Chapter 6

1. (e)

2. (b) A higher borrowing rate is a consequence of the risk of the borrowers' default. In perfect markets with no additional cost of default, this increment would equal the value of the borrower's option to default, and the Sharpe measure, with appropriate treatment of the default option, would be the same. However, in reality there are costs to default so that this part of the increment lowers the Sharpe ratio. Also, notice that answer (c) is not correct because doubling the expected return with a fixed risk-free rate will more than double the risk premium and the Sharpe ratio.

4. a. The expected cash flow is: $(0.5 \times \$70{,}000) + (0.5 \times 200{,}000) = \$135{,}000$

   With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

   $\$135{,}000/1.14 = \$118{,}421$

   b. If the portfolio is purchased for \$118,421, and provides an expected cash inflow of \$135,000, then the expected rate of return [E(r)] is as follows:

   $\$118{,}421 \times [1 + E(r)] = \$135{,}000$

   Therefore, E(r) = 14%. The portfolio price is set to equate the expected rate of return with the required rate of return.

   c. If the risk premium over T-bills is now 12%, then the required return is:

   $6\% + 12\% = 18\%$

   The present value of the portfolio is now:

   $\$135{,}000/1.18 = \$114{,}407$

   d. For a given expected cash flow, portfolios that command greater risk premia must sell at lower prices. The extra discount from expected value is a penalty for risk.

5. When we specify utility by $U = E(r) - 0.5A\sigma^2$, the utility level for T-bills is: 0.07

   The utility level for the risky portfolio is:

   $U = 0.12 - 0.5 \times A \times (0.18)^2 = 0.12 - 0.0162 \times A$

   In order for the risky portfolio to be preferred to bills, the following must hold:

   $0.12 - 0.0162A > 0.07 \Rightarrow A < 0.05/0.0162 = 3.09$

   A must be less than 3.09 for the risky portfolio to be preferred to bills.

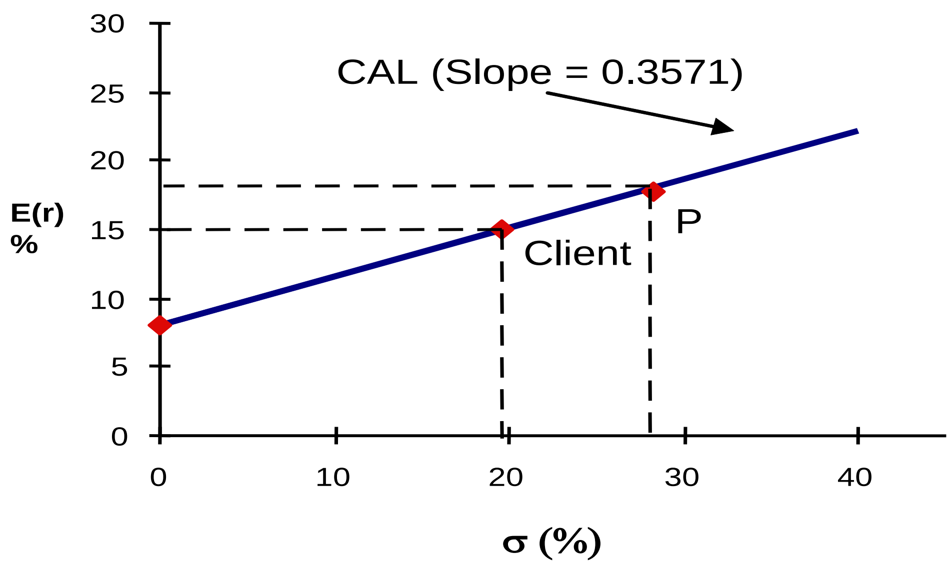
11. Computing utility from $U = E(r) - 0.5 \times A\sigma^2 = E(r) - \sigma^2$, we arrive at the values in the column labeled U(A = 2) in the following table:

W_{Bills}	W_{Index}	$r_{Portfolio}$	$\sigma_{Portfolio}$	$\sigma^2_{Portfolio}$	U(A = 2)	U(A = 3)
0.0	1.0	0.130	0.20	0.0400	0.0900	.0700
0.2	0.8	0.114	0.16	0.0256	0.0884	.0756
0.4	0.6	0.098	0.12	0.0144	0.0836	.0764
0.6	0.4	0.082	0.08	0.0064	0.0756	.0724
0.8	0.2	0.066	0.04	0.0016	0.0644	.0636
1.0	0.0	0.050	0.00	0.0000	0.0500	.0500

The column labeled U(A = 2) implies that investors with A = 2 prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

14. Investment proportions: 30.0% in T-bills
$0.7 \times 25\% =$ 17.5% in Stock A
$0.7 \times 32\% =$ 22.4% in Stock B
$0.7 \times 43\% =$ 30.1% in Stock C

16.



18. a. $\sigma_C = y \times 28\%$

If your client prefers a standard deviation of at most 18%, then:

y = 18/28 = 0.6429 = 64.29% invested in the risky portfolio

b. $E(r_C) = .08 + .1 \times y = .08 + (0.6429 \times .1) = 14.429\%$

19. a. $y^* = \dfrac{E(r_P) - r_f}{A\sigma_P^2} = \dfrac{0.18 - 0.08}{3.5 \times 0.28^2} = \dfrac{0.10}{0.2744} = 0.3644$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 8 + 10 \times y^* = 8 + (0.3644 \times 10) = 11.644\%$

$\sigma_C = 0.3644 \times 28 = 10.203\%$

CFA PROBLEMS

1. Utility for each investment = $E(r) - 0.5 \times 4 \times \sigma^2$

We choose the investment with the highest utility value, Investment 3.

Investment	Expected return E(r)	Standard deviation σ	Utility U
1	0.12	0.30	-0.0600
2	0.15	0.50	-0.3500
3	0.21	0.16	0.1588
4	0.24	0.21	0.1518

2. When investors are risk neutral, then A = 0; the investment with the highest utility is Investment 4 because it has the highest expected return.

3. (b)

8. Expected return for equity fund = T-bill rate + risk premium = 6% + 10% = 16%

Expected rate of return of the client's portfolio = (0.6 × 16%) + (0.4 × 6%) = 12%

Expected return of the client's portfolio = 0.12 × $100,000 = $12,000

(which implies expected total wealth at the end of the period = $112,000)

Standard deviation of client's overall portfolio = 0.6 × 14% = 8.4%

Chapter 7

1. (a) and (e).

3. (a) Answer (a) is valid because it provides the definition of the minimum variance portfolio.

4. The parameters of the opportunity set are:

$E(r_S) = 20\%$, $E(r_B) = 12\%$, $\sigma_S = 30\%$, $\sigma_B = 15\%$, $\rho = 0.10$

From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \rho \times \sigma_S \times \sigma_B$]:

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

The minimum-variance portfolio is computed as follows:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$w_{Min}(B) = 1 - 0.1739 = 0.8261$

The minimum variance portfolio mean and standard deviation are:

$E(r_{Min}) = (0.1739 \times .20) + (0.8261 \times .12) = .1339 = 13.39\%$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{1/2}$$

$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$= 13.92\%$

8. The reward-to-volatility ratio of the optimal CAL is:

$$\frac{E(r_p) - r_f}{\sigma_p} = \frac{.1561 - .08}{.1654} = 0.4601$$ .4601 should be .4603 (rounding)

9. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

$$E(r_C) = r_f + \frac{E(r_p) - r_f}{\sigma_P}\sigma_C = .08 + 0.4601\sigma_C$$ .4601 should be .4603 (rounding)

If $E(r_C)$ is equal to 14%, then the standard deviation of the portfolio is 13.03%.

b. To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1 - y) \times r_f + y \times E(r_P) = r_f + y \times [E(r_P) - r_f] = .08 + y \times (.1561 - .08)$$

Setting $E(r_C) = 14\%$ we find: $y = 0.7881$ and $(1 - y) = 0.2119$ (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7881 × 0.4516 = 0.3559

12. Since Stock A and Stock B are perfectly negatively correlated, a risk-free portfolio can be created and the rate of return for this portfolio, in equilibrium, will be the risk-free rate. To find the proportions of this portfolio [with the proportion w_A invested in Stock A and $w_B = (1 - w_A)$ invested in Stock B], set the standard deviation equal to zero. With perfect negative correlation, the portfolio standard deviation is:

σ_P = Absolute value $[w_A\sigma_A - w_B\sigma_B]$

$0 = 5 \times w_A - [10 \times (1 - w_A)] \Rightarrow w_A = 0.6667$

The expected rate of return for this risk-free portfolio is:

$E(r) = (0.6667 \times 10) + (0.3333 \times 15) = 11.667\%$

Therefore, the risk-free rate is: 11.667%

17. The correct choice is c. Intuitively, we note that since all stocks have the same expected rate of return and standard deviation, we choose the stock that will result in lowest risk. This is the stock that has the lowest correlation with Stock A.

More formally, we note that when all stocks have the same expected rate of return, the optimal portfolio for any risk-averse investor is the global minimum variance portfolio (G). When the portfolio is restricted to Stock A and one additional stock, the objective is to find G for any pair that includes Stock A, and then select the combination with the lowest variance. With two stocks, I and J, the formula for the weights in G is:

$$w_{Min}(I) = \frac{\sigma_J^2 - Cov(r_I, r_J)}{\sigma_I^2 + \sigma_J^2 - 2Cov(r_I, r_J)}$$

$$w_{Min}(J) = 1 - w_{Min}(I)$$

Since all standard deviations are equal to 20%:

$$Cov(r_I, r_J) = \rho\sigma_I\sigma_J = 400\rho \text{ and } w_{Min}(I) = w_{Min}(J) = 0.5$$

This intuitive result is an implication of a property of any efficient frontier, namely, that the covariances of the global minimum variance portfolio with all other assets on the frontier are identical and equal to its own variance. (Otherwise, additional diversification would further reduce the variance.) In this case, the standard deviation of G(I, J) reduces to:

$$\sigma_{Min}(G) = [200 \times (1 + \rho_{IJ})]^{1/2}$$

This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is Stock D. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is 17.03%.

Proportion of bonds in complete portfolio = $0.7881 \times 0.5484 = 0.4322$

19. Yes, the answers to Problems 17 and 18 would change. The efficient frontier of risky assets is horizontal at 8%, so the optimal CAL runs from the risk-free rate through G. This implies risk-averse investors will just hold Treasury Bills.

CFA PROBLEMS

5. c.

6. d.

8. a.

10. Since we do not have any information about expected returns, we focus exclusively on reducing variability. Stocks A and C have equal standard deviations, but the correlation of Stock B with Stock C (0.10) is less than that of Stock A with Stock B (0.90). Therefore, a portfolio comprised of Stocks B and C will have lower total risk than a portfolio comprised of Stocks A and B.

Chapter 4

3. Open-end funds are obligated to redeem investor's shares at net asset value, and thus must keep cash or cash-equivalent securities on hand in order to meet potential redemptions. Closed-end funds do not need the cash reserves because there are no redemptions for closed-end funds. Investors in closed-end funds sell their shares when they wish to cash out.

4. Balanced funds keep relatively stable proportions of funds invested in each asset class. They are meant as convenient instruments to provide participation in a range of asset classes. Life-cycle funds are balanced funds whose asset mix generally depends on the age of the investor. Aggressive life-cycle funds, with larger investments in equities, are marketed to younger investors, while conservative life-cycle funds, with larger investments in fixed-income securities, are designed for older investors. Asset allocation funds, in contrast, may vary the proportions invested in each asset class by large amounts as predictions of relative performance across classes vary. Asset allocation funds therefore engage in more aggressive market timing.

6. Advantages of an ETF over a mutual fund:

   - ETFs are continuously traded and can be sold or purchased on margin
   - There are no Capital Gains Tax triggers when an ETF is sold (shares are just sold from one investor to another)

- Investors buy from Brokers, thus eliminating the cost of direct marketing to individual small investors. This implies lower management fees

Disadvantages of an ETF over a mutual fund:

- Prices can depart from NAV (unlike an open-end fund)
- There is a Broker fee when buying and selling (unlike a no-load fund)

12. $$\frac{NAV_1 - NAV_0 + \text{ Distributions}}{NAV_0} = \frac{\$12.10 - \$12.50 + \$1.50}{\$12.50} = 0.088 = 8.8\%$$

15. NAV_0 = \$200,000,000/10,000,000 = \$20

Dividends per share = \$2,000,000/10,000,000 = \$0.20

NAV_1 is based on the 8% price gain, less the 1% 12b-1 fee:

NAV_1 = \$20 × 1.08 × (1 – 0.01) = \$21.384

Rate of return = $\frac{\$21.384 - \$20 + \$0.20}{\$20}$ = 0.0792 = 7.92%

18. As an initial approximation, your return equals the return on the shares minus the total of the expense ratio and purchase costs: 12% – 1.2% – 4% = 6.8%

But the precise return is less than this because the 4% load is paid up front, not at the end of the year.

To purchase the shares, you would have had to invest: \$20,000/(1 – 0.04) = \$20,833

The shares increase in value from \$20,000 to: \$20,000 × (1.12 – 0.012) = \$22,160

The rate of return is: (\$22,160 – \$20,833)/\$20,833 = 6.37%

20. a. $\frac{\$450,000,000 - \$10,000000}{44,000,000} = \$10$

b. The redemption of 1 million shares will most likely trigger capital gains taxes which will lower the remaining portfolio by an amount greater than \$10,000,000 (implying a remaining total value less than \$440,000,000). The outstanding shares fall to 43 million and the NAV drops to below \$10.