Natchaya Laokham 6104641425

3. Using the data in RDCHEM, the following equation was obtained by OLS:

$$\widehat{rdintens} = 2.613 + .00030\, sales - .0000000070\, sales^2$$
$$(.429) \quad (.00014) \qquad (.0000000037)$$
$$n = 32,\ R^2 = .1484.$$

i. At what point does the marginal effect of *sales* on *rdintens* become negative?

The marginal effect of sales on rdintens is given by:

$$\frac{\partial \widehat{rdintens}}{\partial sales} = 0.00030 - 0.000000014\, sales$$

For the marginal effect to be negative, $\frac{\partial \widehat{rdintens}}{\partial sales} < 0$ holds

This implies, $0.00030 - 0.000000014\, sales < 0$

$$0.000000014\, sales > 0.0003$$

$$sales > \frac{0.0003}{0.000000014}$$

$$sales > 21{,}428.5714$$

Hence, at sales = 21,428.5714 the marginal effect of sales on rdintens become negative.

ii. Would you keep the quadratic term in the model? Explain.

Probably. The t-statistic on $\hat{\beta}_{sales^2}$ is $-\frac{.00000007}{.000000037} = -1.89$,

which is significant against the one-sided alternative $H_1: \beta_{sales^2} < 0$

iii. Define *salesbil* as sales measured in billions of dollars: $\boldsymbol{salesbil} = \boldsymbol{sales}/1{,}000$. Rewrite the estimated equation with *salesbil* and $\boldsymbol{salesbil^2}$ as the independent variables. Be sure to report standard errors and the *R*-squared. [*Hint*: Note that $\boldsymbol{salesbil^2} = \boldsymbol{sales^2}/(1{,}000)^2$.]

$$
\begin{aligned}
rdintens &= 2.613 + 0.0003\,sales - 0.00000007\,sales^2 \\
&= 2.613 + 0.0003\,(1000 \times salesbil) - 0.000000007\,(1000 \times salesbil)^2 \\
&= \underset{(.429)}{2.613} + \underset{(.14)}{.3\,salesbil} - \underset{(.0037)}{0.007\,salesbil^2}
\end{aligned}
$$

Recall that $se(\hat{\beta}_j) = \dfrac{\hat{\sigma}}{[SST_j(1-R^2)]^{1/2}}$ (3.58)

Rescaling sales will have no effect on $\hat{\sigma}$ or R^2 since it does not change the fit of the regression. It will, however, affect SST_{sales} and SST_{sale^2}. Specifically,

$$SST_{salesbil} = \sum_{i=1}^{n} (salesbil - \overline{salebil})^2 = \sum_{i=1}^{n} \frac{(sales - \overline{sales})^2}{1000^2} = \frac{SST_{sales}}{1000^2}$$

Similarly, $SST_{salesbil^2} = \dfrac{SST_{sales^2}}{1000^4}$

Therefore, we need to scale the standard errors of $\hat{\beta}_{salesbil}$ and $\hat{\beta}_{salesbil^2}$ up by 1000 and 1000^2 respectively.

iv. For the purpose of reporting the results, which equation do you prefer?

The equation in part III is easier to read because it contains fewer zeros to the right of the decimal. Of course the interpretation of the two equations in identical once the different scales are accounted for.