% + 6%)/2 = 5.5%
- Interest rate for five-year bond today:
  (5% + 6% + 7% + 8% + 9%)/5 = 7%
- Interest rate for one- to five-year bonds today:
  5%, 5.5%, 6%, 6.5% and 7%

Expectations Theory and Term Structure Facts

- Explains why yield curve has different slopes
  1. When short rates are expected to rise in future, average of future short rates = i_{nt} is above today's short rate; therefore yield curve is upward sloping.
  2. When short rates expected to stay same in future, average of future short rates same as today's, and yield curve is flat.
  3. Only when short rates expected to fall will yield curve be downward sloping.

Expectations Theory and Term Structure Facts

- Pure expectations theory explains fact 1—that short and long rates move together
  1. Short rate rises are persistent
  2. If $i_t \uparrow$ today, i^e_{t+1}, i^e_{t+2} etc. $\uparrow \Rightarrow$ average of future rates $\uparrow \Rightarrow i_{nt} \uparrow$
  3. Therefore: $i_t \uparrow \Rightarrow i_{nt} \uparrow$
     (i.e., short and long rates move together)

Expectations Theory and Term Structure Facts

- Explains fact 2—that yield curves tend to have steep slope when short rates are low and downward slope when short rates are high
  1. When short rates are low, they are expected to rise to normal level, and long rate = average of future short rates will be well above today's short rate; yield curve will have steep upward slope.
  2. When short rates are high, they will be expected to fall in future, and long rate will be below current short rate; yield curve will have downward slope.

Expectations Theory and Term Structure Facts

- Doesn't explain fact 3—that yield curve usually has upward slope
  – Short rates are as likely to fall in future as rise, so average of expected future short rates will not usually be higher than current short rate: therefore, yield curve will not usually slope upward.

Market Segmentation Theory

- **Key Assumption:** Bonds of different maturities are not substitutes at all
- **Implication:** Markets are completely segmented; interest rate at each maturity are determined separately

Market Segmentation Theory

- Explains fact 3—that yield curve is usually upward sloping
  – People typically prefer short holding periods and thus have higher demand for short-term bonds, which have higher prices and lower interest rates than long bonds
- Does not explain fact 1or fact 2 because its assumes long-term and short-term rates are determined independently.

Liquidity Premium Theory

- **Key Assumption:** Bonds of different maturities are substitutes, but are not perfect substitutes
- **Implication:** Modifies Pure Expectations Theory with features of Market Segmentation Theory

Liquidity Premium Theory

- Investors prefer short-term rather than long-term bonds. This implies that investors must be paid positive liquidity premium, i_{nt}, to hold long term bonds.

Liquidity Premium Theory

- Results in following modification of Expectations Theory, where $\boldsymbol{l_{nt}}$ is the liquidity premium.

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \ \ldots \ + i^e_{t+(n-1)}}{n} + l_{nt}$$

- We can also see this graphically…

Liquidity Premium Theory

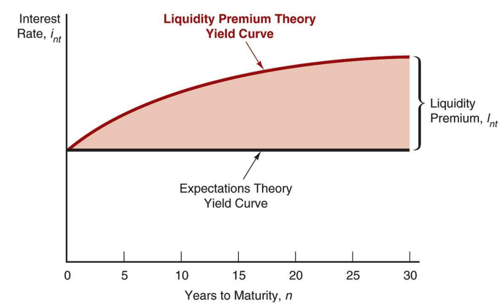


FIGURE 5.5 The Relationship Between the Liquidity Premium and Expectations Theory

Because the liquidity premium is always positive and grows as the term to maturity increases, the yield curve implied by the liquidity premium theory is always above the yield curve implied by the expectations theory and has a steeper slope. For simplicity, the yield curve implied by the expectations theory is drawn under the scenario of unchanging future one-year interest rates.

Numerical Example

1. One-year interest rate over the next five years: 5%, 6%, 7%, 8%, and 9%
2. Investors' preferences for holding short-term bonds so liquidity premium for one- to five-year bonds: 0%, 0.25%, 0.5%, 0.75%, and 1.0%

Numerical Example

- Interest rate on the two-year bond:
  $0.25\% + (5\% + 6\%)/2 = 5.75\%$
- Interest rate on the five-year bond:
  $1.0\% + (5\% + 6\% + 7\% + 8\% + 9\%)/5 = 8\%$
- Interest rates on one to five-year bonds:
  5%, 5.75%, 6.5%, 7.25%, and 8%
- Comparing with those for the pure expectations theory, liquidity premium theory produces yield curves more steeply upward sloped

Liquidity Premium Theory: ~~Term Structure Facts~~

- Explains All 3 Facts
  - Explains fact 3—that usual upward sloped yield curve by liquidity premium for long-term bonds
  - Explains fact 1 and fact 2 using same explanations as pure expectations theory because it has average of future short rates as determinant of long rate

Market Predictions of Future Short Rates

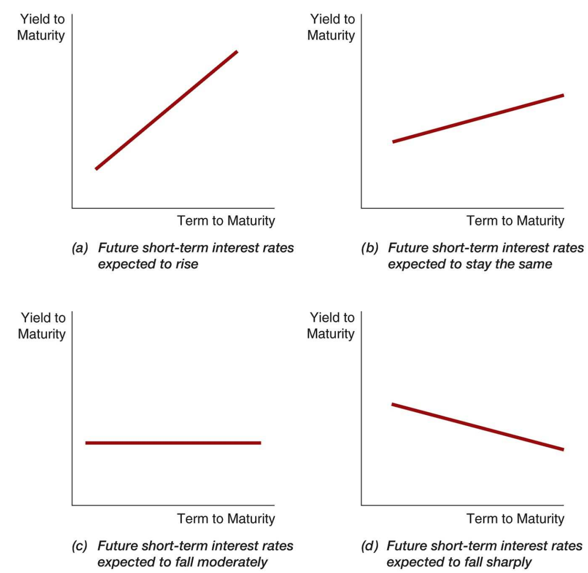


FIGURE 5.6 Yield Curves and the Market's Expectations of Future Short-Term Interest Rates According to the Liquidity Premium Theory

Case: Interpreting Yield Curves

- The picture on the next slide illustrates several yield curves that we have observed for U.S. Treasury securities in recent years.
- What do they tell us about the public's expectations of future rates?

Case: Interpreting Yield Curves, 1980–2010

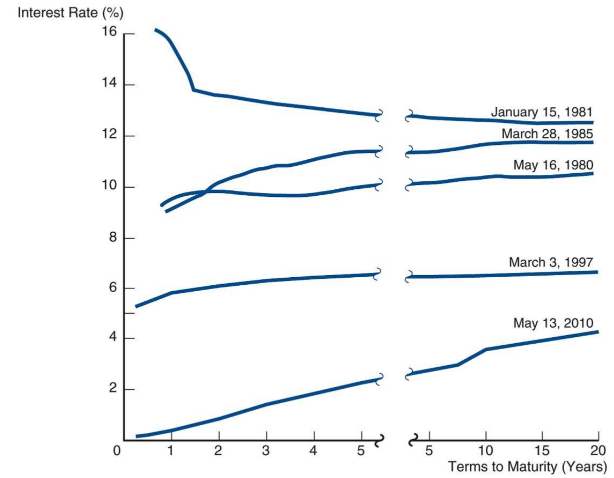


FIGURE 5.7 Yield Curves for U.S. Government Bonds

Sources: Federal Reserve Bank of St. Louis; *U.S. Financial Data*, various issues; *Wall Street Journal*, various dates.

The Practicing Manager: Forecasting Interest Rates with the Term Structure

- Pure Expectations Theory: Invest in 1-period bonds or in two-period bond ⇒

$$(1 + i_t)(1 + i^e_{t+1}) - 1 = (1 + i_{2t})(1 + i_{2t}) - 1$$

- Solve for forward rate, i^e_{t+1}

$$i^e_{t+1} = \frac{(1 + i_{2t})^2}{1 + i_t} - 1$$

- Numerical example: $i_{1t} = 5\%$, $i_{2t} = 5.5\%$

$$i^e_{t+1} = \frac{(1 + 0.055)^2}{1 + 0.05} - 1 = 0.06 = 6\%$$

Forecasting Interest Rates with the Term Structure

- Compare 3-year bond versus 3 one-year bonds

$$(1 + i_t)(1 + i^e_{t+1})(1 + i^e_{t+2}) - 1 = (1 + i_{3t})(1 + i_{3t})(1 + i_{3t}) - 1$$

- Using i^e_{t+1} derived in (4), solve for i^e_{t+2}

$$i^e_{t+2} = \frac{(1 + i_{3t})^3}{(1 + i_{2t})^2} - 1$$

Forecasting Interest Rates with the Term Structure

- Generalize to:

$$i^e_{t+n} = \frac{(1 + i_{n+1t})^{n+1}}{(1 + i_{nt})^n} - 1$$

- Liquidity Premium Theory: $i_{nt} - l_{nt}$ = same as pure expectations theory; replace i_{nt} by $i_{nt} - l_{nt}$ in (5) to get adjusted forward-rate forecast

$$i^e_{t+n} = \frac{(1 + i_{n+1t} - l_{n+1t})^{n+1}}{(1 + i_{nt} - l_{nt})^n} - 1$$

Forecasting Interest Rates with the Term Structure

- Numerical Example
  $l_{2t} = 0.25\%$, $l_{1t} = 0$, $i_{1t} = 5\%$, $i_{2t} = 5.75\%$

$$i^e_{t+1} = \frac{(1 + 0.0575 - 0.0025)^2}{1 + 0.05} - 1 = 0.06 = 6\%$$

- Example: 1-year loan next year
  T-bond + 1%, $l_{2t} = .4\%$, $i_{1t} = 6\%$, $i_{2t} = 7\%$

$$i^e_{t+1} = \frac{(1 + 0.07 - 0.004)^2}{1 + 0.06} - 1 = 0.072 = 7.2\%$$

  Loan rate must be > 8.2%