Production in the Long-Run

EE 211

The Long Run: No Fixed Factors

All inputs are **variable**.

Profit-maximizing firms strive for **technical efficiency** -- maximize output with a given number of inputs.

When a given number of inputs are combined in such a way as to maximize the level of output.

Profit Maximization and Cost Minimization

- Any firm that is trying to maximize its profits in the long run should select the production method that produces its output at the **lowest possible cost**
- This implication of the hypothesis of profit maximization is called cost minimization

Long-Run Cost Minimization

A firm is not minimizing costs if it is possible to substitute one factor for another to keep output constant while reducing total cost:

➔ The firm should substitute one factor for another factor as long as the marginal product of one factor **per dollar spent on it** is greater than the marginal product of the other factor **per dollar spent on it**.

Using ***K*** and ***L*** to represent capital and labor, and p_L and p_K to represent the prices for the two factors, cost is minimized when:

$$\frac{MP_K}{P_K} = \frac{MP_L}{P_L}$$

Whenever the ratio of the MP of each factor to its price is not equal for all factors, there are possibilities for factor substitutions that will reduce costs (for a given level of output)

Example

Suppose the marginal product of capital is 40 units of output and the price of one unit of capital is $10. The marginal product of labor is 20 units of output and the price of one unit of labor is $2.

$$\frac{MP_K}{P_K} = \frac{40}{10} = 4 < \frac{MP_L}{P_L} = \frac{20}{2} = 10$$

In this case, the firm can reduce the cost of producing its current level of output by using more labor and less capital.

Another Interpretation

Rearranging terms:

$$\frac{MP_K}{P_K} = \frac{MP_L}{P_L} \rightarrow \frac{MP_K}{MP_L} = \frac{P_K}{P_L}$$

The ratio of the marginal products on the left side compares the contribution of output to the last unit of capital and the last unit of labor. The right hand side shows how the cost of an additional unit of capital compares to the cost of an additional unit of labor.

Example

$$\frac{MP_K}{MP_L} = \frac{40}{20} = 2 \qquad \frac{P_K}{P_L} = \frac{10}{2} = 5$$

The left side of the equation equals 2 but the right hand side equals to 5. The last unit of capital is twice as productive as the last unit of labor but it is five times as expensive.

It will pay the firm to switch to a method of production that uses less capital and more labor.

Only when the ratio of marginal products is exactly equal to the ratio of factor prices is the firm using the cost minimizing production method

The Principle of Substitution

The principle of substitution: firms adjust the quantities of factors in response to changing relative factor prices.

– Firms use more of the cheaper factor and less of the more expensive factor.

The principle plays a central role in resource allocation because it relates to the way in which individual firms respond to changes in relative factor prices that are caused by the changing relative scarcities of factors in the economy as a whole.

Long-Run Cost Curves

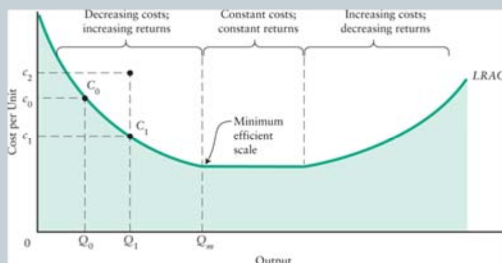
When all factors of production can be varied, consider the least-cost method of producing any level of output.

The long-run average cost (***LRAC***) curve shows the lowest possible cost of producing each level of output when all inputs can be varied.

The ***LRAC*** curve separates unattainable and attainable cost levels, given technology and factor prices.

The ***LRAC*** curve is usually U-shaped.

A "Saucer-Shaped" Long-Run Average Cost Curve



Decreasing costs

- Over the range of output from zero to Q_m
- The firm has falling long-run average costs- an expansion of output permits a reduction of average costs
- When long-run average costs fall as output rises, the firm is said to have **economies of scale**
- The decreasing-cost firm is often said to enjoy long-run **increasing return**

Constant Costs

- The firm's long-run average costs fall until output reaches Q_m
- **The firm's minimum efficient scale**
- LRAC reaches its minimum
- The firm would be encountering constant costs over the relevant range of output- the firm's long run average costs do not change as its output changes
- Factor prices are assumed to be fixed, the firm's output must be increasing exactly in proportion to the increase in inputs. (**constant returns**)

Increasing Costs

- When the LRAC curve is rising, a long-run expansion in production is accompanied by a rise in average costs
- If factor prices are constant, the firm's output must be increasing less than in proportion to the increase in inputs and this increasing cost firm is said to encounter long-run **decreasing returns**
- Decreasing returns imply that the firm suffers some **diseconomies of scale**

The Long-Run Average Cost Curve and Returns to Scale

Falling LRAC	=	Increasing returns to scale
Constant LRAC	=	Constant returns to scale
Rising LRAC	=	Decreasing returns to scale
Q_M	=	Minimum efficient scale

Returns To Scale

Increasing returns to scale – output increases more than in proportion to inputs as the scale of a firm's production increases.

Minimum efficient scale – the smallest output at which LRAC reaches its minimum.

Constant returns to scale – output increases in proportion to inputs as the scale of a firm's production increases.

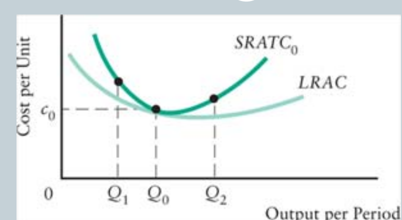
Decreasing returns to scale – output increases less than in proportion to inputs as the scale of a firm's production increases.

The relationship between LR and SR costs

- The LRAC curve shows the lowest cost of producing any output when all factors are variable
- Each SRATC (Short-Run Average Total Cost) curve shows the lowest cost of producing any output when one or more factors are fixed

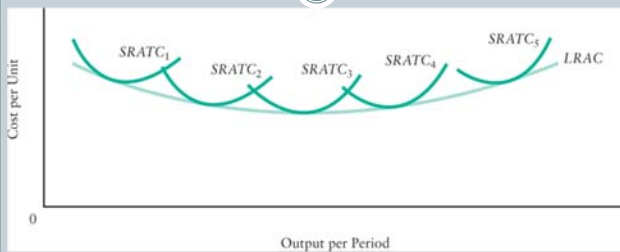
No SR cost curve can fall below the LR cost curve because the LRAC curve represents the lowest attainable cost for each possible output

LRAC and *SRATC* Curves



Each ***SRATC*** curve is tangent to the ***LRAC*** curve at the level of output for which the quantity of the fixed factor is optimal.

The relationship between the LRAC curve and the SRATC curves



To every point on the LRAC curve, there is an associated SRATC curve tangent at that point. Each short-run curve is drawn for a given plant size, shows how costs vary if output varies (holding constant the size of the plant). The level of output at the tangency between such SRATC curve and the LRAC curve shows the level of output for which the plant size is optimal

Shifts in *LRAC* Curves

- Changes in technology and factor prices cause the long-run cost curve to shift.
- A rise in factor prices shifts the ***LRAC*** curve upward.
- A fall in factor prices or a technological improvement shifts the ***LRAC*** curve downward.

The Very Long Run: Changes in Technology

- In the very long run, there are changes in the available techniques and resources for firms. Such changes shifts the long-run cost curves.
- Technological change refers to all changes in the available techniques of production.
- Economists use the notion of productivity to measure the extent of technological change.

Technological Change

Three kinds of changes in the very long run:

1. New techniques — process innovation
2. Improved inputs
3. New products — product innovation

New Techniques

Also called process innovation, which was dramatic throughout the nineteenth and twentieth centuries.

Examples:

- Electricity replaced burning fossil fuels
- Gas combustion and wind-powered turbines replaced nuclear, hydro, or fossil fuel-burning generating stations.

Improved Inputs

Improvements in health and education raise the quality of labor services.

Improvements in material inputs are also constantly occurring.

New production techniques and new and better inputs are important aspects of technological improvement.

- They lead to reductions in firm costs and a downward shift in *LRAC* curves.

New Products

The process is also called product innovation.

Examples:

- iPhone 5.
- New Ipad

The development of new products is a crucial part of the steady increase in living standards.

Firms' Choices in the Very Long Run

Faced with increases in the price of an input, firms may either substitute away (LR) or innovate away (VLR) from the input.

These two options can involve different actions and can have different implications for productivity.

Isoquant Analysis

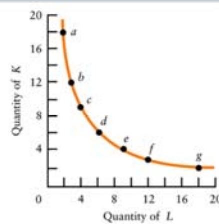
Production In The Long Run

- ***Isoquant:*** the set of all input combinations that yield a given level of output.
- ***Marginal rate of technical substitution (MRTS):*** the rate at which one input can be exchanged for another without altering the total level of output.

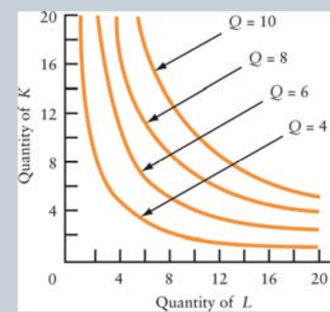
An Isoquant

Alternative Methods of Producing a Given Level of Output

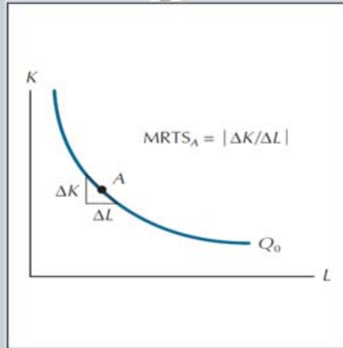
Method	K	L	ΔK	ΔL	Marginal Rate of Substitution (absolute value of $\Delta K/\Delta L$)
a	18	2	−6	1	6.00
b	12	3	−3	1	3.00
c	9	4	−3	2	1.50
d	6	6	−2	3	0.67
e	4	9	−1	3	0.33
f	3	12	−1	6	0.17
g	2	18			



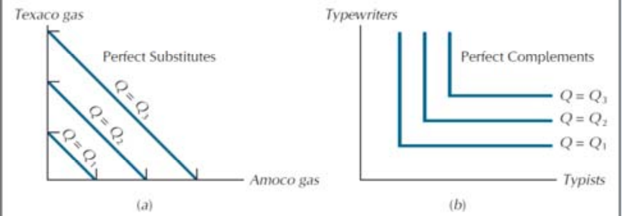
An Isoquant Map



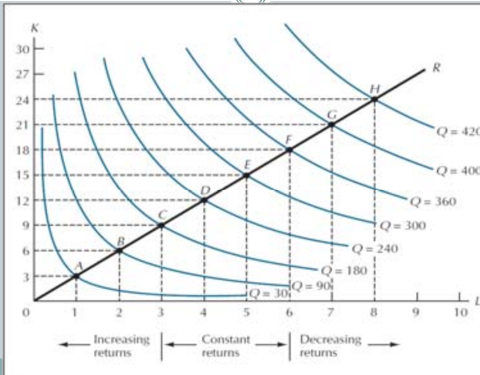
The Marginal Rate of Technical Substitution



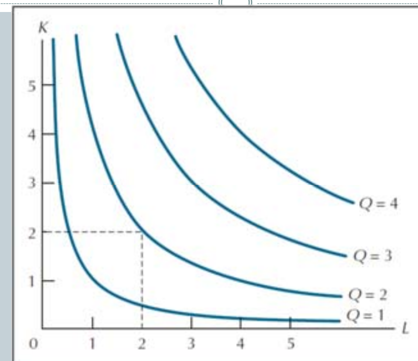
Isoquant Maps for Perfect Substitutes and Perfect Complements



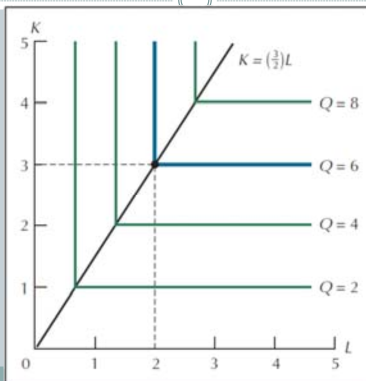
Returns to Scale Shown on the Isoquant Map



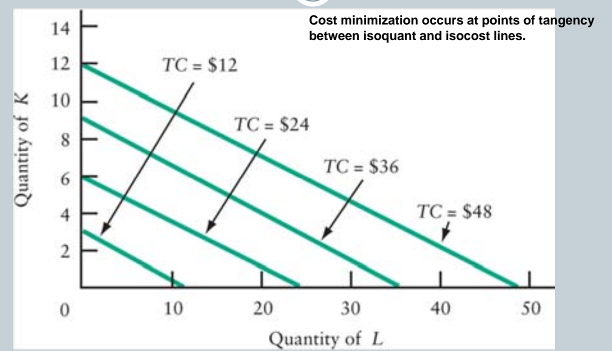
Example: Isoquant Map for the Cobb-Douglas Production Function Q = $K^{1/2}L^{1/2}$



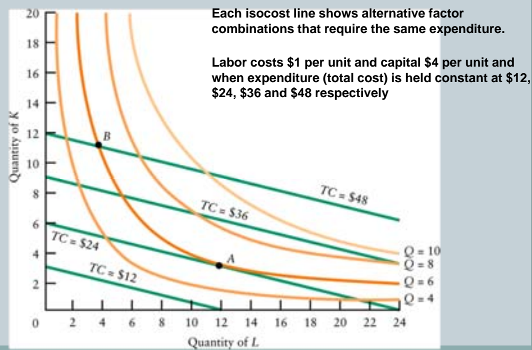
Example: Isoquant Map for the Leontief Production Function Q = min (2K,3L)



Isocost Lines



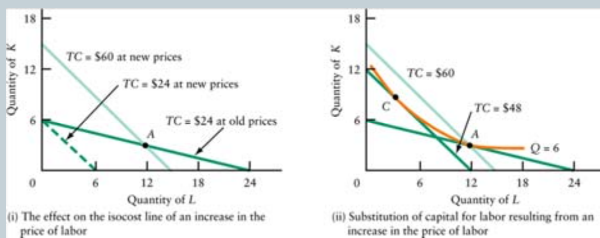
Cost Minimization



The principle of substitution

Suppose that with technology unchanged, the price of one factor changes. In particular, suppose that with the price of capital unchanged at $4 per unit, the price of labor rises from $1 to $4 per unit.

The Effects of a Change in Factor Prices on Costs and Factor Proportions



(i) The effect on the isocost line of an increase in the price of labor

(ii) Substitution of capital for labor resulting from an increase in the price of labor

Changes in relative factor prices will cause a partial replacement of factors that have become relatively more expensive by factors that have become relatively cheaper.

A rise in the price of one factor with all other factor prices held constant will

(1) Shift the cost curves of products that use that factor upward.

(2) Lead to a substitution of factors that are now relatively cheaper for the factor whose price has risen.

Sources:

- Lipsey, Ragan, and Storer (2008)
- Frank, R.H. (2010)