EXERCISES

Questions

5.1. State with reason whether the following statements are true, false, or uncertain. Be precise.

a. The t test of significance discussed in this chapter requires that the sampling distributions of estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ follow the normal distribution.

b. Even though the disturbance term in the CLRM is not normally distributed, the OLS estimators are still unbiased.

c. If there is no intercept in the regression model, the estimated $u_i (= \hat{u}_i)$ will not sum to zero.

d. The p value and the size of a test statistic mean the same thing.

e. In a regression model that contains the intercept, the sum of the residuals is always zero.

f. If a null hypothesis is not rejected, it is true.

g. The higher the value of σ^2, the larger is the variance of $\hat{\beta}_2$ given in Eq. (3.3.1).

h. The conditional and unconditional means of a random variable are the same things.

i. In the two-variable PRF, if the slope coefficient β_2 is zero, the intercept β_1 is estimated by the sample mean $\bar{Y}$.

j. The conditional variance, $\operatorname{var}(Y_i \mid X_i) = \sigma^2$, and the unconditional variance of Y, $\operatorname{var}(Y) = \sigma_Y^2$, will be the same if X had no influence on Y.

5.2. Set up the ANOVA table in the manner of Table 5.4 for the regression model given in Eq. (3.7.2) and test the hypothesis that there is no relationship between food expenditure and total expenditure in India.

5.3. Refer to the demand for cell phones regression given in Eq. (3.7.3).

a. Is the estimated intercept coefficient significant at the 5 percent level of significance? What is the null hypothesis you are testing?

b. Is the estimated slope coefficient significant at the 5 percent level? What is the underlying null hypothesis?

c. Establish a 95 percent confidence for the true slope coefficient.

d. What is the mean forecast value of cell phones demanded if the per capita income is \$9,000? What is the 95 percent confidence interval for the forecast value?

5.4. Let ρ^2 represent the true population coefficient of determination. Suppose you want to test the hypothesis that $\rho^2 = 0$. Verbally explain how you would test this hypothesis. *Hint:* Use Eq. (3.5.11). See also Exercise 5.7.

5.5. What is known as the **characteristic line** of modern investment analysis is simply the regression line obtained from the following model:

$$r_{it} = \alpha_i + \beta_i r_{mt} + u_t$$

where r_{it} = the rate of return on the ith security in time t
r_{mt} = the rate of return on the market portfolio in time t
u_t = stochastic disturbance term

In this model β_i is known as the **beta coefficient** of the ith security, a measure of market (or systematic) risk of a security.*

*See Haim Levy and Marshall Sarnat, *Portfolio and Investment Selection: Theory and Practice,* Prentice Hall International, Englewood Cliffs, NJ, 1984, Chap. 12.

On the basis of 240 monthly rates of return for the period 1956–1976, Fogler and Ganapathy obtained the following characteristic line for IBM stock in relation to the market portfolio index developed at the University of Chicago:*

$$\hat{r}_{it} = 0.7264 + 1.0598 r_{mt} \qquad r^2 = 0.4710$$
$$\text{se} = (0.3001)\ (0.0728) \qquad \text{df} = 238$$
$$F_{1,238} = 211.896$$

a. A security whose beta coefficient is greater than one is said to be a volatile or aggressive security. Was IBM a volatile security in the time period under study?

b. Is the intercept coefficient significantly different from zero? If it is, what is its practical meaning?

5.6. Equation (5.3.5) can also be written as

$$\Pr[\hat{\beta}_2 - t_{\alpha/2}\text{se}(\hat{\beta}_2) < \beta_2 < \hat{\beta}_2 + t_{\alpha/2}\text{se}(\hat{\beta}_2)] = 1 - \alpha$$

That is, the weak inequality ($\leq$) can be replaced by the strong inequality ($<$). Why?

5.7. R. A. Fisher has derived the sampling distribution of the correlation coefficient defined in Eq. (3.5.13). If it is assumed that the variables X and Y are jointly normally distributed, that is, if they come from a bivariate normal distribution (see Appendix 4A, Exercise 4.1), then under the assumption that the population correlation coefficient ρ is zero, it can be shown that $t = r\sqrt{n-2}/\sqrt{1-r^2}$ follows Student's t distribution with $n - 2$ df.** Show that this t value is identical with the t value given in Eq. (5.3.2) under the null hypothesis that $\beta_2 = 0$. Hence establish that under the same null hypothesis $F = t^2$. (See Section 5.9.)

5.8. Consider the following regression output:†

$$\hat{Y}_i = 0.2033 + 0.6560 X_t$$
$$\text{se} = (0.0976)\ (0.1961)$$
$$r^2 = 0.397 \qquad \text{RSS} = 0.0544 \qquad \text{ESS} = 0.0358$$

where Y = labor force participation rate (LFPR) of women in 1972 and X = LFPR of women in 1968. The regression results were obtained from a sample of 19 cities in the United States.

a. How do you interpret this regression?

b. Test the hypothesis: H_0: $\beta_2 = 1$ against H_1: $\beta_2 > 1$. Which test do you use? And why? What are the underlying assumptions of the test(s) you use?

c. Suppose that the LFPR in 1968 was 0.58 (or 58 percent). On the basis of the regression results given above, what is the mean LFPR in 1972? Establish a 95 percent confidence interval for the mean prediction.

d. How would you test the hypothesis that the error term in the population regression is normally distributed? Show the necessary calculations.

*H. Russell Fogler and Sundaram Ganapathy, *Financial Econometrics*, Prentice Hall, Englewood Cliffs, NJ, 1982, p. 13.

**If ρ is in fact zero, Fisher has shown that r follows the same t distribution provided either X or Y is normally distributed. But if ρ is not equal to zero, both variables must be normally distributed. See R. L. Anderson and T. A. Bancroft, *Statistical Theory in Research*, McGraw-Hill, New York, 1952, pp. 87–88.

†Adapted from Samprit Chatterjee, Ali S. Hadi, and Bertram Price, *Regression Analysis by Example*, 3d ed., Wiley Interscience, New York, 2000, pp. 46–47.