B.E. International Program

Faculty of Economics, Thammasat University

Semester: 2/2013
EE325 Introductory Econometrics
Solution for Quiz#2

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1. **[100pts]** Suppose the indifferent curve from consumption of goods X and Y is as follows:

$$\ln(Y_i / X_i) = \beta_1 + \ln(\beta_2 X_i{}^{\beta_3})$$

Quantity consumed of X	1	2	3	4	5
Quantity consumed of Y	4	3.5	2.8	1.9	0.8

1.1) [30pts] Transform this model into regression model

$$\ln\left(Y / X_{ii}\right) = \beta_1 + \ln\left(\beta_2 X_i{}^{\beta_3}\right)$$

$$\ln Y_i - \ln X_i = \beta_1 + \ln\beta_2 + \beta_3 \ln X_i$$

$$\ln Y_i = \beta_1 + \ln\beta_2 + \beta_3 \ln X_i + \ln X_i$$

$$\ln Y_i = \beta_1 + \ln\beta_2 + \left(\beta_3 + 1\right)\ln X_i$$

$$\ln Y_i = \alpha_0 + \alpha_1 \ln X_i$$

where $\alpha_0 = \beta_1 + \ln\beta_2$ and $\alpha_1 = \beta_3 + 1$

Therefore, we can rewrite this indifference curve as following:

$\ln Y_i = \alpha_0 + \alpha_1 \ln X_i + u_i$ where u_i is the disturbance term for regression function.

1.2) [70pts] By using information from the table, <u>estimate</u> value of the parameters in the model from 1.1. <u>Interpret</u> the value of slope coefficient and <u>test</u> hypothesis whether the slope is equal to zero at 95% confidence interval.

<u>1. Estimate the value of parameters</u>

Y_i	X_i	$\ln Y_i$	$\ln X_i$	y_i	x_i	x_i^2	$x_i y_i$
4	1	1.39	0	0.594	-0.958	0.918	-0.914
3.5	2	1.25	0.69	0.454	-0.268	0.072	-0.122
2.8	3	0.92	1.10	0.124	0.142	0.02	0.018
1.9	4	0.64	1.39	-0.156	0.432	0.187	-0.673
0.8	5	-0.22	1.61	-1.016	0.652	0.425	-0.662
		$\bar{Y} = \frac{3.98}{5} = 0.796$	$\bar{X} = \frac{4.79}{5} = 0.958$			$\sum_{i=1}^{5} x_i^2 = 1.622$	$\sum_{i=1}^{5} x_i y_i = -2.353$

Thus, $\hat{\alpha}_1 = \frac{-2.353}{1.622} = -1.45068$

and $\hat{\alpha}_0 = 0.796 - (1.45068)(0.958) = -0.59085$

We can write our regression equation:

$$(\ln Y_i) = -0.59085 - 1.45068(\ln X_i)$$

2. Interpret the value of slope coefficient

I. If X increases by 1 percent, Y will decrease by 1.45068 percent. With this relationship, we *probably* conclude that these goods X and Y are imperfect *substitutes*.

II. If (ln X) increases by 1 unit, (ln Y) will decrease by 1.45068 units.

3. Test hypothesis whether the slope is equal to zero at 95% confidence interval.

I. Calculate standard error:

$$Var(\hat{\alpha}_1) = \frac{13.1735}{1.622} = 8.1218$$

$$se(\hat{\alpha}_1) = \sqrt{Var(\hat{\alpha}_1)} = \sqrt{8.1218} = 2.8499$$

II. Conduct null hypothesis for slope coefficient:

$$H_o : \alpha_1 = 0$$
$$H_a : \alpha_1 \neq 0$$

With 0.05 level of significance,

$$t = \frac{\hat{\alpha}_1 - \alpha_1}{se(\hat{\alpha}_1)} : \ t_{\frac{\alpha}{2}, n-k}$$

$$t_{\frac{0.05}{2}, 5-2} = t_{0.025,3} = \pm 3.182$$

If calculated t is greater than 3.182 or less than -3.182, we will reject the null hypothesis at 0.05 level of significance and we find that calculated t is

$$t = \frac{-1.45068 - 0}{2.8499} = -0.509$$

Since the probability to spot *t* being less than -3.182 is small (0.05) and we still observe *t* greater than -3.182. Therefore, we cannot reject the null hypothesis that slope coefficient is equal to zero, that is, the quantity X consumed does not influence quantity Y consumed.