Estimation Methods

Nonparametric Estimation Methods

- No assumption of distribution
- i.e. Linear Programming

Parametric Estimation Methods

- Assume distribution
  - Least Squares Methods (LS)
  - Maximum Likelihood (ML)
  - Generalized Method of Moments (GMM)

Least Squares Estimation Methods

Ordinary Least Squares (OLS)

Generalized Least Squares (GLS)

Feasible Generalized Least Squares (FGLS)

- Weighted Least Squares (WLS)

  Heteroscedasticity

- Cochrane-Orcutt Technique

  Autocorrelation

Other Least Squares Methods

- Nonlinear Least Squares (NLS)
- System Equation Estimation Methods

OLS Matrix Approach

Scalar Notation

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \cdots + \beta_k x_{ki} + u_i$$

$$where \quad i = 1, 2, 3, \ldots, n$$

$$y_1 = \beta_1 + \beta_2 x_{21} + \beta_3 x_{31} + \cdots + \beta_k x_{k1} + u_1$$
$$y_2 = \beta_1 + \beta_2 x_{22} + \beta_3 x_{32} + \cdots + \beta_k x_{k2} + u_2$$
$$\vdots \qquad\qquad \vdots$$
$$y_n = \beta_1 + \beta_2 x_{2n} + \beta_3 x_{3n} + \cdots + \beta_k x_{kn} + u_n$$

$$\underset{n\times 1}{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}} = \underset{n\times k}{\begin{bmatrix} 1 & x_{21} & x_{31} & \cdots & x_{k1} \\ 1 & x_{22} & x_{32} & \cdots & x_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{2n} & x_{3n} & \cdots & x_{kn} \end{bmatrix}} \cdot \underset{k\times 1}{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}} + \underset{n\times 1}{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}}$$

OLS Matrix Approach

$$\underset{n\times 1}{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}} = \underset{n\times k}{\begin{bmatrix} 1 & x_{21} & x_{31} & \cdots & x_{k1} \\ 1 & x_{22} & x_{32} & \cdots & x_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{2n} & x_{3n} & \cdots & x_{kn} \end{bmatrix}} \cdot \underset{k\times 1}{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}} + \underset{n\times 1}{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}}$$

Matrix Notation

$$\underset{n\times 1}{Y} = \underset{n\times k}{X} \underset{k\times 1}{\beta} + \underset{n\times 1}{u}$$

Least Squares

$$\hat{u}'\hat{u} = \begin{bmatrix} \hat{u}_1 & \hat{u}_2 & \cdots & \hat{u}_n \end{bmatrix} \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \vdots \\ \hat{u}_n \end{bmatrix} = \sum_{i=1}^{n} \hat{u}_i^2$$

OLS Matrix Approach

From
$$Y = X\beta + u$$
$$\hat{u} = Y - X\hat{\beta}$$

Least Squares
$$\hat{u}'\hat{u} = \left(Y - X\hat{\beta}\right)'\left(Y - X\hat{\beta}\right)$$
$$= Y'Y - 2\hat{\beta}'X'Y + \hat{\beta}'X'X\hat{\beta}$$
$$\frac{\partial \hat{u}'\hat{u}}{\partial \hat{\beta}} = -2X'Y + 2X'X\hat{\beta} = 0$$
$$\underset{k\times 1}{\hat{\beta}} = (\underset{k\times k}{X'X})^{-1}\underset{k\times n}{X'}\,\underset{n\times 1}{Y}$$

Assume normal distribution: $u \sim N(0, \sigma^2 I)$

OLS Matrix Approach

Variance-Covariance Matrix

$$uu' = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix} = \begin{bmatrix} u_1u_1 & u_1u_2 & \cdots & u_1u_n \\ u_2u_1 & u_2u_2 & \cdots & u_2u_n \\ \vdots & \vdots & \ddots & \vdots \\ u_nu_1 & u_nu_2 & \cdots & u_nu_n \end{bmatrix}$$

$$E\left(uu'\right) = \begin{bmatrix} E\left(u_1^2\right) & E\left(u_1u_2\right) & \cdots & E\left(u_1u_n\right) \\ E\left(u_2u_1\right) & E\left(u_2^2\right) & \cdots & E\left(u_2u_n\right) \\ \vdots & \vdots & \ddots & \vdots \\ E\left(u_nu_1\right) & E\left(u_nu_2\right) & \cdots & E\left(u_n^2\right) \end{bmatrix}$$

OLS Matrix Approach

Variance-Covariance Matrix

$$E\left(uu'\right) = \underset{n\times n}{\Sigma} = \begin{bmatrix} E\left(u_1^2\right) & E\left(u_1u_2\right) & \cdots & E\left(u_1u_n\right) \\ E\left(u_2u_1\right) & E\left(u_2^2\right) & \cdots & E\left(u_2u_n\right) \\ \vdots & \vdots & \ddots & \vdots \\ E\left(u_nu_1\right) & E\left(u_nu_2\right) & \cdots & E\left(u_n^2\right) \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n3} & \cdots & \sigma_{\mathrm{n}}^2 \end{bmatrix}$$

Variance-Covariance Matrix

Under OLS Assumptions

$$\sum_{n\times n} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n3} & \cdots & \sigma_{\mathrm{n}}^2 \end{bmatrix} = \begin{bmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{bmatrix} = \sigma^2 I$$

Under Heteroscedasticity Problem

$$\sum_{n\times n} = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_n^2 \end{bmatrix}$$

Weighted Least Squares (WLS)

Scalar $$y_i = \beta_1 + \beta_2 x_i + u_i$$

$$\frac{y_i}{\hat{\sigma}_i} = \beta_1\left(\frac{1}{\hat{\sigma}_i}\right) + \beta_2\left(\frac{x_i}{\hat{\sigma}_i}\right) + \left(\frac{u_i}{\hat{\sigma}_i}\right)$$

$$y_i^* = \beta_1^* x_{0i}^* + \beta_2^* x_i^* + u_i^*$$

Matrix

$$\underset{k\times 1}{\hat{\beta}} = (\underset{k\times n}{X'} \underset{\substack{n\times n \\ k\times k}}{\hat{\Sigma}^{-1}} \underset{n\times k}{X})^{-1} \underset{k\times n}{X'} \underset{\substack{n\times n \\ k\times 1}}{\hat{\Sigma}^{-1}} \underset{n\times 1}{Y}$$

Autocorrelation: Cochrane-Orcutt

Scalar $$y_t = \beta_1 + \beta_2 x_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

1. Estimate model using OLS and obtain estimated u_t $\hat{u}_t = \hat{\rho}\hat{u}_{t-1} + \upsilon_t$
2. Compute
3. $$\left(y_t - \hat{\rho} y_{t-1}\right) = \beta_1\left(1 - \hat{\rho}\right) + \beta_2\left(x_t - \hat{\rho} x_{t-1}\right) + \varepsilon_t$$
$$y_t^* = \beta_1^* + \beta_2^* x_t^* + \varepsilon_t^*$$
4. Iterative procedure to estimate ρ

Autocorrelation: Cochrane-Orcutt

Variance-Covariance Matrix $\underset{n\times n}{\Sigma} = \sigma^2 \underset{n\times n}{\Omega}$

where

$$\underset{\text{n}\times\text{n}}{\Omega} = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{n-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & \rho & 1 \end{bmatrix}$$

Matrix

$$\underset{k\times 1}{\hat{\beta}} = (\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Omega}^{-1}} \underset{n\times k}{X}}_{k\times k})^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Omega}^{-1}} \underset{n\times 1}{Y}}_{k\times 1}$$

OLS vs GLS vs FGLS

OLS Estimation $\underset{n\times n}{\Sigma} = \sigma^2 \underset{n\times n}{I}$	$\underset{k\times 1}{\hat{\beta}} = (\underset{k\times k}{X'X})^{-1} \underset{k\times n}{X'} \underset{n\times 1}{Y}$
WLS Estimation $\underset{n\times n}{\hat{\Sigma}} = \hat{\sigma}_i^2 \underset{n\times n}{I}$	$\underset{k\times 1}{\hat{\beta}} = \big(\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}}^{-1} \underset{n\times k}{X}}_{k\times k}\big)^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}}^{-1} \underset{n\times 1}{Y}}_{k\times 1}$
Cochrane-Orcutt $\underset{n\times n}{\hat{\Sigma}} = \sigma^2 \underset{n\times n}{\hat{\Omega}}$	$\underset{k\times 1}{\hat{\beta}} = \big(\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Omega}}^{-1} \underset{n\times k}{X}}_{k\times k}\big)^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Omega}}^{-1} \underset{n\times 1}{Y}}_{k\times 1}$
GLS Estimation $\underset{n\times n}{\Sigma}$ *is known*	$\underset{k\times 1}{\hat{\beta}} = \big(\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\Sigma}^{-1} \underset{n\times k}{X}}_{k\times k}\big)^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\Sigma}^{-1} \underset{n\times 1}{Y}}_{k\times 1}$
FGLS Estimation $\underset{n\times n}{\Sigma}$ *is not known*	$\underset{k\times 1}{\hat{\beta}} = \big(\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}}^{-1} \underset{n\times k}{X}}_{k\times k}\big)^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}}^{-1} \underset{n\times 1}{Y}}_{k\times 1}$

Methodology of Econometrics

1. Statement of theory or hypothesis
2. Specification of mathematical model of the theory
3. Specification of econometric model of theory
4. Obtaining the data
5. Estimation of the parameters of the econometric model
6. Hypothesis testing
7. Forecasting or prediction
8. Using model for control or policy purposes

Assumptions of Least Squares

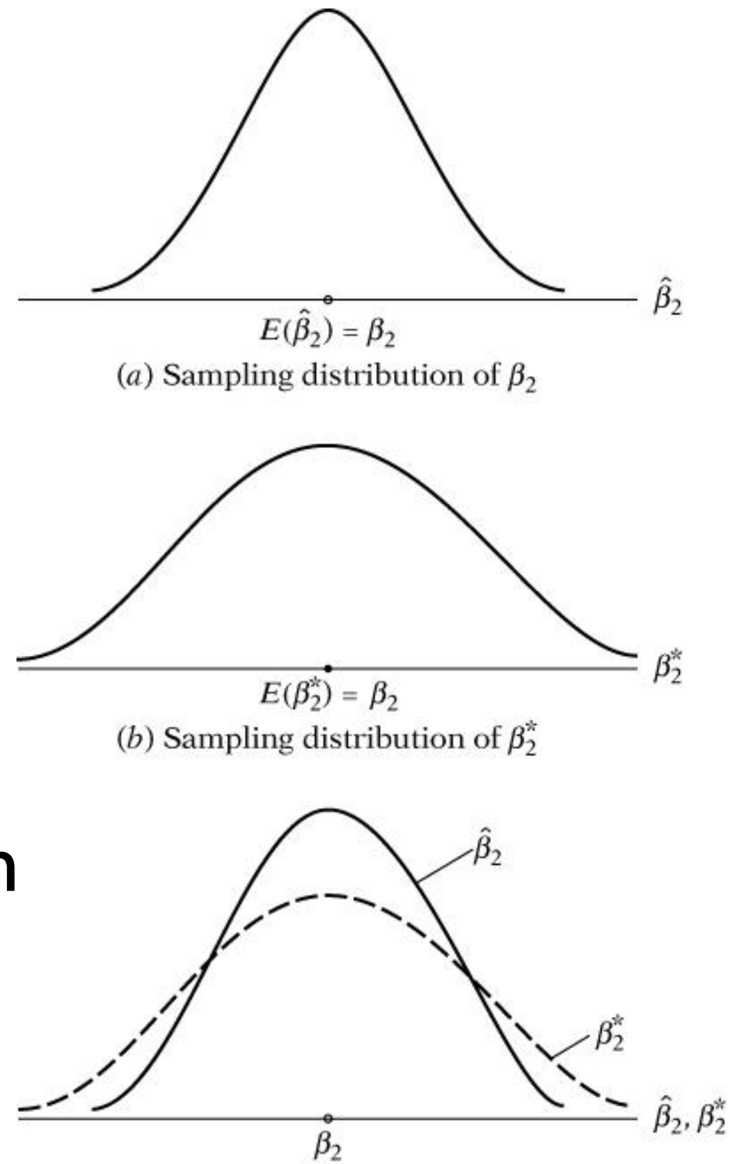
1. Linear Regression Model
2. X_i values are fixed in repeated sampling
3. Zero mean value of disturbance u_i
4. Homoscedasticity or equal variance of u_i
5. No autocorrelation between the disturbance
6. Zero covariance between u_i and X_i
7. Number of observations must be greater than number of parameters to be estimated
8. Variability in X_i values
9. Regression model is correctly specified
10. No perfect multicollinearity
11. Normal Distribution

Properties of Least-Squares Estimator

1. Linear
2. Unbiased
3. Efficient estimator

Gauss-Markov Theorem:

Given assumptions of CLRM, the least-squares estimators, in the class of unbiased linear estimators, have minimum variance -- they are Best Linear Unbiased Estimators (BLUE).



(c) Sampling distributions of β_2 and β_2^*

Choosing Estimated Results

1. Violation of OLS Assumptions.
 - Autocorrelation Problem.
 - Heteroscedasticity Problem.
 - Multicollinearity Problem.
 - Specification Error Problem.

Issues of Concern
 - Cause of the Problem.
 - Consequence of the Problem.
 - Detection of the Problem.
 - Remedial Measure.

Choosing Estimated Results

2. Specific Test.

- Nested vs Non-nested Model.
- Restricted vs Unrestricted F-test.
- Chow-test.
- Dummy Variable.
    - Intercept Dummy
    - Slope Dummy

Evaluating Estimated Results

1. Sign and Meaning of the Coefficients.
   - Whether the estimated results are according to the theory.
2. Overall Test – F-test.
   - Whether all explanatory variables can be used in explaining the dependent variable.
3. R-Squares.
   - How well does the model explain the dependent variable.
4. Individual Test – t-test.
   - Whether each explanatory variables can explain the dependent variable.